Discussion of the JBAR term
- Derivation, Interpretation
and Application to the
Northeastern Atlantic Shelf

JBAR-Term

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Northeastern Atlantic Shelf

by

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1 Introduction

Since its first description in 1971 by Sakisyan and Ivanov (1971) and its more extensive discussion in "The Sea" by Sakisyan (1977) the JBAR effect has been studied in numerous publications. Although it can be reduced to only one single term in the vorticity equations a large number of authors have focussed on this term (e.g. Rattray, 1982; Huntnance, 1984; Csanady, 1985; Mertz and Wright, 1992, Cane, Kamenkovich and Krupitsky, 1998). It can be suspected that also the interesting sounding acronym has played a role to increase the popularity of the JBAR effect. JBAR stands for "Joint Effect of Baroclinicity and Relief", which in some respect is misleading as will be shown in section 2.2. The report is split into two parts. It starts with general theoretical considerations concerning the JBAR effect; in the second part its distribution on the northwestern European shelf including the adjacent shelf break will be presented and discussed.

2 Theoretical Considerations

The theoretical considerations start with the derivation of the vorticity equations from the depth averaged equations of motion. From this derivation the JBAR term directly evolves (section 2.1). In section 2.2 an attempt is made to interpret the JBAR term. This interpretation is carried out in two steps. Firstly, the vorticity equation is deduced in an alternative way, i.e. the vorticity equation is calculated directly from the equations of motion and vertical averaging is performed afterwards. The difference between this vorticity equation and the former one gives a first insight into the meaning of the JBAR term. Secondly, it is demonstrated that the JBAR term can directly be deduced from the hydrostatic equation. This procedure provides additional information about the nature of the JBAR term. Based on these gained interpretations the commonly adopted utilization of the JBAR term will be discussed and criticised.

2.1 Derivation of the JBAR term

The most obvious way to obtain the JBAR term is through derivation of the vorticity equation from the linearised horizontal equations of motion:

$$\frac{du}{dt} - fv = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} + \frac{1}{\rho_0} \frac{\partial \tau_x}{\partial z} \quad \text{resp.} \quad \frac{dv}{dt} + fu = -\frac{1}{\rho_0} \frac{\partial p}{\partial y} + \frac{1}{\rho_0} \frac{\partial \tau_y}{\partial z}. \quad (1)$$

Moreover, we apply the hydrostatic approximation $\frac{\partial p}{\partial z} = -g \cdot \rho$, the Boussinesq and the rigid lid approximation $\zeta(x, y, t) = 0$ as well as the continuity equation for incompressible media:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \quad (2)$$

In order to simplify the problem in the following only the velocity components driven by the pressure gradients $(u_p, v_p)$ will be considered in the equations of motion (1). Averaging this part of the equation of motion over the water column gives:

$$\frac{1}{H} \int_{-H}^{0} \frac{\partial u_p}{\partial t} \, dz - \frac{1}{H} \int_{-H}^{0} f v_p \, dz = -\frac{1}{\rho_0 H} \int_{-H}^{0} \frac{\partial p}{\partial x} \, dz \quad (3a)$$

resp.
\[
\frac{1}{H} \int_{-H}^{0} \frac{\partial u_p}{\partial t} \, dz + \frac{1}{H} \int_{-H}^{0} f u_p \, dz = \frac{1}{\rho_0 H} \int_{-H}^{0} \frac{\partial p}{\partial y} \, dz. \tag{3b}
\]

Application of Leibniz' theorem to (3) leads to:

\[
\frac{1}{H} \left( \frac{\partial}{\partial t} \int_{-H}^{0} u_p \, dz + \frac{\partial (-H)}{\partial t} \cdot (u_p)_{-H} \right) - \frac{1}{H} \int_{-H}^{0} f v_p \, dz = -\frac{1}{\rho_0 H} \left( \frac{\partial}{\partial x} \int_{-H}^{0} p \, dz + \frac{\partial (-H)}{\partial x} \cdot (p)_{-H} \right) \tag{4a}
\]

resp.

\[
\frac{1}{H} \left( \frac{\partial}{\partial t} \int_{-H}^{0} v_p \, dz + \frac{\partial (-H)}{\partial t} \cdot (v_p)_{-H} \right) + \frac{1}{H} \int_{-H}^{0} f u_p \, dz = -\frac{1}{\rho_0 H} \left( \frac{\partial}{\partial y} \int_{-H}^{0} p \, dz + \frac{\partial (-H)}{\partial y} \cdot (p)_{-H} \right). \tag{4b}
\]

Applying the rigid lid assumption, the respective derivatives at the upper boundary \( z = 0 \) vanish. Assuming further that the temporal change of the bathymetry can be neglected \( \frac{\partial H}{\partial t} = 0 \) and defining the depth mean pressure driven velocity as:

\[
\bar{u}_p = \frac{1}{H} \int_{-H}^{0} u_p \, dz \quad \text{resp.} \quad \bar{v}_p = \frac{1}{H} \int_{-H}^{0} v_p \, dz \tag{5a, b}
\]

leads to the simplification of equations (4):

\[
\frac{\partial \bar{u}_p}{\partial t} - f \cdot \bar{v}_p = -\frac{1}{\rho_0 H} \left( \frac{\partial}{\partial x} \int_{-H}^{0} p \, dz + \frac{\partial (-H)}{\partial x} \cdot (p)_{-H} \right) \tag{6a}
\]

resp.

\[
\frac{\partial \bar{v}_p}{\partial t} + f \cdot \bar{u}_p = -\frac{1}{\rho_0 H} \left( \frac{\partial}{\partial y} \int_{-H}^{0} p \, dz + \frac{\partial (-H)}{\partial y} \cdot (p)_{-H} \right). \tag{6b}
\]

Cross-wise differentiation of equations (6) leads to:

\[
\frac{\partial}{\partial y} \left( \frac{\partial \bar{u}_p}{\partial t} \right) - \frac{\partial}{\partial y} (f \cdot \bar{v}_p) = -\frac{1}{\rho_0 H} \frac{\partial}{\partial y} \left( \frac{\partial}{\partial x} \int_{-H}^{0} p \, dz + \frac{\partial (-H)}{\partial x} \cdot (p)_{-H} \right) \tag{7a}
\]

\[
-\frac{\partial}{\partial y} \left( \frac{1}{\rho_0 H} \right) \left( \frac{\partial}{\partial x} \int_{-H}^{0} p \, dz + \frac{\partial (-H)}{\partial x} \cdot (p)_{-H} \right)
\]

resp.

\[
\frac{\partial}{\partial x} \left( \frac{\partial \bar{v}_p}{\partial t} \right) + \frac{\partial}{\partial x} (f \cdot \bar{u}_p) = -\frac{1}{\rho_0 H} \frac{\partial}{\partial x} \left( \frac{\partial}{\partial y} \int_{-H}^{0} p \, dz + \frac{\partial (-H)}{\partial y} \cdot (p)_{-H} \right) \tag{7b}
\]

\[
-\frac{\partial}{\partial x} \left( \frac{1}{\rho_0 H} \right) \left( \frac{\partial}{\partial y} \int_{-H}^{0} p \, dz + \frac{\partial (-H)}{\partial y} \cdot (p)_{-H} \right).
\]
Introduction of the hydrostatic approximation \( p(z) = p_0 + g \int_0^z \rho \, dz \) in the third term on the r.h.s. of (7), which not cancel out like the first term on the right hand side and neglect of the atmospheric pressure \( \rho_0 = 0 \) leads to:

\[
\frac{\partial}{\partial y} \left( \frac{\partial \vec{u}_p}{\partial t} \right) - \frac{\partial}{\partial y} \left( f \cdot \vec{v}_p \right) = -\frac{1}{\rho_0 H} \cdot \frac{\partial}{\partial y} \left( \frac{\partial}{\partial x} \int_0^z p \, dz + \frac{\partial}{\partial x} \left( \frac{\partial H}{\partial x} \right) \cdot (p)_{-H} \right) 
\]

\[
- \frac{\partial}{\partial y} \left( \frac{1}{\rho_0 H} \right) \cdot \left( g \cdot \frac{\partial}{\partial x} \int_{z=H}^{z=H} \left( 1 \cdot \int_{z=H}^{z} \rho \, dz' \right) \, dz \right) + \frac{\partial}{\partial y} \left( \frac{\partial H}{\partial y} \right) \cdot (p)_{-H} 
\]

resp.

\[
\frac{\partial}{\partial x} \left( \frac{\partial \vec{v}_p}{\partial t} \right) + \frac{\partial}{\partial x} \left( f \cdot \vec{u}_p \right) = -\frac{1}{\rho_0 H} \cdot \frac{\partial}{\partial x} \left( \int_0^z p \, dz + \frac{\partial}{\partial y} \left( \frac{\partial H}{\partial y} \right) \cdot (p)_{-H} \right) 
\]

\[
- \frac{\partial}{\partial x} \left( \frac{1}{\rho_0 H} \right) \cdot \left( g \cdot \frac{\partial}{\partial y} \int_{z=H}^{z=H} \left( 1 \cdot \int_{z=H}^{z} \rho \, dz' \right) \, dz \right) + \frac{\partial}{\partial x} \left( \frac{\partial H}{\partial y} \right) \cdot (p)_{-H} 
\]

whereby the extension of the integrand with 1, which is necessary for the partial integration, has already been performed.

The partial integration, the employment of the product rule to the Coriolis term, and the inter-change of differentiation for the time dependent term result in:

\[
\frac{\partial}{\partial t} \left( \frac{\partial \vec{u}_p}{\partial y} \right) - \frac{\partial}{\partial y} \int_{z=H}^{z=H} \left( f \cdot \vec{v}_p \right) = -\frac{1}{\rho_0 H} \cdot \frac{\partial}{\partial y} \left( \int_0^z p \, dz + \frac{\partial}{\partial x} \left( \frac{\partial H}{\partial x} \right) \cdot (p)_{-H} \right) 
\]

\[
- \frac{\partial}{\partial y} \left( \frac{1}{\rho_0 H} \right) \cdot \left( g \cdot \frac{\partial}{\partial x} \int_{z=H}^{z=H} \left( 1 \cdot \int_{z=H}^{z} \rho \, dz' \right) \, dz \right) + \frac{\partial}{\partial y} \left( \frac{\partial H}{\partial x} \right) \cdot (p)_{-H} 
\]

resp.

\[
\frac{\partial}{\partial t} \left( \frac{\partial \vec{v}_p}{\partial x} \right) + \frac{\partial}{\partial x} \int_{z=H}^{z=H} \left( f \cdot \vec{u}_p \right) + \frac{\partial}{\partial x} \int_{z=H}^{z=H} \left( \vec{u}_p \cdot \vec{v}_p \right) = -\frac{1}{\rho_0 H} \cdot \frac{\partial}{\partial x} \left( \int_0^z p \, dz + \frac{\partial}{\partial y} \left( \frac{\partial H}{\partial y} \right) \cdot (p)_{-H} \right) 
\]

\[
- \frac{\partial}{\partial x} \left( \frac{1}{\rho_0 H} \right) \cdot \left( g \cdot \frac{\partial}{\partial y} \int_{z=H}^{z=H} \left( 1 \cdot \int_{z=H}^{z} \rho \, dz' \right) \, dz \right) + \frac{\partial}{\partial x} \left( \frac{\partial H}{\partial y} \right) \cdot (p)_{-H} .
\]

Insertion into the boundaries leads to:

\[
\frac{\partial}{\partial t} \left( \frac{\partial \vec{u}_p}{\partial y} \right) - \frac{\partial}{\partial y} \int_{z=H}^{z=H} \left( f \cdot \vec{v}_p \right) = -\frac{1}{\rho_0 H} \cdot \frac{\partial}{\partial y} \left( \int_0^z p \, dz - \frac{\partial H}{\partial x} \cdot (p)_{-H} \right) 
\]

\[
- \frac{\partial}{\partial y} \left( \frac{1}{\rho_0 H} \right) \cdot \left( g \cdot \frac{\partial}{\partial x} \int_{z=H}^{z=H} \left( 1 \cdot \int_{z=H}^{z} \rho \, dz' \right) \, dz \right) + \frac{\partial}{\partial y} \left( \frac{\partial H}{\partial x} \right) \cdot (p)_{-H} 
\]

resp.
\[
\frac{\partial}{\partial t} \left( \frac{\partial \nu_p}{\partial x} \right) + \frac{\partial f}{\partial x} \nu_p + f \frac{\partial u_p}{\partial y} = -\frac{1}{\rho_0 H} \left( \frac{\partial}{\partial y} \int_{-H}^{0} p \, dz - \frac{\partial H}{\partial y} \cdot (p)_{-H} \right) 
\]

\[
- \frac{\partial}{\partial x} \left( \frac{1}{\rho_0 H} \right) \left( \frac{\partial}{\partial y} \int_{-H}^{0} z \cdot \rho \, dz + H \frac{\partial}{\partial y} \int_{-H}^{0} \rho \, dz - \frac{\partial H}{\partial y} \cdot (p)_{-H} \right).
\]

Using the hydrostatic assumption and again neglecting the atmospheric pressure \( p_0 \), the bottom pressure \((p)_{-H}\) is given by: \((p)_{-H} = g \int_{-H}^{0} \rho \, dz\). After employment of the product rule to this bottom pressure term, the last term in equation (10) cancels out. Defining the potential energy at the bottom as \( \chi = \frac{g}{\rho_0} \int_{-H}^{0} z \cdot \rho \, dz \), employment of the chain rule and interchange of differentiation for the first two terms on the right hand side of (10) result in:

\[
\frac{\partial}{\partial y} \left( \frac{\partial u_p}{\partial t} \right) - \frac{\partial f}{\partial y} \nu_p - f \frac{\partial v_p}{\partial y} = -\frac{1}{\rho_0 H} \left( \frac{\partial^2}{\partial x \partial y} \int_{-H}^{0} p \, dz - \frac{\partial^2 H}{\partial x \partial y} \cdot (p)_{-H} - \frac{\partial H}{\partial y} \cdot \frac{\partial (p)_{-H}}{\partial y} \right) 
\]

\[
+ \frac{\partial}{\partial y} \left( \frac{1}{H} \right) \left( \frac{\partial \chi}{\partial x} - \frac{H}{\rho_0} \cdot \frac{\partial (p)_{-H}}{\partial x} \right).
\]

resp.

\[
\frac{\partial}{\partial t} \left( \frac{\partial v_p}{\partial x} + \frac{\partial u_p}{\partial x} \right) + \frac{\partial f}{\partial x} \nu_p + f \frac{\partial u_p}{\partial x} = -\frac{1}{\rho_0 H} \left( \frac{\partial^2}{\partial x \partial y} \int_{-H}^{0} p \, dz - \frac{\partial^2 H}{\partial x \partial y} \cdot (p)_{-H} - \frac{\partial H}{\partial y} \cdot \frac{\partial (p)_{-H}}{\partial y} \right) 
\]

\[
+ \frac{\partial}{\partial x} \left( \frac{1}{H} \right) \left( \frac{\partial \chi}{\partial y} - \frac{H}{\rho_0} \cdot \frac{\partial (p)_{-H}}{\partial y} \right).
\]

Subtraction of equation (11a) from (11b) gives the vorticity equation:

\[
\frac{\partial}{\partial t} \left( \frac{\partial \nu_p}{\partial x} - \frac{\partial u_p}{\partial y} \right) + \frac{\partial f}{\partial x} \nu_p + \frac{\partial f}{\partial y} \nu_p + f \cdot \left( \frac{\partial u_p}{\partial x} + \frac{\partial \nu_p}{\partial y} \right) 
\]

\[
= -\frac{1}{\rho_0 H} \left( \frac{\partial H}{\partial x} \cdot \frac{\partial (p)_{-H}}{\partial y} - \frac{\partial H}{\partial y} \cdot \frac{\partial (p)_{-H}}{\partial x} \right) 
\]

\[
+ \left[ \frac{\partial}{\partial x} \left( \frac{1}{H} \right) \left( \frac{\partial \chi}{\partial y} - \frac{H}{\rho_0} \cdot \frac{\partial (p)_{-H}}{\partial y} \right) - \frac{\partial}{\partial y} \left( \frac{1}{H} \right) \left( \frac{\partial \chi}{\partial x} - \frac{H}{\rho_0} \cdot \frac{\partial (p)_{-H}}{\partial x} \right) \right].
\]

Using the definition \( \xi_p = \frac{\partial \nu_p}{\partial x} - \frac{\partial u_p}{\partial y} \) for the pressure driven relative vorticity of the depth averaged circulation and \( J(A,B) = \frac{\partial A}{\partial x} \cdot \frac{\partial B}{\partial y} - \frac{\partial A}{\partial y} \cdot \frac{\partial B}{\partial x} \) for the Jacobian operator it follows:

\[
\frac{\partial}{\partial t} \xi_p + \frac{\partial f}{\partial x} \nu_p + \frac{\partial f}{\partial y} \nu_p + f \cdot \left( \frac{\partial u_p}{\partial x} + \frac{\partial \nu_p}{\partial y} \right) = -\frac{1}{\rho_0 H} \left( \frac{\partial H}{\partial x} \cdot \frac{\partial (p)_{-H}}{\partial y} - \frac{\partial H}{\partial y} \cdot \frac{\partial (p)_{-H}}{\partial x} \right) 
\]
\begin{equation}
+ J \left( \chi, \frac{1}{H} \right) + \left[ \frac{\partial}{\partial y} \left( \frac{1}{\rho_0 H} \right) \right] \left( H \cdot \frac{\partial (p)_{-H}}{\partial x} \right) - \frac{\partial}{\partial x} \left( \frac{1}{\rho_0 H} \right) \left( H \cdot \frac{\partial (p)_{-H}}{\partial y} \right) \right].
\end{equation}

Rearrangement of terms in (13) leads to:

\begin{equation}
\frac{\partial}{\partial t} \xi_p + \frac{\partial f}{\partial x} \overline{u}_p + \frac{\partial f}{\partial y} \overline{v}_p + f \left( \frac{\partial \overline{u}_p}{\partial x} + \frac{\partial \overline{v}_p}{\partial y} \right) = J \left( \chi, \frac{1}{H} \right)
\end{equation}

\begin{equation}
+ \frac{1}{\rho_0 H} \frac{\partial H}{\partial y} \frac{\partial (p)_{-H}}{\partial x} + \frac{1}{\rho_0 H} \frac{\partial (p)_{-H}}{\partial y} \frac{\partial H}{\partial x} - \frac{\partial H}{\partial x} \frac{\partial (p)_{-H}}{\partial y} - \frac{\partial}{\partial x} \left( \frac{1}{\rho_0 H} \right) H \cdot \frac{\partial (p)_{-H}}{\partial y}.
\end{equation}

Factoring out the bottom pressure derivatives and employment of the product rule in the last line of (14) yield:

\begin{equation}
\frac{\partial}{\partial t} \xi_p + \frac{\partial f}{\partial x} \overline{u}_p + \frac{\partial f}{\partial y} \overline{v}_p + f \left( \frac{\partial \overline{u}_p}{\partial x} + \frac{\partial \overline{v}_p}{\partial y} \right) = J \left( \chi, \frac{1}{H} \right)
\end{equation}

\begin{equation}
+ \frac{\partial}{\partial y} \left( \frac{H}{\rho_0} \right) \frac{\partial (p)_{-H}}{\partial x} - \frac{\partial}{\partial x} \left( \frac{H}{\rho_0} \right) \frac{\partial (p)_{-H}}{\partial y}.
\end{equation}

Since \( \rho_0 = const \), the derivations of \( \frac{1}{\rho_0} \) in x- and y direction are zero. Hence the last line in equation (15) cancels and the vorticity equation reduces to:

\begin{equation}
\frac{\partial}{\partial t} \xi_p + \frac{\partial f}{\partial x} \overline{u}_p + \frac{\partial f}{\partial y} \overline{v}_p + f \left( \frac{\partial \overline{u}_p}{\partial x} + \frac{\partial \overline{v}_p}{\partial y} \right) = J \left( \chi, \frac{1}{H} \right).
\end{equation}

Subsequently, the terms containing the Coriolis parameter are transformed. For this purpose the continuity equation (2) will be averaged over depth, analogously to the equation of motion. For the pressure driven part of the circulation one obtains:

\begin{equation}
\frac{1}{H} \left( \int_{-H}^{0} \frac{\partial u_p}{\partial x} dz + \int_{-H}^{0} \frac{\partial v_p}{\partial y} dz + \int_{-H}^{0} \frac{\partial w_p}{\partial z} dz \right) = 0.
\end{equation}

Employment of Leibniz' theorem to (17) yields:

\begin{equation}
\frac{1}{H} \left( \frac{\partial}{\partial x} \int_{-H}^{0} u_p dz + \frac{\partial (-H)}{\partial x} \cdot (u_p)_{-H} + \frac{\partial}{\partial y} \int_{-H}^{0} v_p dz + \frac{\partial (-H)}{\partial y} \cdot (v_p)_{-H} + (w_p)_0 - (w_p)_{-H} \right) = 0.
\end{equation}

Making use of the rigid lid assumption at the sea surface and the kinematic boundary condition at the bottom:

\begin{align}
(w)_0 = 0 \quad \text{and} \quad (w_p)_{-H} = \frac{\partial (-H)}{\partial x} \cdot (u_p)_{-H} + \frac{\partial (-H)}{\partial y} \cdot (v_p)_{-H} \quad (19a, b)
\end{align}

all derivatives with respect to the water depth \( H \) in (18) cancel out and one obtains:

\begin{equation}
\frac{1}{H} \left( \frac{\partial}{\partial x} \int_{-H}^{0} u_p dz + \frac{\partial}{\partial y} \int_{-H}^{0} v_p dz \right) = 0.
\end{equation}

Application of the product rule to (20) yields:
\[ \frac{\partial}{\partial x} \left( \frac{1}{H} \int_{-H}^{0} u_p \, dz \right) + \frac{\partial}{\partial y} \left( \frac{1}{H} \int_{-H}^{0} v_p \, dz \right) - \frac{\partial}{\partial x} \left( \frac{1}{H} \right) \cdot \int_{-H}^{0} u_p \, dz - \frac{\partial}{\partial y} \left( \frac{1}{H} \right) \cdot \int_{-H}^{0} v_p \, dz = 0. \] (21)

Using the definitions (5a, b) for the depth averaged pressure driven velocities it follows:

\[ \frac{\partial u_p}{\partial x} + \frac{\partial v_p}{\partial y} = \frac{\partial}{\partial x} \left( \frac{1}{H} \right) \cdot H \cdot u_p + \frac{\partial}{\partial y} \left( \frac{1}{H} \right) \cdot H \cdot v_p. \] (22)

Insertion of (22) into the vorticity equation (16) leads to:

\[ \frac{\partial}{\partial t} \tilde{\omega}_p + \frac{\partial f}{\partial x} u_p + \frac{\partial f}{\partial y} v_p + f \left( \frac{\partial}{\partial x} \left( \frac{1}{H} \right) \cdot H \cdot u_p + \frac{\partial}{\partial y} \left( \frac{1}{H} \right) \cdot H \cdot v_p \right) = J \left( \chi, \frac{1}{H} \right) \] (23)

or:

\[ \frac{\partial}{\partial t} \tilde{\omega}_p + \frac{\partial f}{\partial x} u_p + \frac{\partial f}{\partial y} v_p - f \left( \frac{\partial H}{\partial x} \cdot u_p + \frac{\partial H}{\partial y} \cdot v_p \right) = J \left( \chi, \frac{1}{H} \right). \] (24)

after the employment of the quotient rule.

Using the vector notation \( \vec{v}_p = (u_p, v_p) \) the vorticity equation (24) gets its final form:

\[ \frac{\partial}{\partial t} \tilde{\omega}_p + \vec{v}_p \cdot \vec{\nabla}_h f - f \left( \frac{\partial H}{\partial x} \cdot u_p + \frac{\partial H}{\partial y} \cdot v_p \right) = J \left( \chi, \frac{1}{H} \right). \] (25)

According to (25) in the pure pressure driven case relative vorticity of the depth averaged circulation (term 1) is influenced by: a) currents perpendicular to lines of constant planetary vorticity (term 2), b) topographically induced stretching or squeezing of the water body containing planetary vorticity (term 3), and c) the JBAR effect (term 4). The latter takes into account the interaction between baroclinicity and topography. It is obtained by multiplication of the gradient of the topography with perpendicular gradients of the potential energy at the bottom. This interpretation of the JBAR term makes use of the quotient rule, which explicitly written has the form:

\[ \frac{\partial}{\partial x} \left( \frac{1}{H} \right) = -\frac{1}{H^2} \cdot \frac{\partial H}{\partial x} \text{ resp. } \frac{\partial}{\partial y} \left( \frac{1}{H} \right) = -\frac{1}{H^2} \cdot \frac{\partial H}{\partial y}. \]

Please note, (25) reduces to the vorticity equation for the pure barotropic case, when the JBAR term is neglected.

### 2.2 Interpretation of the JBAR-Term

A reasonable explanation of the nature of the JBAR term can only be achieved, when it is considered in connection with the underlying basic equations and simplifying assumptions from which it evolves.

#### 2.2.1 Employment of the alternative vorticity equation

The most obvious physical interpretation of the JBAR term is given by Mertz and Wright (1992). Their argumentation starts with a derivation of the vorticity equation in two ways. One way is to average the equations of motion (1a,b) first over depth, differentiate them crosswise and finally subtract them from each other. It results an equation for the change of relative vorticity of
the depth averaged circulation $\xi_p$. This procedure corresponds to the procedure carried out in the previous section. Hence, Mertz and Wright's (1992) solution corresponds to equation (25). The other possibility to derive the vorticity equation starts with a crosswise differentiation of the equation of motion (1a,b) followed by a subtraction of (1b) from (1a), which leads preliminarily to the vorticity equation in the velocity form. Final vertical averaging gives an equation for the change of the depth averaged relative vorticity $\tilde{\varsigma}_p$. Renouncing the lengthy calculation, this equation reads:

$$\frac{\partial}{\partial t} \tilde{\varsigma}_p + \tilde{\mathbf{v}}_p \cdot \nabla_h \tilde{f} - \frac{f}{H} \cdot (\tilde{\mathbf{v}}_p \cdot \nabla_h H) = 0.$$  \hspace{1cm} (26)

In this case the JBAR term is not evolving, which means that the baroclinicity must be included implicitly. It will be shown that now the bottom velocity $(\mathbf{v}_{p})_{-H}$ contains the information of the baroclinic field. Another difference between equation (26) and equation (25) lies in the vortex stretching term, in which the vertical averaged velocity is replaced by the bottom velocity. The difference between equation (25) and (26) leads to the following formulation of the JBAR term:

$$\int \left( \chi, \frac{1}{H} \right) = \frac{\partial}{\partial t} (\tilde{\xi}_p - \tilde{\varsigma}_p) - \frac{f}{H} \cdot (\tilde{\mathbf{v}}_p \cdot \nabla_h H),$$  \hspace{1cm} (27)

with $\tilde{\mathbf{v}}_p = (\tilde{u}_p, \tilde{v}_p) \equiv (\mathbf{u}_p - (u_p)_{-H}, \mathbf{v}_p - (v_p)_{-H})$ as the difference between the vertical mean velocity and the bottom velocity. For stationary or quasi-stationary conditions $\left( \frac{\partial \tilde{\xi}_p}{\partial t} = \frac{\partial \tilde{\varsigma}_p}{\partial t} = 0 \right)$

the time dependent term in (27) cancels out, which also means that a pure geostrophic equilibrium is considered. Obviously, in this pure geostrophic case the JBAR term has the function of a correction term. Without its introduction in equation (25) the depth averaged horizontal velocity would be responsible for vortex stretching. If, as usual, a vertical shear of the velocity is present this assumption would produce an error. Because from the kinematic boundary condition it can be concluded, that the bottom velocity is responsible for a topographically induced stretching of the water column, instead of the depth averaged velocity. Hence, the JBAR term corrects an error in the vorticity equation, replacing the depth averaged velocity by the bottom velocity in the vortex stretching term. It can be shown that also in the non-stationary case, i.e. under no purely geostrophic conditions, the same argumentation holds. The only difference is the occurrence of an additional term. Using equation (27) one obtains the following difference between $\xi_p$ and $\tilde{\varsigma}_p$:

$$\xi_p - \tilde{\varsigma}_p = \frac{\partial \tilde{\mathbf{v}}_p}{\partial x} - \frac{\partial \tilde{\mathbf{u}}_p}{\partial y} - \left[ \frac{1}{H} \frac{\partial \mathbf{v}_p}{\partial y} dz - \frac{1}{H} \frac{\partial \mathbf{u}_p}{\partial y} dz \right]$$  \hspace{1cm} (28)

$$= \frac{\partial \tilde{\mathbf{v}}_p}{\partial x} - \frac{\partial \tilde{\mathbf{u}}_p}{\partial y} - \left[ \frac{1}{H} \frac{\partial (\tilde{\mathbf{v}}_p dz)}{\partial x} - \frac{\partial (\mathbf{v}_p dz)}{\partial x} - \frac{\partial H}{\partial y} \cdot (\mathbf{v}_p dz) \right] - \frac{1}{H} \frac{\partial (\mathbf{u}_p dz)}{\partial y} - \frac{\partial H}{\partial y} \cdot (\mathbf{u}_p dz)$$

$$= \frac{\partial \tilde{\mathbf{v}}_p}{\partial x} - \frac{\partial \tilde{\mathbf{u}}_p}{\partial y} - \left[ \frac{\partial \tilde{\mathbf{v}}_p}{\partial x} + \frac{1}{H} \frac{\partial \mathbf{v}_p dz}{\partial x} - \frac{\partial \mathbf{u}_p dz}{\partial x} \right] - \frac{1}{H} \frac{\partial H}{\partial x} \cdot (\mathbf{v}_p dz)$$

$$- \frac{\partial H}{\partial x} \cdot (\mathbf{u}_p dz).$$
whereby it is made use of Leibniz' theorem, partial integration and the quotient rule. According to (28) the JBAR term derived in (27) obtains also for the non-stationary case an explicit solution which is:

\[
J^{(\chi, \frac{1}{H})} = \frac{\partial}{\partial t} \left( \frac{1}{H} \left( \nabla_p \cdot \frac{\partial H}{\partial x} - \vec{u}_p \cdot \frac{\partial H}{\partial y} \right) \right) - \frac{f}{H} \left( \vec{v}_p \cdot \nabla_h H \right).
\]  

(29)

The additional evolving term on the r.h.s. also contains the differences \( \vec{u}_p \) and \( \vec{v}_p \) between vertical mean and the bottom velocities. Obviously, this term corrects the temporal deviation of relative vorticity \( \frac{\partial \vec{\xi}_p}{\partial t} \) in (25).

2.2.2 Employment of the hydrostatic equation

An alternative derivation of the JBAR term can be achieved by utilization of the hydrostatic equation:

First the hydrostatic equation is integrated from \(-H\) to \(z\) and subsequently vertically averaged over the entire water column:

\[
\frac{1}{H} \int_{-H}^{0} \int dp \cdot dz = \frac{g}{H} \int_{z=-H}^{0} \rho \cdot dz.
\]

(30)

Partial integration results in:

\[
\frac{1}{H} \int_{-H}^{0} \left( p(z) - (p)_{-H} \right) dz = \frac{g}{H} \left( \int_{z=-H}^{0} \rho \cdot dz + z \left( \int_{z=-H}^{0} \rho \cdot dz \right) \right).
\]

(31)

Insertion into the boundaries, definition of the depth averaged pressure \( \bar{p} = \frac{1}{H} \int_{-H}^{0} p(z) \cdot dz \) and employment of the definition for the potential energy at the bottom \( \chi \) yield:

\[
\bar{p} - (p)_{-H} = \frac{1}{H} \left( -\rho_0 \cdot \chi + 0 \cdot g \cdot \int_{z=-H}^{0} \rho \cdot dz - (H) \cdot \int_{z=-H}^{0} \rho \cdot dz \right).
\]

(32)

Because both last terms of (32) cancel out, (32) simplifies to:

\[
\rho_0 \cdot \chi = H \cdot (\bar{p} - (p)_{-H} )
\]

(33)

Multiplication of (33) with the horizontal nabla operator \( \nabla_h \) and application of the product rule give:

\[
\rho_0 \cdot \nabla_h \chi = H \cdot \left( \nabla_h \bar{p} - \nabla_h (p)_{-H} \right) + \nabla_h H \cdot (\bar{p} - (p)_{-H} ).
\]

(34)
In the component notation the $x$-component gets the form:

$$\rho_0 \frac{\partial \chi}{\partial x} = H \left[ \frac{\partial}{\partial x} \left( \frac{1}{H} \int_{-H}^{0} p \, dz \right) - \frac{\partial (p)_{-H}}{\partial x} \right] + \frac{\partial H}{\partial x} \cdot (\bar{p} - (p)_{-H})$$

(35a)

$$= H \left[ \frac{\partial}{\partial x} \left( \frac{1}{H} \right) \cdot \int_{-H}^{0} p \, dz + \frac{1}{H} \frac{\partial}{\partial x} \int_{-H}^{0} p \, dz - \frac{\partial (p)_{-H}}{\partial x} \right] + \frac{\partial H}{\partial x} \cdot (\bar{p} - (p)_{-H})$$

$$= H \left[ \frac{\partial}{\partial x} \left( \frac{1}{H} \right) \cdot \int_{-H}^{0} p \, dz + \frac{1}{H} \left( \int_{-H}^{0} \frac{\partial p}{\partial x} \, dz + \frac{\partial H}{\partial x} \cdot (p)_{-H} \right) - \frac{\partial (p)_{-H}}{\partial x} \right] + \frac{\partial H}{\partial x} \cdot (\bar{p} - (p)_{-H})$$

$$= H \left[ \frac{\partial}{\partial x} \left( \frac{1}{H} \right) \cdot \int_{-H}^{0} p \, dz + \frac{1}{H} \left( \int_{-H}^{0} \frac{\partial p}{\partial x} \, dz \right) - \frac{\partial (p)_{-H}}{\partial x} \right] + \frac{\partial H}{\partial x} \cdot (\bar{p} - (p)_{-H})$$

$$= H \left[ \frac{\partial}{\partial x} \left( \frac{1}{H} \right) \cdot \int_{-H}^{0} p \, dz + \frac{1}{H} \left( \int_{-H}^{0} \frac{\partial p}{\partial x} \, dz \right) - \frac{\partial (p)_{-H}}{\partial x} \right] + \frac{\partial H}{\partial x} \cdot (\bar{p} - (p)_{-H})$$

$$= H \left[ \frac{1}{H} \left( \int_{-H}^{0} \frac{\partial p}{\partial x} \, dz \right) - g \cdot \left( \frac{\partial}{\partial x} \int_{-H}^{0} \rho \, dz \right) \right]$$

$$= H \left[ \frac{1}{H} \left( \int_{-H}^{0} \frac{\partial p}{\partial x} \, dz \right) - \int_{-H}^{0} \frac{\partial}{\partial x} \left( g \cdot \rho \right) \, dz - g \cdot \frac{\partial h}{\partial x} \cdot (\rho)_{-H} \right]$$

$$= H \left[ \frac{1}{H} \left( \int_{-H}^{0} \frac{\partial p}{\partial x} \, dz \right) + \int_{-H}^{0} \frac{\partial}{\partial x} \left( \frac{\partial p}{\partial x} \right) \, dz - g \cdot \frac{\partial h}{\partial x} \cdot (\rho)_{-H} \right]$$

$$= H \left[ \frac{1}{H} \left( \int_{-H}^{0} \frac{\partial p}{\partial x} \, dz \right) + \left( \frac{\partial p}{\partial x} \right)_{-H} - g \cdot \frac{\partial h}{\partial x} \cdot (\rho)_{-H} \right]$$

$$= H \left[ \frac{\partial p}{\partial x} - \left( \frac{\partial p}{\partial x} \right)_{-H} \right] - g \cdot H \cdot \frac{\partial h}{\partial x} \cdot (\rho)_{-H}$$

whereby the upper bar indicates the vertical average of the gradient.

For the $y$-component one obtains respectively:

$$\rho_0 \frac{\partial \chi}{\partial y} = H \left[ \frac{\partial p}{\partial y} - \left( \frac{\partial p}{\partial y} \right)_{-H} \right] - g \cdot H \cdot \frac{\partial h}{\partial y} \cdot (\rho)_{-H}$$

(35b)

Combination of (35a) and (35b) in the vector notation gives instead of (34):

$$\rho_0 \vec{\nabla}_h \chi = H \cdot \left( \vec{\nabla}_h p - \left( \vec{\nabla}_h p \right)_{-H} \right) - g \cdot H \cdot \vec{\nabla}_h H \cdot (\rho)_{-H}$$

(36)

The cross product between (36) and $\vec{\nabla}_h \left( \frac{1}{H} \right)$ leads to:

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\[ \left\{ \vec{\nabla}_h \times \nabla_h \left( \frac{1}{H} \right) \right\} \cdot \vec{k} = \left\{ \frac{H}{\rho_0} \cdot \left[ \vec{\nabla}_h p - \vec{\nabla}_h (p)_{-H} \right] \times \vec{\nabla}_h \left( \frac{1}{H} \right) \right\} \cdot \vec{k} . \]  

(37)

Employment of the quotient rule yields:

\[ \left\{ \vec{\nabla}_h \times \nabla_h \left( \frac{1}{H} \right) \right\} \cdot \vec{k} = \left\{ -\frac{1}{\rho_0 H} \cdot \left[ \vec{\nabla}_h p - \vec{\nabla}_h (p)_{-H} \right] \times \vec{\nabla}_h H \right\} \cdot \vec{k} . \]  

(38)

Differentiation of the cross products gives:

\[ \left( \frac{\partial \chi}{\partial x} - \frac{\partial \chi}{\partial y} \left( \frac{1}{H} \right) \right) \left( \frac{\partial (\vec{p} \cdot \vec{u})}{\partial x} - \frac{\partial (\vec{p} \cdot \vec{u})}{\partial y} \left( \frac{1}{H} \right) \right) = -\frac{1}{\rho_0 H} \cdot \left( \frac{\partial \chi}{\partial x} \left( \frac{\partial H}{\partial y} \right) - \frac{\partial \chi}{\partial y} \left( \frac{\partial H}{\partial x} \right) \right) \cdot \frac{\partial H}{\partial x} . \]  

(39)

Using the geostrophic equilibrium from the equations of motion (1a, b)

\[ -\frac{1}{\rho_0} \cdot \frac{\partial p}{\partial x} = -fv, \]  

resp.

\[ -\frac{1}{\rho_0} \cdot \frac{\partial p}{\partial y} = fu, \]

the pressure gradients arriving in (39) get the form:

\[ \frac{\partial p}{\partial x} = \rho_0 f \cdot \vec{v}, \]  

resp.

\[ \frac{\partial p}{\partial y} = -\rho_0 f \cdot \vec{u}, \]

and

\[ \left( \frac{\partial \chi}{\partial x} \right)_{-H} = \rho_0 f \cdot \left( \vec{v} \right)_{-H} \]  

resp.

\[ \left( \frac{\partial \chi}{\partial y} \right)_{-H} = -\rho_0 f \cdot \left( \vec{u} \right)_{-H} . \]

Insertion into (39) and employment of the Jacobian operator leads to:

\[ J \left( \chi, \frac{1}{H} \right) = -\frac{f}{H} \left( \vec{u} \cdot \left( \vec{v} \right)_{-H} \right) \frac{\partial H}{\partial x} + \left( \vec{v} \cdot \left( \vec{v} \right)_{-H} \right) \frac{\partial H}{\partial y} . \]  

(40)

In vector notation the JBAR term finally gets the form:

\[ J \left( \chi, \frac{1}{H} \right) = -\frac{f}{H} \left( \vec{v} \cdot \vec{v} \right) H , \]  

(41)

with \( \vec{v} = (\vec{u}, \vec{v}) = (\vec{u} - (u)_{-H}, \vec{v} - (v)_{-H}) \).

Thus, this derivation of the JBAR term in (41) clearly shows, that it is only the geostrophic part of the difference between depth averaged and bottom velocities \( \vec{v} \) which enters the JBAR term. According to equation (29) all non-geostrophic parts in the differences between those two velocities enter the addition time-dependent term \( \frac{\partial}{\partial t} \left\{ -\left( \frac{1}{H} \right) \cdot \left( \vec{v} \cdot \frac{\partial H}{\partial x} - \vec{u} \cdot \frac{\partial H}{\partial y} \right) \right\} \) of this equation. This seems to be reasonable, since the only term, that disturbs the geostrophic balance in the initial equations for the pressure induced motion (3a,b), is the time-dependent term.

2.3 Criticism related to the utility of the JBAR term

In many publications of the past the influence of topography on the baroclinic circulation was inferred directly from the JBAR term. Frequently, this has resulted in the incorrect finding
that the influence of topography is of same or even larger influence as the rest of the forcing terms. From the above discussion it is clear that the JBAR term has the character of a correction term, which corrects an error in the vorticity equation obtained from the vertically averaged equations of motion (Mertz and Wright, 1992). It guarantees that the bottom velocity is responsible for the vortex stretching, instead of the depth averaged velocity.

Even if the bottom velocity is zero, i.e. no influence of the topography is possible, it can be concluded from equation (41) that the JBAR term attains a value different from zero. According to equation (41) with \( \langle u_x \rangle_H = \langle v_y \rangle_H = 0 \) the JBAR term is given by:

\[
J \left( \chi', \frac{1}{H} \right) = -\frac{f}{H} \cdot \hat{\nabla}_s \cdot \hat{\nabla}_b H.
\]

Thus, it is exactly the negative of the vortex stretching term in the vorticity equation (27). This ensures that in (27) the effect of vortex stretching cancels out, which results from the incorrect use of the depth averaged velocity in this term.

With other words: First of all, the special way of deriving the vorticity equation introduces an inconsistency, which is corrected afterwards by the introduction of the JBAR term. Therefore the JBAR term is strictly speaking, not a forcing but a correction term. In turn, this also means that the JBAR term is getting larger, if the error in the vorticity equation increases. The error is zero, if vertical averaged and bottom velocities are equal. Stronger deviations between these two velocities generate larger values of the JBAR term. Thus, even a vanishing bottom velocity, in general, gives a JBAR value different from zero, although there is no influence of topography. In this case, only the wrongly introduced effect of topography in (27) will be corrected.

In recent years the criticism concerning the utilization of the JBAR term to interpret oceanographic phenomena has increased. Recent publications tend to the conclusion, that, in general, the influence of topography on the baroclinic circulation has been overestimated in the past. For example, Cane et al. (1998) have performed a very instructive experiment with a simple analytic two layer model. They studied a pure wind driven circulation, neglecting the momentum transfer between surface and bottom layer. The pressure field was determined in a way which ensures a motionless bottom layer. In their application with realistic parameter setting they showed that the JBAR term was 2.2 times larger than the wind stress term, although interactions between baroclinicity and topography could not have any influence on the solution at all, due to the special configuration of their experiment.

Despite the fact, that this two layer model used by Cane et al. (1998) is a strong idealisation, it reflects a very important aspect of conditions in the real ocean. Namely the fact, that, in general, the velocities in the deep oceans are very small. As in the two layer model from Cane et al. (1998) this can be explained by the compensation between the baroclinic and the barotropic pressure gradient. The assumption of a compensation depth is supported by the long and successful history of the layer-of-motion-concept, which has been applied in the dynamic method as well as in many theoretical model studies. By means of this concept, it was possible to derive the principle structure of the ocean circulation very accurately without accounting for topography.

As presented by Cane et al. (1998), the discrepancy between the JBAR and the layer-of-no-motion concept is repeated in structural difference between the vorticity equation and the Sverdrup balance. The latter is obtained by vertical integration of the equations of motion up to a depth of negligible motion; thus the topography is not considered. In this case, lines with \( f = \text{const} \) define the characteristics of the streamlines, whereas in the case of the vorticity equation, lines with \( f / H = \text{const} \) form the streamline characteristics.
The above discussion of the JBAR term can be summarised as follows: One should be careful, not to mix up the necessary correction performed in the vorticity equation with a real physical process, i.e. the interaction between the baroclinic field and the topography, as it has happened very frequently in the past. In particular, compensations between baroclinic and barotropic pressure gradients, taking place in most parts of the world ocean reduce the actual influence of topography and consequently, the interaction between topography and baroclinicity, too. By definition, the JBAR term only contains the baroclinic component. Thus, this term is not capable of accounting for the compensation between barotropic and baroclinic pressure gradients in depth.

3 Application to the Northeastern Atlantic Shelf Break

3.1 Description of the model

The hydro-thermodynamical model used in the present investigation is a modified version of the shelf sea circulation model developed by Backhaus (1985). The governing equations are the shallow water equations in combination with the hydrostatic assumption, the equation of continuity, the transport equation for temperature and salinity and the equation of state for sea water. The model feature most relevant for long-term simulations is the sophisticated semi-implicit numerical scheme, which is used to solve this system of differential equations. A detailed discussion of the numerical formulation is presented in Backhaus (1985). This model has extensively been applied to many regions of the world, and to the North Sea in particular. Thus, the general performance of the model with respect to the reproduction of hydrodynamic and thermal conditions in shelf regions and their adjacent oceanic areas has proved to be very reliable.

Modifications carried out by the author mainly aim at a more realistic representation of the thermal stratification in the North Sea. This has been achieved by incorporating turbulent surface and bottom mixed layer processes (Pohlmann, 1996a). The vertical eddy viscosity is calculated with the help of a second order closure scheme, which originally has been proposed by Kochergin (1987). According to this approach, the vertical eddy viscosity is enhanced by vertical velocity shear and reduced by vertical stability. A detailed description of this formulation and of the spatial and temporal distribution of vertical eddy viscosity in the North Sea is given in Pohlmann (1996b).

3.2 Model Simulation

The three-dimensional Northwestern European Shelf Model was used to calculate the hydro- and thermodynamic parameters continuously for the period from 1st January to 31st December 1990. During this period the model was driven by three-hourly wind and atmospheric pressure distributions, that were obtained from an objective analysis of meteorological observations carried out by merchant-ships. At the open boundaries salinity and temperature data as well as sea level elevations were prescribed. The former ones were obtained from climatological monthly means, whereas the latter include the inverse barometric effect, the dynamic heights and the M2-tide.

The Northwestern European Shelf Model covers the entire Baltic Sea, the North Sea and parts of the northeastern Atlantic up to 15° W and from 47,5° to 64° N. The horizontal resolution of this model is 6° in meridional and 10° in zonal direction. The discretisation and the underlying topography of the model domains are given in Fig. 1. In the vertical the model is resolved by a
maximum number of 19 layers, which is necessary for an adequate resolution of that part of the water column, which is influenced by a thermocline. With the exception of the first layer, which has a thickness of 10 m, the upper 50 m of the water column has a resolution of 5 m. The thickness of the lower layers gradually increases from 10 to 2500 m with increasing depth. The time step is chosen to be 10 min, which turns out to be sufficient for a reasonable resolution of all major processes.

3.3 Presentation of model results

In this section the temporal and spatial distribution of the JBAR term will be described and interpreted. On one hand, this interpretation is performed by means of two hydrodynamic quantities, i.e. sea surface elevation and the vertical averaged velocity field. On the other hand, two additional terms of the vorticity equation are employed in order to analyse the development of the JBAR term, namely the correction term \( -\frac{f}{H} (\mathbf{v} \cdot \nabla H) \), which has been discussed in the previous sections and the time derivation of relative vorticity \( \frac{\partial \mathbf{v}}{\partial t} \).

The JBAR term (Figs. 2a-l) shows a very strong spatial variability. Maximum values of more than 4.0 \( \times 10^{-10} \text{s}^2 \) are reached at the shelf break off the English Channel in the southwestern corner of the model domain. In the northern part of the shelf break magnitudes of 2.0 \( \times 10^{-10} \text{s}^2 \) are found. In accordance with the definition of the JBAR term, highest magnitudes can be found along the Continental shelf break, where about six local minima and six local maxima can be distinguished. Their locations are correlated to the topography, but a clear picture of this correlation cannot be derived without considering the definition of the JBAR term. On the shelf, values are about two orders of magnitude smaller. With the exception of the Norwegian trench amplitudes are below 0.05 \( \times 10^{-10} \text{s}^2 \).

The seasonal variability is strongest in the shallower parts of the model domain, i.e. in the Baltic Sea and the North Sea, whereas in the adjacent oceanic areas the JBAR term exhibits only a very small seasonal variability. Here, the locations of the minima and maxima are nearly stationary. This can be explained by the fact that in this part of the model domain the density field reveals an extremely strong seasonal cycle, due to the shallowness of these regions. Thus, in shelf areas, the seasonal cycle of the heat flux can have a much stronger impact on the heat content of the entire water column, compared to the shelf break and oceanic regions. Due to this strong influence of the heat flux on the shelf, the general direction of the density gradient reverses from winter to summer. In winter the density gradient points towards the coast, whereas in summer it is directed to the interior of the shelf sea. The coastal reduction of the salinity due to the inflow of fresh water weakens the above mentioned density gradient in winter and strengthens it in summer.

The potential energy at the bottom, which determines the JBAR term, is obtained by an integration of density over depth. Since over the shelf break and in oceanic regions the major part of the water column is located below the seasonal thermocline, the vertically averaged seasonal variability of the density field is comparatively low, which explains the different seasonal behaviour of shelf and oceanic areas. The model was forced with climatological monthly mean temperature and salinity distributions, which are linearly interpolation between the months. In the present model version both quantities are treated purely diagnostic and are not modified by internal model processes. Since climatological fields are smoothed both in space and time, the current model results represent only a lower limit of the variability, that has to be expected for more realistic conditions.
The relatively small seasonal variability in deeper areas of the model domain cannot only be found for the JBAR term but also for other parameter, i.e. the sea surface elevation (Figs. 3a-l) and the depth averaged velocity fields (Figs. 4a-l). The sea surface elevation shows a gradient from the open ocean into the North Sea and further into the Baltic Sea. Two major reasons explain this phenomenon. On one hand, under the assumption of a zero order hydrostatic equilibrium, elevations have to increase with decreasing salinity in order to compensate for the density gradient. On the other hand the mean direction of the wind over the North Atlantic causes a "windstatu" effect, which also leads to an increase of the sea surface elevation from the Atlantic towards the semi-enclosed Baltic Sea basin. The depth averaged circulation patterns also exhibit generally known features, e.g., a cyclonic circulation in the North Sea, caused by an interplay between wind, density and tidal induced currents (Backhaus, Pohlmann and Hainbucher, 1986), and a northeastward directed jet current, which flows parallel to the shelf break.

Again, both presented hydrodynamical fields show a weaker seasonal variability in the oceanic parts compared to the shelf regions, although compared to the JBAR term this variability is considerably larger. This can be explained by the fact, that the major contribution of the circulation can be found in the wind influenced surface mixed layer. Here, the circulation strongly depends on the variability of the wind forcing field. Assuming a certain influence of geostrophy in the wind driven zone, the variability of the circulation also effects the sea surface elevation. The latter is also effected by the atmospheric pressure at the sea surface, which is another temporal and spatial varying forcing function of the model and therefore introduces an additional variability.

Finally, two other terms of the vorticity equation are considered in order to analyse the JBAR term. The development of the time derivative of the relative vorticity (Figs. 5a-l) reaches values which are at least two orders of magnitude smaller than the JBAR term. Largest values of up to $0.05 \times 10^{-10}$ s$^{-2}$ are reached along the shelf break and in the Norwegian trench. Positive and negative values are approximately equally distributed, but covering distinct separate areas. These areas have spatial scales comparable to those found for the minima and maxima of the JBAR term. This negligible magnitude of the time derivative of relative vorticity is a clear indication, that for temporal and spatial scales considered in the presented model run, the vorticity equation can be treated as being quasi-stationary. As discussed in section 2.2.1, the stationarity of the vorticity equation is a prerequisite for the validity of equation (27), in which the time-dependent term is neglected. Under these circumstances (27) provides a direct connection between the JBAR term and a correction term, which accounts for the difference between the depth averaged and the bottom velocity in the vortex stretching term.

Figs. 6a-l show the distribution of the monthly mean of this correction term in 1991. The seasonal variability of this term is significantly larger than that of the JBAR term. This can be explained by the fact, that according to equation (27), the variability of this correction term is governed by the variability of the circulation; and the latter is influenced by seasonal changing wind fields. Therefore the seasonal changes should be much larger in shallow shelf regions, where wind effects reach down to the bottom. However, apart from shelf regions the locations of the maxima and minima coincide relatively good. For instance, the dipole structure at the Fair Isles and west of Ireland can be found for both quantities. Nevertheless, it has to be noted, that the magnitudes of these extremes sometimes differ by orders of magnitude. As discussed in section 2.2, here the JBAR term at least partially compensates the error introduced by the employment of a vertical mean value in the vortex stretching term. On the shelf the situation is quite different. In general, the JBAR and the correction term are not correlated at all. For example in the Doggerbank area the JBAR term shows no indication of the dipole structure in summer, which can be found for the correction term. In these shallow areas very often both terms act in the same
direction and thus, their contribution to the vorticity equation must be cancelled by other terms in order to obtain a stationary solution of equation (25).

4 Conclusions

As has been demonstrated in the theoretical considerations at the beginning of this report, in general, the JBAR term is not a correct measure of the interaction between topography and baroclinicity as it has been anticipated by most authors in the last three decades. Moreover, the JBAR term is not an independent forcing term, which frequently was proclaimed. Contrarily, this study has emphasised, that the JBAR term is a correction term. It corrects an error, that was introduced into the vorticity equation in consequence of the specific way by which the vorticity equation was derived.

The results of a Northeastern Atlantic model have illustrated, that the simple balance assumed in the vorticity equation, from which the JBAR term evolves, is not existing in such a model. In the off-shelf areas this balance can be found at least partially, whereas on the shelf itself, it is not valid at all. Therefore, even the role of the JBAR term as a correction term could not clearly be confirmed. Not only in most parts of the shelf but also in oceanic areas and over the shelf break, other terms, arising from friction and wind stress are needed in order to close the vorticity balance. These terms already have been included and discussed in many other studies (e.g.: Nihoul and Ronday, 1975; Huthnance, 1982; Dippner, 1995) and, in principle, they could also easily be deduced from the current model results. Since the focus of this study was the utilization of the JBAR term and not the vorticity balance itself, the investigation of such additional terms was out of the scope of the present report.

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Figure captions

Fig. 1: Topography of the Northwestern European Shelf Model (depth in m)

Figs. 2a-l: Monthly mean value of the JBAR term ($10^{-10}$ $s^{-2}$) for the year 1991
(a-l: January to December)

Figs. 3a-l: Monthly mean sea surface elevation (m) for the year 1991
(a-l: January to December)

Figs. 4a-l: Monthly mean vertically averaged circulation pattern (cm s$^{-1}$) for the year 1991
(a-l: January to December)

Figs. 5a-l: Monthly mean value of the time derivation of the relative vorticity ($10^{-10}$ $s^{-2}$)
for the year 1991 (a-l: January to December)

Figs. 6a-l: Monthly mean value of the correction term ($10^{-10}$ $s^{-2}$),
    i.e. the vortex stretching term, driven by the difference between vertical mean
    and bottom velocity, for the year 1991 (a-l: January to December)
List of symbols

$x, y, z$ spatial cartesian coordinates in east, north and vertical (positive upwards) direction

$k$ vertical unity vector (positive upwards)

$t$ temporal coordinate

$u, v, w$ velocities in $x$-, $y$-, $z$-direction

$u_p, v_p, w_p$ pressure induced velocities in $x$-, $y$-, $z$-direction

$\bar{u}_p, \bar{v}_p$ depth averaged velocities induced by the pressure gradient in $x$-, $y$-direction

$\bar{u}_g, \bar{v}_g$ depth averaged velocities induced by geostrophy in $x$-, $y$-direction

$\bar{u}_p, \bar{v}_p$ difference of depth averaged and the bottom velocity both induced by the pressure gradient in $x$-, $y$-direction

$\bar{u}_g, \bar{v}_g$ difference between the depth averaged and the bottom velocities both induced by geostrophy in $x$-, $y$-direction

$\vec{v}_p$ vector of $\bar{u}_p, \bar{v}_p$

$\vec{v}_p$ vector of $\bar{u}_p, \bar{v}_p$

$\vec{v}_g$ vector of $\bar{u}_g, \bar{v}_g$

$\zeta$ sea surface elevation

$\rho$ density

$\rho_0$ reference density

$p$ pressure

$p_0$ pressure at the sea surface

$\bar{p}$ depth averaged pressure

$f$ Coriolis parameter

$g$ gravitational acceleration

$H$ undisturbed water depth

$\dot{\xi}_p$ relative vorticity of the depth averaged, pressure induced velocities

$\bar{\xi}_p$ depth averaged relative vorticity of the pressure induced equations of motion

$\chi$ potential energy at the bottom

$J$ Jacobian operator

$\vec{V}_h$ horizontal nabla operator
Reference


Fig. 1: Topography of the Northwestern European Shelf Model (depth in m)
Fig. 1: Topography of the Northwestern European Shelf Model (depth in m)
Fig. 2a-l: Monthly mean value of the JBAR term ($10^{-10}$ s$^{-2}$) for the year 1991 (a-l: January to December)
Fig. 2a-l: Monthly mean value of the JBAR term \(s^2 \times 10^{10}\) for the year 1991
(a-l: January to December)
BAM-Modelldaten
JBAR gem./skal.

JBAR91-November.L15

[Map of geographical region with contour lines and color scale]

November

[Color scale indicating log. Verlauf: -2.08 to 5.50]

10^-10
Fig. 3a-l: Monthly mean sea surface elevation (m) for the year 1991
(a-l: January to December)
BAM-Modelldaten
Wasserstand - Monatsmittelwerte

Fig. 3a-1: Monthly mean sea surface elevation (m) for the year 1991 (a-1: January to December)
BAM-Modelldaten
Wasserstand - Monatsmittelwerte

ZZ91-Dezember

-1.2 -1.1 -1.0 -0.9 -0.8 -0.7 -0.6 -0.5 -0.4 -0.3 -0.2 -0.1 0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 m
Fig. 4a-l: Monthly mean vertically averaged circulation pattern (cm/s) for the year 1991
(a-l: January to December)
BAM-Modelldaten
Strömung - Monatsmittelwerte

vectorM91-März

März

cm/s

0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50
BAM-Modelldaten
Strömung - Monatsmittelwerte

vectorM91-Mai

cm/s

0  2  4  6  8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50
BAM-Modelldaten
Strömung - Monatsmittelwerte

vectorM91-Juli

cm/s
BAM-Modelldaten
Strömung - Monatsmittelwerte

vectorM91-November

November
BAM-Modelldaten
Strömung - Monatsmittelwerte

vectorM91-Dezember

Dezember

cm/s

0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50
Fig. 5a-l: Monthly mean value of the time derivation of the relative vorticity \(10^{-10} \text{ s}^{-2}\) for the year 1991
(a-l: January to December)
Fig. 5a-l: Monthly mean value of the time derivation of the relative vorticity \( (s^{-2} \times 10^{-10}) \) for the year 1991
(a-l: January to December)
BAM-Modelldaten
VorDif gem./skal.

VORDif91-Juni.L15

15°W 10°W 5°W 0° 5°E 10°E 15°E 20°E 25°E 30°E

64°N 62°N 60°N 58°N 56°N 54°N 52°N 50°N 48°N

log. Verlauf

-2.0 -1.62 -1.25 -0.95 -0.72 -0.54 -0.39 -0.27 -0.18 -0.10 -0.05 -0.01 0.00 0.01 0.05 0.10 0.18 0.27 0.39 0.54 0.72 0.95 1.25 1.62 2.08 2.66 3.42 4.33 5.50

10^{-10}
BAM-Modelldaten
VorDif gem./skal.

VORDif91-Oktober.L15

Oktober

log. Verlauf

10^{-10}

-2.08 -1.62 -1.25 -0.95 -0.72 -0.54 -0.39 -0.27 -0.18 -0.10 -0.05 -0.01 0.00 0.01 0.05 0.10 0.18 0.27 0.39 0.54 0.72 0.95 1.25 1.62 2.08 2.66 3.42 4.33 5.50
Fig. 6a-l: Monthly mean value of the correction term \( (10^{-10} \, \text{s}^{-2}) \),
i.e. the vortex stretching term, driven by the difference between
vertical mean and bottom velocity, for the year 1991
(a-l: January to December)
BAM-Modelldaten
Korrekt gem./skal.

Korrekt91-Januar.L15

Fig. 6a1: Monthly mean value of the correction term ($G \cdot 10^{-10}$), i.e. the vortex stretching term, driven with the difference between vertical median and bottom velocity, for the year 1991.
BAM-Modellldaten
Korrekt gem./skal.

Korrekt91-März.L15
BAM-Modelldaten
Korrekt gem./skal.

Korrekt91-November.L15

November
Since its first description in 1971 by Sakisyan and Ivanov (1971) and its more extensive discussion in „The Sea“ by Sarkisyan (1977) the JBAR effect has been studied in numerous publications. Although it can be reduced to only one single term in the vorticity equations a large number of authors have focussed on this term. It can be suspected that also the interesting sounding acronym has played a role to increase the popularity of the JBAR effect. JBAR stands for „Joint Effect of Baroclinicity and Relief“, which in some respect is misleading as will be shown. The report is split into two parts. It starts with general theoretical considerations concerning the JBAR effect, whereas in the second part its distribution on the northwestern European shelf including the adjacent shelf break will be presented and discussed.

Already the theoretical considerations at the beginning will clearly demonstrated that in general the JBAR term is not a correct measure of the interaction between topography and baroclinicity as it has been anticipated by most authors in the last three decades. Moreover, it is not even a totally independent forcing term, which always was proclaimed. The calculations demonstrate, that the JBAR term is a correction term, which corrects an error introduced into the vorticity equation caused by the specific method by which the vorticity equation is derived.

Finally, the results of a Northeastern Atlantic model, that simulates the actual phenomena in a realistic form will be analysed. The results illustrate, that the simple balance assumed in the vorticity equation from which the JBAR term evolves, is not existent in such a model. In the off-shelf areas this balance can be found at least partially, whereas on the shelf itself, it is not valid at all. Therefore not even the role of the JBAR term as being a correction term could clearly be confirmed. Other terms, like the friction term or the wind stress term are needed in order to close the vorticity balance, not only in most parts of the shelf but also in the oceanic areas and over the shelf break..
BAM-Modelldaten
JBAR gem./skal.

JBAR91-Januar.L15

Fig 2a-1: Monthly mean value of the JBAR term ($g_2 \times 10^{-10}$ for the year 1991)
Fig. 3a-1: Monthly mean sea surface elevation (m) for the year 1991
(a-1: January to December)
BAM-Modelldaten
Strömung - Monatsmittelwerte

vectorM91-Januar

Fig. 4.1: Monthly mean vertically averaged circulation pattern (cm/s) for the year 1991 (a-l: January to December)
Fig. 5a-l: Monthly mean value of the time derivation of the relative vorticity ($s^2 \times 10^{-10}$) for the year 1991 (a-l: January to December)
Fig. 6a-l: Monthly mean value of the correction term ($s^2 \times 10^{10}$), i.e. the vortex stretching term, driven with the difference between vertical mean and bottom velocity, for the year 1991 (a-l: January to December)