Adaptivity with Simplex Elements in EUROPLEXUS

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1. Introduction

This report is a sequel to reports and publications [1-12] on mesh adaptivity in fast transient dynamics and presents the formulation and implementation of mesh adaptivity for simplex elements (triangles in 2D, tetrahedra in 3D) in fast transient dynamics. The algorithms are implemented in the EUROPLEXUS code.

EUROPLEXUS [13] is a computer code for fast explicit transient dynamic analysis of fluid-structure systems jointly developed by the French Commissariat à l’Energie Atomique et aux Energies Alternatives (CEA Saclay) and by the Joint Research Centre of the European Commission (JRC Ispra).

Reference [1] presented the first implementation in EUROPLEXUS of an adaptive mesh refinement and un-refinement procedure, in two space dimensions (element shape QUA4) for solid mechanics. The procedure was extended to fluid mechanics (FE formulation) in 2D in reference [2]. Then, reference [3] applied a similar refinement and un-refinement procedure in three space dimensions to the CUB8 element shape, both in solids mechanics and in fluid mechanics (FE formulation).

All numerical examples presented in references [1-3] with a variable mesh used a so-called “manual” mesh adaptation directive, the \texttt{WAVE} directive (see the code manual in reference [13]), first introduced in reference [1]. This directive refines the mesh along “wavefronts” that are specified by the user, e.g. according to a known analytical solution to the problem considered. This technique was used with success to simulate a bar problem (in solid mechanics) and a shock tube problem (in fluid mechanics) both in 2D and in 3D [1-3].

However, those solutions cannot be qualified as “true” adaptive solutions, because in (true) adaptivity mesh refinement and un-refinement should be completely automatic, based upon suitable error estimators or error indicators. The formulation of error estimators in fast transient dynamics is challenging and is still a subject of research. The use of so-called error indicators, however, is much simpler. For this reason, subsequent work in EUROPLEXUS focused on error indicators. References [4] and [5] document a first prototype implementation of adaptivity based upon error indicators in EUROPLEXUS, limited to 2D problems in continuum and fluid mechanics. An extension of the indicator technique to 3D is under development but has not been completed and documented yet.

Publications [6-7] focus on the natural quantities of interest in goal-oriented error assessment and adaptivity, but limited to the case of linear elasto-dynamics.

The adaptive technique was then applied to Cell-Centred Finite Volumes (CCFV) for the description of the fluid domain, first in 2D (see [8]) and then also in 3D [9]. More recently, the technique has also been extended for use with the CDEM combustion model which makes use of the CCFV formulation [10]. A complete description of the element refinement and un-refinement techniques used in
mesh adaptation has been published in a paper [11]. Finally, reference [12] shows the combination of mesh adaptivity with Fluid-Structure Interaction, i.e. the automatic fluid mesh refinement and un-refinement near a moving and deforming structure.

The present work extends mesh adaptivity to simplex element shapes, i.e. the 3-node triangle (TRI3) in 2D and the 4-node tetrahedron (TET4) in 3D. These elements are useful in fully general unstructured meshing of complex geometries.

This document is organized as follows:

• Section 2 presents the geometrical aspects of adaptivity with TRI3 element shapes in 2D.
• Section 3 presents the geometrical aspects of adaptivity with TET4 element shapes in 3D.
• Section 4 presents some numerical examples in order to validate the proposed formulations.
• Some conclusions are given in Section 5.

The Appendix contains a listing of all the input files mentioned in the present report.
2. Adaptivity for the TRI3

The geometrical aspects of mesh adaptivity for the TRI3 element shape, i.e. the 3-node triangle in 2D, have been developed in close analogy to what had been done in reference [1] for the QUA4 element (4-node quadrilateral).

The refinement (one level only) of a generic triangle is shown in Figure 1 and compared with that of a quadrilateral.

In both cases, four descendents are generated upon refinement of a parent element (indicated as element 0 in the Figure). For the triangle, up to three new nodes are generated (all on the boundary), while for the quadrilateral up to 5 new nodes are generated (four on the boundary and one internal).

![Figure 1 - Adaptive refinement of a QUA4 and of a TRI3](image-url)
As concerns Cell-Centred Finite Volumes, for the quadrilateral four new internal interfaces are always generated, for the triangle three new internal interfaces are always generated. These are indicated by arrows in Figure 2 and numbered in red.

Then for the quadrilateral up to 8 new external interfaces are generated, for the triangle up to 6 new external interfaces are generated (two interfaces for each external face of these elements).

**Figure 2 - Adaptive refinement of a QUA4 and of a TRI3: new internal VFCC interfaces (in red)**

<table>
<thead>
<tr>
<th>Connectivity (face: nodes)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1: 1-2</td>
<td>5: 5-9</td>
<td></td>
</tr>
<tr>
<td>2: 2-3</td>
<td>6: 6-9</td>
<td></td>
</tr>
<tr>
<td>3: 3-4</td>
<td>7: 7-9</td>
<td></td>
</tr>
<tr>
<td>4: 4-1</td>
<td>8: 8-9</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Connectivity (face: nodes)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1: 1-2</td>
<td>4: 5-6</td>
<td></td>
</tr>
<tr>
<td>2: 2-3</td>
<td>5: 6-4</td>
<td></td>
</tr>
<tr>
<td>3: 3-1</td>
<td>6: 4-5</td>
<td></td>
</tr>
</tbody>
</table>
3. Adaptivity for the TET4

The geometrical aspects of mesh adaptivity for the TET4 element shape, i.e. the 4-node tetrahedron in 3D, have been developed in close analogy to what had been done in reference [3] for the CUB8 element (8-node hexahedron).

The refinement (one level only) of a (linear) cube is shown in Figure 3 (from reference [3]) and can be compared with that of the (linear) tetrahedron, which is shown in Figure 4.

In both cases, eight descendent are generated upon refinement of a parent element. For the tetrahedron, up to six new nodes are generated (all on the boundary), while for the hexahedron up to 19 new nodes are generated (eighteen on the boundary and one internal).

Linear tetrahedra in 3D have four nodes, four faces and six corners. The splitting process occurs as shown in Figure 5.

In a first phase, the 0-level element $A$ is split into four sub-tetrahedra, one for each “corner” of the original tetrahedron, plus an octahedron. With the numbering shown in Figure 5, the sub-tetrahedra are: $D_1(1,5,7,8)$, $D_2(5,2,6,9)$, $D_3(7,6,3,10)$ and $D_4(8,9,10,4)$. The octahedron is numbered $(5,6,7,8,9,10)$ and may be viewed as the union of two pyramids: $(5,7,10,9,6)$ and $(5,9,10,7,8)$. The first-step splitting is shown in an “exploded” view in the lower part of Figure 5, where the octahedron is colored in red for clarity. Then, we must obtain more tetrahedra from the octahedron. There are two alternatives: splitting the octahedron in 8 or in 4 tetrahedra. While the first alternative is unique, the second one can be realized in three different ways.

With the first alternative, the second and last phase is as follows. First, the centroid of the quadrilateral $(5,7,10,9)$, i.e. the base of the two pyramids that form the octahedron, is computed. Let this be point 11 in the upper part of Figure 6. This may be obtained as the mid-point of segment $a-b$ or, equivalently, as the mid-point of segment $c-d$, where $a, b, c, d$ are the mid-points of the four sides of the quadrilateral $(5,7,10,9)$, as indicated in the Figure. Finally, from each of the two mentioned pyramids plus the newly computed point 11, we get the eight desired tetrahedra. These are: $D_5(5,8,9,11)$, $D_6(6,9,10,11)$, $D_7(5,6,7,11)$, $D_8(7,10,8,11)$, $D_9(8,10,9,11)$, $D_{10}(5,6,9,11)$, $D_{11}(5,7,8,11)$ and $D_{12}(6,10,7,11)$. These tetrahedra are represented in “exploded” view in the lower part of Figure 6. Thus, with this first strategy altogether the splitting of a linear tetrahedron generates twelve sub-tetrahedra, the four shown in Figure 5 plus the eight shown in Figure 6.

With the second alternative, the second and last phase is as follows. The octahedron is split in four tetrahedra only, instead of eight tetrahedra. First, the three diagonals of the octahedron, 9-7, 5-10 and 6-8 are evaluated and the shortest one is considered (in order to obtain final tetrahedra with the best aspect ratios, i.e. to avoid the appearance of slivers). If the shortest diagonal is 9-7, then the splitting
is as shown in Figure 7 (left). If the shortest diagonal is 5-10, then the splitting is as shown in Figure 7 (right). If the shortest diagonal is 6-8, then the splitting is as shown in Figure 8. Thus, with this strategy altogether the splitting of a linear tetrahedron generates eight sub-tetrahedra, the four shown in Figure 5 plus the four shown in Figure 7 or Figure 8.

The second strategy is preferred because it is likely to lead to better-shaped descendents, although the presence of three alternatives could in principle make the programming more complicated. However, by noting that each of the four “internal” tetrahedra (originating from the octahedron) has one and only one “external” face (lying on the surface of the parent tetrahedron), it is possible to simplify the programming (by making it as similar as possible to that of the CUB8 element) if one numbers and arranges the internal tetrahedra in such a way that the two following conditions are always met (irrespective of the particular case, A, B or C, of the octahedron splitting):

I The (local) index of the descendent must be equal to the local index of the corresponding parent’s face, plus 4.

II The (local) number of the “external” face of the descendent must be equal to the (local) number of the corresponding parent’s face.

With the local numbering shown in Figures 7 and 8 for the three alternative cases (A, B, C) both of the above conditions are satisfied.
Figure 3 - Adaptive refinement of a 8-node hexahedron (CUB8 element shape), from [3]
Figure 4 - Adaptive refinement of a 3D 4-node tetrahedron (TET4 element shape)

Constants

Faces: 1-3-2 ; 1-2-4 ; 1-4-3 ; 2-3-4.
Corners: 1-2 ; 2-3 ; 3-1 ; 4-1 ; 4-2 ; 4-3 = COR2NOD(1:2,1:6).
Faces to corners: 3-2-1 ; 1-5-4 ; 4-6-3 ; 2-6-5 = FAC2COR(1:3,1:4).
Corners to faces: 1-2 ; 1-4 ; 1-3 ; 2-3 ; 4-2 ; 3-4 = COR2FAC(1:2,1:6).
Corners to mid-side nodes: 1-2 ; 2-3 ; 3-1 ; 4-1 ; 4-2 ; 4-3 = COR2DN(1:2,1:6)
(1st entry is descendent index, 2nd entry is descendent’s node).
Faces to mid-face nodes: N/A to tetrahedron.
Corners to descendents: 1-2 ; 2-3 ; 3-1 ; 4-1 ; 4-2 ; 4-3 = COR2DES(1:2,1:6).
Note that, with the above numbering of the tetrahedron, COR2NOD = COR2DN = COR2DES.

Variables

Nodes: 1-4 parent element (4) always reused,
5:10 corners (6) new or reused = NN(1:10).
Small neighbours: “descendent” neighbours (0=none) for each face, first three in the same order as face nodes,
fourth one is “central” neighbor=SNEI(1:4,1:4).
Small neighbours’ faces: SNEI’s face adjacent to this element’s descendent (same org. as SNEI)=SFAC(1:4,1:4).
Descendents D1:D4 are the smaller tetrahedra containing nodes 1:4, respectively. D5:D8 are built from the remaining octahedron. There are three possibilities (A, B, C) depending on the shortest diagonal (9-7, 5-10, 6-8 respectively).

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1=1-5-7-8</td>
<td>D5=5-6-7-9</td>
<td>D5=5-6-7-10</td>
<td>D5=5-6-7-8</td>
</tr>
<tr>
<td>D2=5-2-6-9</td>
<td>D6=8-5-7-9</td>
<td>D6=8-5-10-9</td>
<td>D6=8-5-6-9</td>
</tr>
<tr>
<td>D3=7-6-3-10</td>
<td>D7=8-9-7-10</td>
<td>D7=8-5-7-10</td>
<td>D7=8-6-7-10</td>
</tr>
<tr>
<td>D4=8-9-10-4</td>
<td>D8=7-9-6-10</td>
<td>D8=5-9-6-10</td>
<td>D8=8-9-6-10</td>
</tr>
</tbody>
</table>

Figure 4 - Adaptive refinement of a 3D 4-node tetrahedron (TET4 element shape)
Figure 5 - Splitting of a TET4 element shape, first phase

Exploded view of level 1, first phase
Figure 6 - Splitting of a TET4 element shape, first strategy, second and last phase

Exploded view of level 1, second phase
Faces: 1-3-2 ; 1-2-4 ; 1-4-3 ; 2-3-4.

Alternative A : diagonal 9-7 shorter than 5-10 and 6-8

Alternative B : diagonal 5-10 shorter than 9-7 and 6-8

Figure 7 - Splitting of a TET4 element shape, second strategy, second and last phase
Alternative C: diagonal 6-8 shorter than 5-10 and 9-7

Faces: 1-3-2 ; 1-2-4 ; 1-4-3 ; 2-3-4.

Exploded view of level 1, second phase
As concerns Cell-Centred Finite Volumes, for the hexahedron twelve new internal interfaces are always generated, for the tetrahedron eight new internal interfaces are always generated. The latter are shown in Figures 9, 10, 11 for the three different splittings of the octahedron (case A, B, C, respectively).

Then for the hexahedron up to 24 new external interfaces are generated, for the tetrahedron up to 16 new external interfaces are generated (four interfaces for each external face of these elements).
Figure 9 - Adaptive refinement of a TET4: new internal VFCC interfaces in case A
Figure 10 - Adaptive refinement of a TET4: new internal VFCC interfaces in case B
Figure 11 - Adaptive refinement of a TET4: new internal VFCC interfaces in case C
4. Numerical examples

We present some numerical examples in order to test the algorithms described in the previous Sections.

4.1 Wave propagation in 2D

The first example is that of the propagation of two spherical waves in a 2D quadrangular region by means of the WAVE directive, see Figure 12 and reference [1]. This test verifies only the geometric aspects of adaptivity because no real stress wave is generated in the model.

First, a reference solution is obtained by means of quadrilaterals, see reference [1]. Then, an adaptive solution with triangles is obtained. All performed calculations are summarized in Table 1.

![Figure 12 - Definition of the wave propagation problem](image)

<table>
<thead>
<tr>
<th>Case</th>
<th>Base Mesh</th>
<th>Notes</th>
<th>Steps</th>
<th>CPU [s]</th>
<th>Els*step</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEST10QUA</td>
<td>100 FL24</td>
<td>WAVE 2 SPHE MAXL 4</td>
<td>114</td>
<td>1.92</td>
<td>183,634</td>
</tr>
<tr>
<td>TEST10TRI</td>
<td>200 FL23</td>
<td>WAVE 2 SPHE MAXL 4</td>
<td>182</td>
<td>4.02</td>
<td>587,214</td>
</tr>
</tbody>
</table>

Table 1 - Calculations for the wave propagation problem

**TEST10QUA**

This test uses a very coarse base fluid mesh, of just $10 \times 10$ FL24 quadrilateral fluid elements. The evolution of the adapted mesh is shown in Figure 13.

**TEST10TRI**

This test uses a very coarse base fluid mesh, of just 200 FL23 triangular fluid elements. The evolution of the adapted mesh is shown in Figure 14 and is very similar to the case with quadrilaterals. However, this solution was slightly unstable and a safety coefficient CSTA 0.5 had to be used here.
Figure 13 - Evolution of the adapted mesh for case TEST10QUA
Figure 14 - Evolution of the adapted mesh for case TEST10QUA
4.2 Bar impact in 2D

The next example is that of the impact of an elastic bar on a rigid obstacle. An elastic bar of square unit cross-section and length 100 units impacts at an initial velocity of 100 m/s on a rigid wall. The material is steel-like. The calculation is performed until 40 ms, when rebound of the bar starts.

First, a reference solutions (without adaptivity) are obtained by means of triangles (TRIA element). Then, equivalent solutions with adaptivity are obtained. All performed calculations are summarized in Table 2.

<table>
<thead>
<tr>
<th>Case</th>
<th>Base Mesh</th>
<th>Notes</th>
<th>Steps</th>
<th>CPU [s]</th>
<th>Els*step</th>
</tr>
</thead>
<tbody>
<tr>
<td>BITR01</td>
<td>200 TRIA</td>
<td>No adaptivity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BITR02</td>
<td>200 TRIA</td>
<td>WAVE 2 PLAN MAXL 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BITT03</td>
<td>200 TRIA</td>
<td>WAVE 2 PLAN MAXL 3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2 - Calculations for the bar impact problem in 2D

**BITR01**

This test uses a mesh of just 200 TRIA triangular elements. The solutions is considered as a (rough, because of the coarse mesh) reference, since no adaptivity is used. Figure 16 shows the spatial distribution of the velocity every 10 ms.

**BITE02**

This test is similar to BITR01 but uses adaptivity. Two WAVE directives are specified, the first starting at the right end of the bar and at time 0 (incident wave), when impact occurs, the second starting at the left end of the bar at time 20 ms (reflected wave). A maximum refinement level of MAXL 2 is prescribed (recall that level 1 is the base mesh), so elements are refined just once. Figure 17 shows the spatial distribution of the velocity every 10 ms.
**BITR03**

This test is similar to BITR02 but uses MAXL 3. Figure 18 shows the spatial distribution of the velocity every 10 ms. Figures 19 and 20 compare the three solutions with triangles at 10 ms (incident wave) and at 30 ms (reflected wave), respectively. As the mesh is progressively refined, the solution becomes steeper and the oscillations amplitude diminishes.

*Figure 16 - Spatial distribution of the velocity every 10 ms in case BITR01*
Figure 17 - Spatial distribution of the velocity every 10 ms in case BITR02

Figure 18 - Spatial distribution of the velocity every 10 ms in case BITR03
Figure 19 - Comparison of three solutions at 10 ms (incident wave) with triangles

Figure 20 - Comparison of three solutions at 30 ms (reflected wave) with triangles
4.3 Shock tube in 2D with Finite Elements

The next example is a classical shock tube, similar to those considered in reference [2]. The adaptive mesh refinement is once again piloted by the WAVE directive.

First, a reference solution is obtained by means of a fine mesh of (non-adaptive) triangles. Then, an adaptive solution with triangles is obtained. All performed calculations are summarized in Table 1.

<table>
<thead>
<tr>
<th>Case</th>
<th>Base Mesh</th>
<th>Notes</th>
<th>Steps</th>
<th>CPU [s]</th>
<th>Els*step</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHOT00</td>
<td>12,800 FL23</td>
<td>Non-adaptive fine mesh</td>
<td>1,053</td>
<td>19.1</td>
<td>13,491,200</td>
</tr>
<tr>
<td>SHOT01</td>
<td>200 FL23</td>
<td>WAVE 4 PLAN MAXL 4</td>
<td>1,051</td>
<td>3.12</td>
<td>1,286,716</td>
</tr>
</tbody>
</table>

Table 3 - Calculations for the shock tube problem with Finite Elements

**SHOT00**

This test uses a very fine non-adaptive fluid mesh, of $800 \times 8 \times 2 = 12800$ triangular fluid elements FL23. The solution is not particularly good (i.e. not as regular as with quadrilaterals, but this may happen with triangles since the solution is mesh-sensitive) and is shown in Figures 21 (pressure), 22 (density), 23 (specific internal energy) and 24 (velocity). This is taken as a reference for the subsequent adaptive solution.

**SHOT01**

This solution is adaptive and uses a very coarse base fluid mesh, of just 200 triangular fluid elements FL23. The solution is shown in Figures 21 (pressure), 22 (density), 23 (specific internal energy) and 24 (velocity) and is in relatively good agreement with the reference.
Figure 21 - Pressure in case SHOT00

Figure 22 - Density in case SHOT00
Figure 23 - Specific internal energy in case SHOT00

Figure 24 - Velocity in case SHOT00
Figure 25 - Pressure in case SHOT01

Figure 26 - Density in case SHOT01
Figure 27 - Specific internal energy in case SHOT01

Figure 28 - Velocity in case SHOT01
4.4 Shock tube in 2D with Finite Volumes

The next example is the same shock tube problem considered in Section 4.3, but now solved with the Cell-Centred Finite Volume method, using element T3VF.

First, a reference solution is obtained by means of a fine mesh of (non-adaptive) triangles. Then, an adaptive solution with triangles is obtained. All performed calculations are summarized in Table 4.

<table>
<thead>
<tr>
<th>Case</th>
<th>Base Mesh</th>
<th>Notes</th>
<th>Steps</th>
<th>CPU [s]</th>
<th>Els*step</th>
</tr>
</thead>
<tbody>
<tr>
<td>FVTU00</td>
<td>1,600 T3VF</td>
<td>Non-adaptive fine mesh</td>
<td>1,495</td>
<td>3.65</td>
<td>2,393,600</td>
</tr>
<tr>
<td>FVTU01</td>
<td>200 T3VF</td>
<td>WAVE 4 PLAN MAXL 4</td>
<td>1,492</td>
<td>4.38</td>
<td>1,876,681</td>
</tr>
</tbody>
</table>

Table 4 - Calculations for the shock tube problem with Finite Volumes

**FVTU00**

This test uses a fine non-adaptive fluid mesh, of 1,600 triangular fluid volumes T3VF. The solution is shown in Figures 21 (pressure), 22 (density), 23 (specific internal energy) and 24 (velocity), compared with the analytical solution (in red), with which it is in very good agreement. This is taken as a reference for the subsequent adaptive solution.

**FVTU01**

This solution is adaptive and uses a very coarse base fluid mesh, of just 200 triangular fluid volumes T3VF. The solution is shown in Figures 21 (pressure), 22 (density), 23 (specific internal energy) and 24 (velocity) and is in relatively good agreement with the reference (not shown) and with the analytical solution (in red).
Figure 29 - Pressure in case FVTU00

Figure 30 - Density in case FVTU00
Figure 31 - Velocity in case FVTU00

Figure 32 - Pressure in case FVTU01
Figure 33 - Density in case FVTU01

Figure 34 - Velocity in case FVTU01
4.5 Testing CLxx boundary conditions in 2D

The next example verifies the use of natural boundary conditions by means of CLxx elements in 2D.

The test is similar to case TWAD11 of reference [1]. A bar of constant cross section is loaded at the left end by an applied pressure, represented by a CL22 element with an IMPE PIMP material. The pressure is constant in time, and generates a stress wave in the bar.

All performed calculations are summarized in Table 5.

<table>
<thead>
<tr>
<th>Case</th>
<th>Base Mesh</th>
<th>Notes</th>
<th>Steps</th>
<th>CPU [s]</th>
<th>Els*step</th>
</tr>
</thead>
<tbody>
<tr>
<td>TWAD21</td>
<td>20 TRIA</td>
<td>CL22 WAVE 1 PLAN MAXL 4</td>
<td>228</td>
<td>0.8</td>
<td>42,990</td>
</tr>
<tr>
<td>TWAD22</td>
<td>40 TRIA</td>
<td>CL22 WAVE 1 PLAN MAXL 4</td>
<td>323</td>
<td>1.4</td>
<td>121,002</td>
</tr>
</tbody>
</table>

Table 5 - Calculations for the pressure-loaded bar in 2D

TWAD21

This test uses a very coarse base mesh, of only 20 triangular elements TRIA, obtained by splitting along a diagonal a mesh of 10 regular quadrangles (thus the triangles mesh is not symmetric). The solution is shown in Figures 35 (displacements), 36 (velocity) and 37 (velocities of two symmetric points in time). The two curves in the last Figure should be superposed, but they show a small lack of symmetry instead. This is thought to be due to the fact that the triangular mesh is not symmetric.

TWAD22

This solution identical to the previous one, but uses a symmetric base mesh of 40 triangles, obtained from a uniform mesh of 10 quadrilaterals by subdividing each quadrilateral into four (rather than just two) triangles, by inserting a central node, by means of the Cast3m procedure pxq42t34.proc. The solution is shown in Figures 38 (displacements), 39 (velocities) and 40 (velocities of two symmetric points in time) and is now perfectly symmetric.
Figure 35 - Displacements in case TWAD21

Figure 36 - Velocities in case TWAD21
Figure 37 - Velocity of two symmetric points in case TWAD21

Figure 38 - Displacements in case TWAD22
Figure 39 - Velocities in case TWAD22

Figure 40 - Velocities of two symmetric points in case TWAD22
4.6 Bar impact in 3D

The first 3D example is that of the impact of an elastic bar on a rigid obstacle. An elastic bar of square unit cross-section and length 100 units impacts at an initial velocity of 100 m/s on a rigid wall. The material is steel-like. The calculation is performed until 40 ms, when rebound of the bar starts.

First, reference solutions (without and with adaptivity) are obtained by means of hexahedra. Then, equivalent solutions with tetrahedra (without and with adaptivity) are obtained. All performed calculations are summarized in Table 6.

![Figure 41 - Definition of the bar impact problem in 3D](image)

First, reference solutions (without and with adaptivity) are obtained by means of hexahedra. Then, equivalent solutions with tetrahedra (without and with adaptivity) are obtained. All performed calculations are summarized in Table 6.

<table>
<thead>
<tr>
<th>Case</th>
<th>Base Mesh</th>
<th>Notes</th>
<th>Steps</th>
<th>CPU [s]</th>
<th>Els*step</th>
</tr>
</thead>
<tbody>
<tr>
<td>BICU01</td>
<td>100 CUBE</td>
<td>No adaptivity</td>
<td>293</td>
<td>0.2</td>
<td>29,400</td>
</tr>
<tr>
<td>BICU02</td>
<td>100 CUBE</td>
<td>WAVE 2 PLAN MAXL 2</td>
<td>585</td>
<td>0.5</td>
<td>96,463</td>
</tr>
<tr>
<td>BICU03</td>
<td>100 CUBE</td>
<td>WAVE 2 PLAN MAXL 3</td>
<td>1,169</td>
<td>2.3</td>
<td>751,634</td>
</tr>
<tr>
<td>BITE01</td>
<td>100 TETR</td>
<td>No adaptivity</td>
<td>511</td>
<td>0.6</td>
<td>614,400</td>
</tr>
<tr>
<td>BITE02</td>
<td>100 TETR</td>
<td>WAVE 2 PLAN MAXL 2</td>
<td>1,013</td>
<td>4.0</td>
<td>2,003,313</td>
</tr>
<tr>
<td>BITE03</td>
<td>100 TETR</td>
<td>WAVE 2 PLAN MAXL 3</td>
<td>2,028</td>
<td>147.6</td>
<td>15,643,835</td>
</tr>
</tbody>
</table>

Table 6 - Calculations for the bar impact problem in 3D

**BICU01**

This test uses a mesh of just 100 CUBE hexahedral elements. The solutions is considered as a (rough, because of the coarse mesh) reference, since no adaptivity is used. Figure 42 shows the spatial distribution of the velocity every 10 ms.

**BICU02**

This test is similar to BICU01 but uses adaptivity. Two WAVE directives are specified, the first starting at the right end of the bar and at time 0 (incident wave), when impact occurs, the second starting...
at the left end of the bar at time 20 ms (reflected wave). A maximum refinement level of \texttt{MAXL} 2 is prescribed (recall that level 1 is the base mesh), so elements are refined just once. Figure 43 shows the spatial distribution of the velocity every 10 ms.

\textbf{BICU03}

This test is similar to BICU02 but uses \texttt{MAXL} 3. Figure 44 shows the spatial distribution of the velocity every 10 ms. Figures 45 and 46 compare the three solutions with hexahedra at 10 ms (incident wave) and at 30 ms (reflected wave), respectively. As the mesh is progressively refined, the solution becomes steeper and the oscillations amplitude diminishes.

\textbf{BITE01}

This test uses a mesh of just 100 TETR tetrahedral elements. The solutions is considered as a (rough, because of the coarse mesh) reference, since no adaptivity is used. Figure 47 shows the spatial distribution of the velocity every 10 ms.

\textbf{BITE02}

This test is similar to BITE01 but uses adaptivity. Two \texttt{WAVE} directives are specified, the first starting at the right end of the bar and at time 0 (incident wave), when impact occurs, the second starting at the left end of the bar at time 20 ms (reflected wave). A maximum refinement level of \texttt{MAXL} 2 is prescribed (recall that level 1 is the base mesh), so elements are refined just once. Figure 48 shows the spatial distribution of the velocity every 10 ms.

\textbf{BITE03}

This test is similar to BITE02 but uses \texttt{MAXL} 3. Figure 49 shows the spatial distribution of the velocity every 10 ms. Figures 50 and 51 compare the three solutions with tetrahedra at 10 ms (incident wave) and at 30 ms (reflected wave), respectively. As the mesh is progressively refined, the solution becomes steeper and the oscillations amplitude diminishes.

\textit{Comparison of solutions with hexahedra and tetrahedra}

Finally, Figures 52 and 53 compare all six solutions (three with hexahedra and three with tetrahedra) at 10 ms (incident wave) and at 30 ms (reflected wave), respectively. Very good agreement is observed.
Figure 42 - Spatial distribution of the velocity every 10 ms in case BICU01

Figure 43 - Spatial distribution of the velocity every 10 ms in case BICU02
Figure 44 - Spatial distribution of the velocity every 10 ms in case BICU03

Figure 45 - Comparison of three solutions at 10 ms (incident wave) with hexahedra
Figure 46 - Comparison of three solutions at 30 ms (reflected wave) with hexahedra

Figure 47 - Spatial distribution of the velocity every 10 ms in case BITE01
Figure 48 - Spatial distribution of the velocity every 10 ms in case BITE02

Figure 49 - Spatial distribution of the velocity every 10 ms in case BITE03
Figure 50 - Comparison of three solutions at 10 ms (incident wave) with tetrahedra

Figure 51 - Comparison of three solutions at 30 ms (reflected wave) with tetrahedra
Figure 52 - Comparison of all six solutions at 10 ms (incident wave)

Figure 53 - Comparison of all six solutions at 30 ms (reflected wave)
4.7 Shock tube in 3D with Finite Elements

The next example is a classical shock tube, similar to those considered in reference [3]. The adaptive mesh refinement is once again piloted by the \texttt{WAVE} directive.

Two versions of the FL34 element (JRC’s 4-node fluid tetrahedron) are available in the code. The old version (before 2007) can be selected by specifying the \texttt{OPTI F34} option, but its use is discouraged and is retained only to repeat some old tests. The new version (after 2007) is now by default (or can be chosen explicitly by the \texttt{OPTI NF34} option). Only the new version of the element can be subjected to adaptivity.

The FL34 faces had a different numbering from those of TETR and TEVF, prior to the present work: faces 2 and 4 were interchanged. In order to simplify programming, the face numbering of FL34 has been changed and made identical to that of TETR and TEVF.

First, a reference solution is obtained by means of a fine mesh of (non-adaptive) tetrahedra. Then, an adaptive solution with tetrahedra is obtained. All performed calculations are summarized in Table 7.

<table>
<thead>
<tr>
<th>Case</th>
<th>Base Mesh</th>
<th>Notes</th>
<th>Steps</th>
<th>CPU [s]</th>
<th>Els*step</th>
</tr>
</thead>
<tbody>
<tr>
<td>TES411</td>
<td>1,200 FL34</td>
<td>Non-adaptive mesh ALF0 0.5 BET0 0.2</td>
<td>522</td>
<td>1.3</td>
<td>627,600</td>
</tr>
<tr>
<td>TES412</td>
<td>1,200 FL34</td>
<td>Non-adaptive mesh ALF0 1 BET0 1</td>
<td>500</td>
<td>1.1</td>
<td>601,200</td>
</tr>
<tr>
<td>TES413</td>
<td>1,200 FL34</td>
<td>WAVE 4 PLAN MAXL 2 ALF0 1 BET0 1</td>
<td>1,473</td>
<td>8.3</td>
<td>2,907,070</td>
</tr>
<tr>
<td>TES418</td>
<td>1,200 FL34</td>
<td>WAVE 4 PLAN MAXL 2 ALF0 0.5 BET0 0.2</td>
<td>[475]</td>
<td>[2.7]</td>
<td>[908,768]</td>
</tr>
</tbody>
</table>

Table 7 - Calculations for the shock tube problem in 3D with Finite Elements

**TES411**

This test uses a relatively fine non-adaptive fluid mesh, of $100 \times 12 = 1200$ tetrahedral fluid elements FL34, obtained from a uniform mesh of 100 hexahedra, each of which is split into 12 tetrahedra by the \texttt{pxhex2te.proc} procedure in Cast3m. The upwinding parameters are chosen as follows: ALF0 0.5 BET0 0.2, leading to a reasonably sharp solution with few oscillations. The solution is shown in Figures 54 (pressure), 55 (density) and 56 (velocity). The solution is not very precise, as can be seen by the comparison with the analytical curves. Better agreement can be obtained by even smaller values of the upwinding, and by using also a smaller artificial viscosity. The solution with tetrahedra seems to be very sensitive to all these parameters, even more than solutions with other element types.
This solution is identical to case TES411 but uses ALF0 1 BET0 1 in order to get a smoother solution. The result is indeed quite smooth, but not in good agreement with the analytical solution, see Figures 57, 58 and 59.

This solution is identical to case TES412 but uses adaptivity (four WAVE directives like in previous examples) with MAXL 2 (just one level of mesh refinement). The result is stable but very poor, see Figures 60, 61 and 62. This cannot be considered a satisfactory solution. The problem should be investigated. Maybe there is already a problem in the basic (non-adaptive) FL34 element, which becomes worse when adaptivity is applied.

This solution is an adaptive solution like in case TES413 but with the upwind parameters of case TES411. The solution is unstable. High velocities build up and the calculation stops after 0.14 ms instead of the final 0.6 ms.

The above solutions are potentially non-symmetric because of the subdivision of hexahedra into 12 tetrahedra to generate the base mesh (procedure pxhex2te.proc). In order to check this aspect, an alternative procedure pxhex2t2.proc has been written that splits each hexahedron into 24 tetrahedra, by adding not only a central node into the element volume, but also a central node in each element face, thus resulting into a symmetric base mesh. However, solutions with this mesh (not presented here for brevity) have shown the same instability problems as the ones with the unsymmetrical mesh.

As a result of these tests, the use of FL34 in adaptivity is not advisable until problems will have been solved. It is much better to use the VFCC element TEVF, which seems to give very good results both without and with adaptivity.
Figure 54 - Pressure in case TES411

Figure 55 - Density in case TES411
Figure 56 - Velocity in case TES411

Figure 57 - Pressure in case TES412
Figure 58 - Density in case TES412

Figure 59 - Velocity in case TES412
Figure 60 - Pressure in case TES413

Figure 61 - Density in case TES413
4.8 Shock tube in 3D with Finite Volumes

The next example is the same shock tube problem considered in Section 4.7, but now solved with the Cell-Centred Finite Volume method, using the tetrahedron element TEVF. The problem formulation is taken from reference [9].

First, the problem is solved with CCFV hexahedra, element CUVF, without and with mesh adaptivity, for ease of comparison. Then, a reference tetrahedral solution is obtained by means of a fine mesh of (non-adaptive) tetrahedra. Finally, an adaptive solution with tetrahedra is obtained. All performed calculations are summarized in Table 8.

<table>
<thead>
<tr>
<th>Case</th>
<th>Base Mesh</th>
<th>Notes</th>
<th>Steps</th>
<th>CPU [s]</th>
<th>Els*step</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUSH01</td>
<td>100 CUVF</td>
<td>Non-adaptive mesh, first-order in space and time</td>
<td>119</td>
<td>0.1</td>
<td>12,000</td>
</tr>
<tr>
<td>CUSH07</td>
<td>100 CUVF</td>
<td>WAVE 4 PLAN MAXL 3, first-order in space and time</td>
<td>412</td>
<td>1.1</td>
<td>262,696</td>
</tr>
</tbody>
</table>

Table 8 - Calculations for the shock tube problem in 3D with Finite Volumes
This test uses a relatively coarse non-adaptive fluid mesh, of 100 cubic fluid volumes CUVF. A first-order formulation in both space and time is assumed. The solution is shown in Figures 63 (pressure), 64 (density) and 65 (velocity), compared with the analytical solution (in red), with which it is in good agreement (for the coarse mesh chosen). This is taken as a reference for the subsequent solutions.

This solution is adaptive and uses the same base fluid mesh as case CUSH01, of just 100 hexahedral fluid volumes CUVF. Four WAVE directives with MAXL 3 are used to drive mesh refinement and unrefinement. The solution (in green) is shown in Figures 66 (pressure), 67 (density) and 68 (velocity) and is in good agreement with the reference (in black) and with the analytical solution (in red).

This test is identical to CUSH01 but uses a second-order formulation in space and time. The solution is shown in Figures 69 (pressure), 70 (density) and 71 (velocity), compared with the analytical solution (in red), with which it is in very good agreement (for the coarse mesh chosen). The solution is more accurate than the first-order solution, as expected.

This solution is similar to CUSH07 but uses a second-order formulation in space and time. The solution is shown in Figures 72 (pressure), 73 (density) and 74 (velocity), compared with the analytical solution (in red), with which it is in very good agreement (for the coarse mesh chosen).

<table>
<thead>
<tr>
<th>Case</th>
<th>Base Mesh</th>
<th>Notes</th>
<th>Steps</th>
<th>CPU [s]</th>
<th>Els*step</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUSH11</td>
<td>100 CUVF</td>
<td>Non-adaptive mesh, second-order in space and time</td>
<td>119</td>
<td>0.2</td>
<td>12,000</td>
</tr>
<tr>
<td>CUSH17</td>
<td>100 CUVF</td>
<td>WAVE 4 PLAN MAXL 3, second-order in space and time</td>
<td>417</td>
<td>1.7</td>
<td>265,317</td>
</tr>
<tr>
<td>TESH01</td>
<td>100 TEVF</td>
<td>Non-adaptive mesh, first-order in space and time</td>
<td>461</td>
<td>1.0</td>
<td>554,400</td>
</tr>
<tr>
<td>TESH07</td>
<td>100 TEVF</td>
<td>WAVE 4 PLAN MAXL 3, first-order in space and time</td>
<td>1,732</td>
<td>31.1</td>
<td>13,194,822</td>
</tr>
<tr>
<td>TESH11</td>
<td>100 TEVF</td>
<td>Non-adaptive mesh, second-order in space and time</td>
<td>475</td>
<td>2.1</td>
<td>571,200</td>
</tr>
<tr>
<td>TESH17</td>
<td>100 TEVF</td>
<td>WAVE 4 PLAN MAXL 3, second-order in space and time</td>
<td>1,738</td>
<td>58.3</td>
<td>13,219,460</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case</th>
<th>Base Mesh</th>
<th>Notes</th>
<th>Steps</th>
<th>CPU [s]</th>
<th>Els*step</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUSH01</td>
<td>100 CUVF</td>
<td>Non-adaptive mesh, second-order in space and time</td>
<td>119</td>
<td>0.2</td>
<td>12,000</td>
</tr>
<tr>
<td>CUSH17</td>
<td>100 CUVF</td>
<td>WAVE 4 PLAN MAXL 3, second-order in space and time</td>
<td>417</td>
<td>1.7</td>
<td>265,317</td>
</tr>
<tr>
<td>TESH01</td>
<td>100 TEVF</td>
<td>Non-adaptive mesh, first-order in space and time</td>
<td>461</td>
<td>1.0</td>
<td>554,400</td>
</tr>
<tr>
<td>TESH07</td>
<td>100 TEVF</td>
<td>WAVE 4 PLAN MAXL 3, first-order in space and time</td>
<td>1,732</td>
<td>31.1</td>
<td>13,194,822</td>
</tr>
<tr>
<td>TESH11</td>
<td>100 TEVF</td>
<td>Non-adaptive mesh, second-order in space and time</td>
<td>475</td>
<td>2.1</td>
<td>571,200</td>
</tr>
<tr>
<td>TESH17</td>
<td>100 TEVF</td>
<td>WAVE 4 PLAN MAXL 3, second-order in space and time</td>
<td>1,738</td>
<td>58.3</td>
<td>13,219,460</td>
</tr>
</tbody>
</table>

Table 8 - Calculations for the shock tube problem in 3D with Finite Volumes
**TESH01**

This test is similar to CUSH01 but uses tetrahedral finite elements TEVF. The mesh is obtained from the 100-element mesh of case CUSH01 by subdividing each hexahedron into twelve tetrahedra by means of the `pxhex2te.proc` Cast3m procedure listed in Appendix. This (non-adaptive) solution is first-order in space and time and is shown in Figures 75 (pressure), 76 (density) and 77 (velocity), compared with the analytical solution (in red), with which it is in very good agreement (for the coarse mesh chosen).

**TESH07**

This solution is adaptive and uses the same base fluid mesh as case TESH01, of $100 \times 12 = 1200$ tetrahedral fluid volumes TEVF. Four `WAVE` directives with `MAXL 3` are used to drive mesh refinement and unrefinement. The solution (in green) is shown in Figures 78 (pressure), 79 (density) and 80 (velocity) and is in good agreement with the reference (in black) and with the analytical solution (in red).

**TESH11**

This test is similar to TESH01 but is second-order in space and time. The solution is shown in Figures 81 (pressure), 82 (density) and 83 (velocity), compared with the analytical solution (in red), with which it is in very good agreement (for the coarse mesh chosen).

**TESH17**

This test is similar to TESH07 but is second-order in space and time. The solution is shown in Figures 84 (pressure), 85 (density) and 86 (velocity), compared with the analytical solution (in red), with which it is in very good agreement.
Figure 63 - Pressure in case CUSH01

Figure 64 - Density in case CUSH01
Figure 65 - Velocity in case CUSH01

Figure 66 - Pressure in cases CUSH07 and CUSH01
Figure 67 - Density in cases CUSH07 and CUSH01

Figure 68 - Velocity in cases CUSH07 and CUSH01
Figure 69 - Pressure in case CUSH11

Figure 70 - Density in case CUSH11
Figure 71 - Velocity in case CUSH11

Figure 72 - Pressure in cases CUSH17 and CUSH11
Figure 73 - Density in cases CUSH17 and CUSH11

Figure 74 - Velocity in cases CUSH17 and CUSH11
Figure 75 - Pressure in case TESH01

Figure 76 - Density in case TESH01
**Figure 77 - Velocity in case TESH01**

**Figure 78 - Pressure in cases TESH07 and TESH01**
Figure 79 - Density in cases TESH07 and TESH01

Figure 80 - Velocity in cases TESH07 and TESH01
Figure 81 - Pressure in case TESH11

Figure 82 - Density in case TESH11
Figure 83 - Velocity in case TESH11

Figure 84 - Pressure in cases TESH17 and TESH11
Figure 85 - Density in cases TESH17 and TESH11

Figure 86 - Velocity in cases TESH17 and TESH11
4.9 Testing CLxx boundary conditions in 3D

The next example verifies the use of natural boundary conditions by means of CLxx elements in 3D. The test is similar to case TWAD11 of reference [1] and to the 2D test TWAD21/TWAD22 presented above in Section 4.5. A bar of constant cross section is loaded at the left end by an applied pressure, represented by a CL3I element with an IMPE PIMP material. The pressure is constant in time, and generates a stress wave in the bar.

All performed calculations are summarized in Table 9.

<table>
<thead>
<tr>
<th>Case</th>
<th>Base Mesh</th>
<th>Notes</th>
<th>Steps</th>
<th>CPU [s]</th>
<th>Els*step</th>
</tr>
</thead>
<tbody>
<tr>
<td>TWAD31</td>
<td>240 TETR</td>
<td>4 CL3I WAVE 1 PLAN MAXL 2</td>
<td>228</td>
<td>0.8</td>
<td>42,990</td>
</tr>
</tbody>
</table>

Table 9 - Calculations for the pressure-loaded bar in 3D

TWAD31

This test uses a very coarse base mesh, of only 240 tetrahedral elements TETR, obtained by splitting a mesh of 10 regular hexahedra. Each hexahedron is split into 24 tetrahedra by means of the pxhex2t2.proc Gibiane procedure in Cast3m. The mesh is thus symmetric and so should be the solution. The solution is shown in Figures 87 (displacements), 88 (velocity) and 89 (velocities of four symmetric points in time). The waves in the bar are symmetric but not planar. However, this occurs also in a similar test done without adaptivity, so it is thought to be due to the nature of tetrahedra. The solution is indeed perfectly symmetric.
Figure 88 - Velocities in case TWAD31

Figure 89 - Velocity of four symmetric points in case TWAD31
5. Conclusions

In this report the application of mesh adaptivity to simplex elements, the three-node triangle in 2D and the four-node tetrahedron in 3D, has been presented. The formulation is applied to structural (solid) elements as well as to fluid elements, the latter both using Finite Element (FE) and Cell-Centred Finite Volumes (CCFV).

Results for solid elements and for CCFV fluids are very encouraging, as shown in the numerical examples. As concerns fluid FEs, the results for the triangle in 2D (element FL23) are not very accurate, but acceptable. Instead, in 3D the results for the FL34 are very bad. This may be due to problems in the FL34 element itself, which become worse when adaptivity is activated. The subject is under investigation. For the moment, for 3D fluids it is advised to use the TEVF tetrahedron, which gives excellent results.

Among the things that remain to be done, is to allow the possibility of mixing up (contiguous) quadrilaterals and triangles in 2D applications with adaptivity, especially in fluids.
6. References


[12] F. CASADEI, G. VALSAMOS, A. BECCANTINI: “Combination of Mesh Adaptivity with Fluid-

FIN

biteit0a.epx

BITE01A

END

biteit0a.epx

BITE01A

END

biteit02.epx

BITE02

FIN

biteit02.epx

BITE02

FIN

biteit03.epx

BITE03

FIN

biteit03.epx

BITE03

FIN

Page 73
Post-treatment from Alice file

ECHO

REJ ALIC PSC GARD

SORT GRAP

AXES 1.0 'Time [s]'

SCOU 0 'vx_00' VITE COMP 1 T 0.E-3 SANS 1.0 'Abélasses' LECT axe AXIM TERM

SCOU 1 'vx_10' VITE COMP 1 T 10.E-3 SANS 1.0 'Abélasses' LECT axe AXIM TERM

SCOU 2 'vx_20' VITE COMP 1 T 20.E-3 SANS 1.0 'Abélettes' LECT axe AXIM TERM

SCOU 3 'vx_30' VITE COMP 1 T 30.E-3 SANS 1.0 'Abélettes' LECT axe AXIM TERM

SCOU 4 'vx_40' VITE COMP 1 T 40.E-3 SANS 1.0 'Abélettes' LECT axe AXIM TERM

TRA 0 0 0 4 AXES 1.0 'Velocity [m/s]' YZER

COLO ROUG NOIR VERT TURQ ROSE

TRAC 0 1 2 3 4 AXES 1.0 'Velocity [m/s]' YZER

SCOU 1 'vx_00' VITE COMP 1 T 0.E-3 SAXE 1.0 'Abscissa' LECT xaxis TERM

SCOU 2 'vx_10' VITE COMP 1 T 10.E-3 SAXE 1.0 'Abscissa' LECT xaxis TERM

SCOU 3 'vx_20' VITE COMP 1 T 20.E-3 SAXE 1.0 'Abscissa' LECT xaxis TERM

SCOU 4 'vx_30' VITE COMP 1 T 30.E-3 SAXE 1.0 'Abscissa' LECT xaxis TERM

SCOU 0 'vx_00' VITE COMP 1 T 0.E-3 SAXE 1.0 'Abscissa' LECT xaxis TERM

AXTE 1.0 'Time [s]'

SORT GRAP

RESU ALIC PSCR GARD

ECHO

Post-treatment from Alice file

ECHO

REJ ALIC PSC GARD

SORT GRAP

AXES 1.0 'Time [s]'

SCOU 0 'vx_00' VITE COMP 1 T 0.E-3 SANS 1.0 'Abélasses' LECT axe AXIM TERM

SCOU 1 'vx_10' VITE COMP 1 T 10.E-3 SANS 1.0 'Abélettes' LECT axe AXIM TERM

SCOU 2 'vx_20' VITE COMP 1 T 20.E-3 SANS 1.0 'Abélettes' LECT axe AXIM TERM

SCOU 3 'vx_30' VITE COMP 1 T 30.E-3 SANS 1.0 'Abélettes' LECT axe AXIM TERM

SCOU 4 'vx_40' VITE COMP 1 T 40.E-3 SANS 1.0 'Abélettes' LECT axe AXIM TERM

TRA 0 0 0 4 AXES 1.0 'Velocity [m/s]' YZER

COLO ROUG NOIR VERT TURQ ROSE

TRAC 0 1 2 3 4 AXES 1.0 'Velocity [m/s]' YZER

SCOU 1 'vx_00' VITE COMP 1 T 0.E-3 SAXE 1.0 'Abscissa' LECT xaxis TERM

SCOU 0 'vx_00' VITE COMP 1 T 0.E-3 SAXE 1.0 'Abscissa' LECT xaxis TERM

AXTE 1.0 'Time [s]'

SORT GRAP

RESU ALIC PSCR GARD

ECHO
PLAN X 0 Y 0 Z 0 TO 20,20,20
END

COMP COUL VEST LECT bar TERM
BUCG 2 'end' LECT bar TERM COND XT OF 99.9

'naxis' LECT bar TERM
COND LINE X1 0 Y1 0 Z1 200 100 0 TOL 0.01

MATE LINE BD 8000 YOUN 3 ELI WE 0

LECT bar_tri5 TERM

LECT BU CLG LECT bar TERM
INIT VITE 1 100.5 LECT bar DIFF END TERM

ECHO DEPL VITE ACCE FINT FEXT FLIA FDEC CONT ECRO TFRE 10.E-3

FC10 ALICE LECT ALICE 0.8-3

OPTI BAR AUTO NOTE LOG 1
CALC TREC 0 TRED 40.0-3

FIN

ECHO

BTR 03

END

ECHO

BTR 01

END

ECHO
**cush01.dgibi**

**cush01.epx**

**cush07.dgibi**

**cush07.epx**
TERM  
DIME  
EULE TRID  
CAST mesh  
CUSH17  
cush17.epx  
fin;  
trac qual cach mesh;  
tass mesh;  
mesh = bar;  
base = dall c1 c2 c3 c4 plan;  
c 4=p 3d1p 0 ;  
c 2=p 1d1p 2 ;  
c 1=p 0d1p 1 ;  
p2 = 0 dd dd;  
p1 = 0 dd 0;  
n = 100;  
dd = 0.01;  
*=================================================================
ENDPLAY  
TRAC OFFS SIZE 1400 400 FICH BMP REND  
SCEN GEOM NAVI FREE  
TRAC OFFS SIZE 1400 400 FICH BMP REND  
SLER CAM1 1 NFRA 1  
SCEN GEOM NAVI FREE  
CAME 1 EYE 5.00000E-01 5.00000E-03 3.00015E-01  
SORT VISU NSTO 61  
OPTI PRIN  
RESU ALIC GARD PSCR  
SUIT  
*=================================================================
LIST 3 6 AXES 1.0 'VELOC. [M/S]'  
TRAC 3 6 AXES 1.0 'VELOC. [M/S]'  
COLO NOIR NOIR  
TRAC 2 5 AXES 1.0 'DENS. [KG/M3]'  
COLO NOIR NOIR  
COUR 6 'vx_eb' VCVI COMP 1 ELEM LECT eb TERM  
COUR 4 'p_eb' ECRO COMP 1 ELEM LECT eb TERM  
COUR 2 'p_ea' ECRO COMP 2 ELEM LECT ea TERM  
AXTE 1.0 'Time [s]'  
PERF 'cush11t.pun'  
SORT GRAP  
ECHO  
Post-treatment (time curves from alice temps file)  
*=================================================================
*  
COLO NOIR ROUG  
LIST 62 AXES 1.0 'DENS. [KG/M3]'  
LIST 61 AXES 1.0 'PRESS. [PA]'  
COLO NOIR ROUG  
DCOU 75 'v_ana' SHTU GAMM 1.5 ROM 10 ROP 1 EINT 2.0E5 LENM 0.5 LENP 0.5  
DCOU 71 'p_ana' SHTU GAMM 1.5 ROM 10 ROP 1 EINT 2.0E5 LENM 0.5 LENP 0.5  
COLO PAPE  
ISO FILL FIEL VCVI SCAL USER PROG 20 PAS 20 280 TERM  
ECRO COMP 2 LECT eb TERM REFE 1.92754E+0 TOLE 2.E-2  
ECRO COMP 2 LECT ea TERM REFE 9.99419E+0 TOLE 2.E-2  
ECRO COMP 1 LECT eb TERM REFE 2.81236E+5 TOLE 2.E-2  
VCVI COMP 1 LECT eb TERM REFE 2.95348E+2 TOLE 2.E-2  
FOV 5.25000E+01  
UP 0.00000E+00 1.00000E+00 0.00000E+00  
RIGH 1.00000E+00 0.00000E+00 0.00000E+00  
VIEW 0.00000E+00 0.00000E+00 -1.00000E+00  
TIME 0.60E-3 NRAR 30 VARI 5  
TIME 0.60E-3 NRAR 30 VARI 2  
TIME 0.60E-3 NRAR 30 VARI 1  
VCVI COMP 1  
*=================================================================
LIST 3 6 AXES 1.0 'VELOC. [M/S]'  
LIST 2 5 AXES 1.0 'DENS. [KG/M3]'  
LIST 1 4 AXES 1.0 'PRESS. [PA]'  
COLO NOIR NOIR VERT VERT  
TRAC 3 6 13 16 AXES 1.0 'VELOC. [M/S]'  
COLO NOIR NOIR VERT VERT  
TRAC 2 5 12 15 AXES 1.0 'DENS. [KG/M3]'  
RCOU 16 'vx_eb' FICH 'cush11t.pun' RENA 'vx_eb_11'  
RCOU 15 'ro_eb' FICH 'cush11t.pun' RENA 'ro_eb_11'  
RCOU 14 'p_eb' FICH 'cush11t.pun' RENA 'p_eb_11'  
RCOU 12 'ro_ea' FICH 'cush11t.pun' RENA 'ro_ea_11'  
RCOU 11 'p_ea' FICH 'cush11t.pun' RENA 'p_ea_11'  
COUR 5 'ro_eb' ECRO COMP 2 ELEM LECT eb TERM  
COUR 4 'p_eb' ECRO COMP 1 ELEM LECT eb TERM  
COUR 2 'ro_ea' ECRO COMP 2 ELEM LECT ea TERM  
RESU ALIC TEMP GARD PSCR  
ECHO  
SUIT  
*=================================================================
LIST 65 AXES 1.0 'VELOC. [M/S]'  
COLO NOIR VERT ROUG  
LIST 62 AXES 1.0 'DENS. [KG/M3]'  
COLO NOIR VERT ROUG  
TRAC 12 62 72 AXES 1.0 'DENS. [KG/M3]'  
LIST 61 AXES 1.0 'PRESS. [PA]'  
RCOU 15 'vx_61' FICH 'cush11.pun' RENA 'vx_61_11'  
RCOU 12 'ro_61' FICH 'cush11.pun' RENA 'ro_61_11'  
RCOU 11 'p_61' FICH 'cush11.pun' RENA 'p_61_11'  
DCOU 75 'v_ana' SHTU GAMM 1.5 ROM 10 ROP 1 EINT 2.0E5 LENM 0.5 LENP 0.5  
DCOU 71 'p_ana' SHTU GAMM 1.5 ROM 10 ROP 1 EINT 2.0E5 LENM 0.5 LENP 0.5  
SCOU 65 'vx_61' NSTO 61 SAXE 1.0 'curr_abscissa' LECT xaxo TERM  
*=================================================================
SOOT  
Post-treatment (time curves from alice temps file)  
ECRO  
RESU ALIC TEMP GARD SCHC  
SORT GRAP  
PREP 'cush1t.pun'  
ATE 1.0 'Time [s]'  
COUR 1 'p_ea' ECRO COMP 1 ELMN LECT ea TERM  
COUR 2 'ro_ea' ECRO COMP 2 ELMN LECT ea TERM  
COUR 3 'ro_ea' VCVI COMP 3 ELMN LECT ea TERM  
COUR 4 'p_eb' ECRO COMP 3 ELMN LECT eb TERM  
COUR 5 'ro_eb' ECRO COMP 2 ELMN LECT eb TERM  
COUR 6 'vx_eb' VCVI COMP 3 ELMN LECT eb TERM  
TRAC 14 AXES 1.0 'VELOC. [M/S]'  
TRAC 3 AXES 1.0 'DENS. [KG/M3]'  
TRAC 2 AXES 1.0 'PRESS. [PA]'  
TRAC 1 AXES 1.0 'CURV. [M/M]'  
TRAC mesh = bar;  
base = dall c1 c2 c3 c4 plan;  
c 4=p 3d1p 0 ;  
c 2=p 1d1p 2 ;  
c 1=p 0d1p 1 ;  
p2 = 0 dd dd;  
p1 = 0 dd 0;  
n = 100;  
dd = 0.01;  
*=================================================================
ENDPLAY  
TRAC OFFS SIZE 1400 400 FICH BMP REND  
SCEN GEOM NAVI FREE  
TRAC OFFS SIZE 1400 400 FICH BMP REND  
SLER CAM1 1 NFRA 1  
TRAC OFFS SIZE 1400 400 FICH BMP REND  
SCEN GEOM NAVI FREE  
LIST 61 AXES 1.0 'PRESS. [PA]'  
LIST 62 AXES 1.0 'DENS. [KG/M3]'  
COLO NOIR VERT ROUG  
TRAC OFFS SIZE 1400 400 FICH BMP REND  
REPLAY  
FIN  
*=================================================================
VCCY COMP 3  
DCOU 71 'p_ea' SHTU GAMM 1.5 ROM 10 BCP 1 EINT 2.0 DDS LEMN 0.5 LEMP 0.5  
TIME 0.60E-3 HRAR 30 VARI 1  
DCOU 72 'p_ea' SHTU GAMM 1.5 ROM 10 BCP 1 EINT 2.0 DDS LEMN 0.5 LEMP 0.5  
TIME 0.60E-3 HRAR 30 VARI 2  
DCOU 75 'p_ea' SHTU GAMM 1.5 ROM 10 BCP 1 EINT 2.0 DDS LEMN 0.5 LEMP 0.5  
TIME 0.60E-3 HRAR 30 VARI 5  
TRAC 41 AXES 1.0 'PRESS. [PA]'  
TRAC qual cach mesh;  
tass mesh;  
mesh = bar;  
base = dall c1 c2 c3 c4 plan;  
bar = base volu tr n tr(n=dd) 0 1  
weh - bar  
tass mesh;  
auro forme cach mesh;  
fin,
**Material with no elements, then isolated from the rest.**

In this way it is taken into account in the dimension of ECRO in the extension region. One can check that this material does not affect the final result.

---

**CREB**

**PINI 1.2 PREF 1.2 TINI 1.090909090909 KSI0 0.0**

Unburnt region.
SCEN GEOM NAVI FREE
CAME 1 EYE 5.00000E-01 5.00000E-03 3.00015E-01
OPTI PRIN
RESU ALIC GARD PSCR
ECHO
Post treatment (BMPs from alice file)
SUIT

*=================================================================
!COLOR noir roug vert
!COLOR noir roug vert
!TRAC 62 72 22 AXES 1.0 'DENS. [KG/M3]'
!TRAC 61 71 21 AXES 1.0 'PRESS. [PA]'
! RENA 'vx_61_00'
!RCOU 25 'vx_61' FICH 'bm_vfcc_adap_shocktube_2d_00.pun'
! RENA 'ro_61_00'
!RCOU 22 'ro_61' FICH 'bm_vfcc_adap_shocktube_2d_00.pun'
! RENA 'p_61_00'
LIST 65 AXES 1.0 'VELOC. [M/S]'
COLO NOIR ROUG
LIST 62 AXES 1.0 'DENS. [KG/M3]'
COLO NOIR ROUG
LIST 61 AXES 1.0 'PRESS. [PA]'
TRAC 61 71 AXES 1.0 'PRESS. [PA]'
DCOU 75 'v_ana' SHTU GAMM 1.5 ROM 10 ROP 1 EINT 2.0E5 LENM 0.5 LENP 0.5
DCOU 72 'r_ana' SHTU GAMM 1.5 ROM 10 ROP 1 EINT 2.0E5 LENM 0.5 LENP 0.5
SCOU 65 'vx_61' NSTO 61 SAXE 1.0 'curr_abscissa' LECT xaxo TERM
SCOU 62 'ro_61' NSTO 61 SAXE 1.0 'curr_abscissa' LECT xaxo TERM
SCOU 61 'p_61' NSTO 61 SAXE 1.0 'curr_abscissa' LECT xaxo TERM
*
*s'il y a des doutes ...*
* En general, il est toujours une bonne idee de specifier le SUPPort,*
* sur les VFCC)
*
AXTE 1.0 'Time [s]'
!PERF 'bm_vfcc_adap_shocktube_2d_01.pun'
SORT GRAP
COMP NGRO 1 'xaxo' LECT 1 PAS 1 101 TERM
RESU ALIC GARD PSCR
ECHO
Post-treatment (space curves from alice file)
SUIT

*=================================================================
CALC TINI 0. TEND 0.60E-3
!VFCC FCON 12 ! OK
!VFCC FCON 11 ! OK
!VFCC FCON 10 ! OK
!VFCC FCON 9 ! OK
!VFCC FCON 8 ! OK
!VFCC FCON 6 ! OK
!VFCC FCON 3 ! OK
!VFCC FCON 2 VISC 0.75 ! OK
OPTI NOTE STEP IO
ECRI COOR DEPL VITE ACCE FINT FEXT CONT ECRO TFRE 0.3E-3
*
*
pxhex2t2.proc 14 April 2014 10:24 am

ECRO COMP 2 LECT ea TERM REFE 9.95460E+0 TOLE 2.E-2
ECRO COMP 1 LECT eb TERM REFE 2.81384E+5 TOLE 2.E-2
ECRO COMP 1 LECT ea TERM REFE 9.93201E+5 TOLE 2.E-2
FICH ALIC TFRE 1.0E-5
FICH ALIC TEMP FREQ 1
FANT 1.0 LECT bidon TERM
YMAS 0.3 ! Mass fraction if the unburnt region
MMOL 1.0 H0 -4.2 CREA -1.
YMAS 0.7 ! Mass fraction if the unburnt region
CV0 2.5
MMOL 1.0 H0 -4.2 CREA 1.
COMP1
NLHS 1
NESP 2
TMAX 6000.
FOV 5.25000E+01
RIGH 1.00000E+00 0.00000E+00 0.00000E+00
VIEW 0.00000E+00 0.00000E+00 -1.00000E+00
POIN LECT pa pmid pb TERM
TIME 0.60E-3 NRAR 30 VARI 2
TIME 0.60E-3 NRAR 30 VARI 1
VCVI COMP 1
SUPP LECT bar TERM
ECRO COMP 2
ECRO COMP 1
SUPP LECT bar TERM
*
*
* Tie t11 = manu tet4 p6 p7 p12 p9;
* t10 = manu tet4 p7 p3 p12 p9;
* t9 = manu tet4 p3 p2 p12 p9;
* p12 = x12 y12 z12;
*
* y12 = (y2 + y3 + y6 + y7) / 4.0;
* x12 = (x2 + x3 + x6 + x7) / 4.0;
*
* Pyramid # 3
*
* t8 = manu tet4 p1 p5 p11 p9;
* t6 = manu tet4 p6 p2 p11 p9;
* t5 = manu tet4 p2 p1 p11 p9;
* p11 = x11 y11 z11;
*
* y11 = (y1 + y2 + y5 + y6) / 4.0;
*
* Pyramid # 2
*
* t4 = manu tet4 p4 p1 p10 p9;
* t3 = manu tet4 p3 p4 p10 p9;
* t2 = manu tet4 p2 p3 p10 p9;
* t1 = manu tet4 p1 p2 p10 p9;
* p10 = x10 y10 z10;
* z10 = (z1 + z2 + z3 + z4) / 4.0;
* x10 = (x1 + x2 + x3 + x4) / 4.0;
*
* Pyramid # 1
*
* p 9=x 9y 9z 9 ;
* z9 = (z1 + z2 + z3 + z4 + z5 + z6 + z7 + z8) / 8.0;
* y9 = (y1 + y2 + y3 + y4 + y5 + y6 + y7 + y8) / 8.0;
* x9 = (x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8) / 8.0;
* --
* pxhex2t2.proc

pxhex2t2.proc
* Pyramid # 5
* n5 = noeu p5;
* n4 = noeu p4;
* n3 = noeu p3;
* n2 = noeu p2;
* n1 = noeu p1;
* * Pyramid # 4
* nlow = n4; ilow = 1;
* si ( n5 < nlow ) ; nlow = n5; ilow = 2; finsi;
* si ( n6 < nlow ) ; nlow = n6; ilow = 2; finsi;
* si ( ilow ega 1); sinon;
* * Pyramid # 3
* nlow = n5; ilow = 1;
* si ( n6 < nlow ) ; nlow = n6; ilow = 2; finsi;
* si ( n7 < nlow ) ; nlow = n7; ilow = 1; finsi;
* si ( ilow ega 1); sinon;
* * Pyramid # 2
* nlow = n7; ilow = 1;
* si ( n8 < nlow ) ; nlow = n8; ilow = 2; finsi;
* si ( n9 < nlow ) ; nlow = n9; ilow = 2; finsi;
* si ( ilow ega 1); sinon;
* t1 = manu tet4 p1 p2 p3 p4;
* t2 = manu tet4 p1 p2 p5 p6;
* t3 = manu tet4 p1 p2 p7 p8;
* t4 = manu tet4 p1 p2 p9 p10;
* t5 = manu tet4 p11 p12 p13 p14;
* t6 = manu tet4 p11 p12 p15 p16;
* t7 = manu tet4 p11 p12 p17 p18;
* t8 = manu tet4 p11 p12 p19 p20;
* t9 = manu tet4 p11 p12 p21 p22;
* t10 = manu tet4 p11 p12 p23 p24;
* finproc tetr;

pxhex2te.proc

'DEBPROC' pxhex2te hexa*'MAILLAGE';
*
*--------------------------------------------------
* tetr : mesh containing 12 tetrahedra
* -----
* Output :
* hexa : a mesh containing just one hexahedron
* Input:
*------------------------------------------------------------------
* les points ordonnes a partir de P1
* PORDO = objet MAILLAGE de type POI1 (ligne de points) contenant
* Output:
* PLIN = objet MAILLAGE de type POI1 (ligne de points)
* =====
* Input:
* pour ordonner une serie de points PLIN en partant de P1
* *$$PXORDPOI

pxordpoi.proc

'$$$ PXORDPOI
* * pour ordonner une serie de points PLIN en partant de P1
* * Input:
* * Output:
* * $PLIN = objet MAILLAGE de type P011 (ligne de points)
* * $P1 = premier point de la ligne (typ POINT)
* *------------------------------------------------------------------
* $PORDO = objet MAILLAGE de type P011 (ligne de points) contenant
* les points ordonnees a partir de P1
* 'DEBPROC' PXORDPOI $PLIN*'MAILLAGE' P1*'POINT'
* *--------------------------------------------------
* $PORDO-P1;
* $PPA=PLIN 'POIN' 'PROC' $PPA;
* $PLIN= 'DIFF' (($PPA 'ET' $PPA) 'ELEM' 1) $PLIN;
* I=I + 1;
* finproc tetr;

Page 82
**quad0.epx**

**quad01.epx**

**quad02.epx**

**sh3a04.epx**
The shock tube problem is solved twice: discontinuity and the rarefaction wave. The wave speeds are taken adaptivity) but is piloted by the WAVE directive. Several 2D shock tube problem. The element employed is FL24. The results of the two solutions are compared and are in relatively good agreement.
**Tes401.dgibi**

```plaintext
FIN

**Tes401.epx**

```plaintext
FIN

**Tes411.dgibi**

```plaintext
FIN

```
COMP 4 'bar1' LECT bar1 TERM COND XB LT 0.5... (The text continues with various commands and comments related to meshing, element creation, and post-processing.)

```plaintext
mesh = bar1; fin loop1; repe loop1 (nbel bar8); i=0; bar8 = base volu n tran ((n*dd) 0 0); base = dall c1 c2 c3 c4 plan; c 4=p 3d1p 0; c 3=p 2d1p 3; c 2=p 1d1p 2; p3 = 0 0 dd; p2 = 0 dd dd; p1 = 0 dd 0; n = 100; p 0=000; opti sauv form 'tes418.msh'; opti titr 'TES418'; opti dime 3 elem cub8; opti donn 'pxhex2te.proc'; opti echo 1;
```

The document continues with a series of commands related to setting up the simulation, including creating elements, defining materials, and setting up post-processing options. It concludes with a series of notes and comments related to the simulation setup and results.
post-treatment (time curves from alice file)

| FREQ 0 TFRE 1.E-3 |
| SPLI 45 |
| SPLI 44 |
| SPLI 42 |
| SPLI 36 |
| SPLI 35 |
| SPLI 34 |
| SPLI 33 |
| SPLI 32 |
| SPLI 31 |
| SPLI 30 |
| SPLI 27 |
| SPLI 23 |
| SPLI 22 |
| SPLI 18 |
| SPLI 16 |
| SPLI 9 |
| SPLI 4 |
| SPLI 3 |
| SPLI 1 |

| ADAP |
| CALC TINI 0. TEND 1.0E-3 |

| PLAY |
| !ADAP |
| !SPLI 1 |
| !SPLI 2 |
| !SPLI 3 |
| !SPLI 4 |
| !SPLI 5 |
| !SPLI 6 |
| !SPLI 7 |
| !SPLI 8 |
| !SPLI 9 |
| !SPLI 10 |
| !SPLI 11 |
| !SPLI 12 |
| !SPLI 13 |
| !SPLI 14 |
| !SPLI 15 |
| !SPLI 16 |
| !SPLI 17 |
| !SPLI 18 |
| !SPLI 19 |
| !SPLI 20 |
| !SPLI 21 |
| !SPLI 22 |
| !SPLI 23 |
| !SPLI 24 |
| !SPLI 25 |
| !SPLI 26 |
| !SPLI 27 |
| !SPLI 28 |
| !SPLI 29 |
| !SPLI 30 |
| !SPLI 31 |
| !SPLI 32 |
| !SPLI 33 |
| !SPLI 34 |
| !SPLI 35 |
| !SPLI 36 |
| !SPLI 37 |
| !SPLI 38 |
| !SPLI 39 |
| !SPLI 40 |
| !SPLI 41 |
| !SPLI 42 |
| !SPLI 43 |
| !SPLI 44 |

| VELG |
**tesh01.dgibi**

```plaintext
! SPLI 32
! SPLI 33
! SPLI 34
! SPLI 36
! SPLI 37
! SPLI 38
! SPLI 39
! SPLI 40
! SPLI 41
! SPLI 42
! SPLI 43
! SPLI 44
! SPLI 45
! SPLI 46
! SPLI 47
! SPLI 48

*CONV win
ECHO
tes01.epx
fin;
trac qual cach mesh;
sauv form mesh;
mesh = bar;
elim tol bar;
fin loop1;
repe loop1 (nbel bar8);
i=0;
bar8 = base volu n tran ((n*dd) 0 0 0);
base = dall c1 c2 c3 c4 plan;
c4=p 3d1p 0;
c3=p 2d1p 3;
c2=p 1d1p 2;
c1=p 0d1p 1;
p3 = 0 0 dd;
p2 = 0 dd dd;
tol = 0.0001;
n = 100;
tol = 0.001;
p1 = 0 0 0;
p2 = 0 0 0;
p3 = 0 0 dd;
p1 = 0 dd dd;

# QUAL VOUT COMP 1 LECT xa TERM REF 9.87244E+0 TOLE 2.E-2
# QUAL VOUT COMP 1 LECT ya TERM REF 9.75031E+0 TOLE 2.E-2
# QUAL VOUT COMP 1 LECT za TERM REF 1.94570E+0 TOLE 2.E-2
# QUAL VOUT COMP 1 LECT xb TERM REF 2.95611E-0 TOLE 2.E-2
# QUAL VOUT COMP 1 LECT yc TERM REF 2.81434E+0 TOLE 2.E-2
# QUAL VOUT COMP 1 LECT zc TERM REF 9.62835E+0 TOLE 2.E-2
# QUAL VOUT COMP 2 LECT xb TERM REF 2.38935E+0 TOLE 2.E-2
# QUAL VOUT COMP 2 LECT zc TERM REF 9.75031E+0 TOLE 2.E-2
# QUAL VOUT COMP 2 LECT yb TERM REF 1.94570E+0 TOLE 2.E-2
# QUAL VOUT COMP 2 LECT yc TERM REF 2.81434E+0 TOLE 2.E-2
# QUAL VOUT COMP 2 LECT xa TERM REF 2.95611E-0 TOLE 2.E-2

```

**tesh01.epx**

```plaintext
TESHO
ECRO
|CONV win
CAIT mesh
EULE TRID
SCOM VTPF bar TERM
COMP GRID 3 'xaxo' LECT bar TERM COND EXX LT 0.5
'xaxo' LECT bar DPDF bar TERM
exx = LECT bar TERM COND NEAR P0H 0.25 0 0
'xaxo' LECT bar TERM COND NEAR P0H 0.75 0 0
'xaxo' LECT bar TERM COND LINE XI 0 YI 0 ZI 0
X2 1 Y2 0 EZ 0 TOL 1.E-4
CURL ROUG LECT bar TERM
VERLET LECT bar TERM
MATE GASS 25 GAMM 2.5 PRI 1.5 PREP 1.5
LART bar TERM
GASS 1 1 GAPP 1.5 PRI 1.5 PREP 1.5
LERT bar TERM
ECUI COMP DPDF VTPF ACCE PRINT PRTX COMP KVD VTPC TYPE 0.5.E-3
PIC ALDEC TYPE PRE 1
PIC ALDEC TYPE 0.5.E-3
OPTI NODE STEP 20
CSTA 0.5
LOG 1

|VPCF PCOM 1 | OK
|VPCF PCOM 2 VPC 0.75 | OK
|VPCF PCOM 3 | OK
|VPCF PCOM 4 | OK
|VPCF PCOM 5 | OK
|VPCF PCOM 6 | OK
|VPCF PCOM 7 | OK
|VPCF PCOM 8 | OK
|VPCF PCOM 9 | OK

```

**tesh07.dgibi**

```plaintext
opti echo 1;
opti dom 'pahaztxe.proc';
opti dme 3 elem cube;
opti titr 'TESH07';
opti avou form 'tesh07.mesh';
opti trac per fanc 'tesh07_mesh.epx';
p0 = 0 0 0;
tol = 0.0001;
P0H = 0.

```

**tesh03.dgibi**

```plaintext
!VFCC FCON 1 ! OK
!VFCC FCON 2 ! OK
!VFCC FCON 3 ! OK
!VFCC FCON 4 ! OK
!VFCC FCON 5 ! OK
!VFCC FCON 6 ! OK
!VFCC FCON 7 ! OK
!VFCC FCON 8 ! OK
!VFCC FCON 9 ! OK

```

**tesh04.dgibi**

```plaintext
!VFCC FCON 1 ! OK
!VFCC FCON 2 ! OK
!VFCC FCON 3 ! OK
!VFCC FCON 4 ! OK
!VFCC FCON 5 ! OK
!VFCC FCON 6 ! OK
!VFCC FCON 7 ! OK
!VFCC FCON 8 ! OK
!VFCC FCON 9 ! OK
```

**tesh05.dgibi**

```plaintext
!VFCC FCON 1 ! OK
!VFCC FCON 2 ! OK
!VFCC FCON 3 ! OK
!VFCC FCON 4 ! OK
!VFCC FCON 5 ! OK
!VFCC FCON 6 ! OK
!VFCC FCON 7 ! OK
!VFCC FCON 8 ! OK
```

**tesh06.dgibi**

```plaintext
!VFCC FCON 1 ! OK
!VFCC FCON 2 ! OK
!VFCC FCON 3 ! OK
```

**tesh07.dgibi**

```plaintext
!VFCC FCON 1 ! OK
```

**tesh08.dgibi**

```plaintext
!VFCC FCON 1 ! OK
```

**tesh09.dgibi**

```plaintext
!VFCC FCON 1 ! OK
```

**tesh10.dgibi**

```plaintext
!VFCC FCON 1 ! OK
```
tlesh17.epx

cast mesh

dime

acs

tesh17.epx
The graphical production of an animation in the presence of adaptivity is also tested here. The geometrical part of the adaptivity algorithm behaves as expected. and do not induce any stress in the structure. This test verifies only the geometrical aspects of "adaptivity". A square region is subjected to two (user-defined) circular wavefronts that traverse the region. The mesh is refined near the wavefronts and de-refined when the wavefronts have passed. This test verifies mesh refinement and de-refinement ("adaptivity"). A square region is subjected to two (user-defined) circular wavefronts that traverse the region. The mesh is refined near the wavefronts and de-refined when the wavefronts have passed. This test verifies mesh refinement and de-refinement ("adaptivity"). A square region is subjected to two (user-defined) circular wavefronts that traverse the region. The mesh is refined near the wavefronts and de-refined when the wavefronts have passed. This test verifies mesh refinement and de-refinement ("adaptivity"). A square region is subjected to two (user-defined) circular wavefronts that traverse the region. The mesh is refined near the wavefronts and de-refined when the wavefronts have passed. This test verifies mesh refinement and de-refinement ("adaptivity"). A square region is subjected to two (user-defined) circular wavefronts that traverse the region. The mesh is refined near the wavefronts and de-refined when the wavefronts have passed. This test verifies mesh refinement and de-refinement ("adaptivity"). A square region is subjected to two (user-defined) circular wavefronts that traverse the region. The mesh is refined near the wavefronts and de-refined when the wavefronts have passed.

BEGIN DESCRIPTION

TEST10QUA ECHO

END DESCRIPTION

BEGIN DESCRIPTION

FIN

END DESCRIPTION

BEGIN DESCRIPTION

FIN
The geometrical part of the adaptivity algorithm behaves as expected. and do not induce any stress in the structure. are purely geometrical entities specified by the user and is not a true adaptive calculation, since the wavefronts are circular wavefronts that traverse the region. This test verifies only the geometrical aspects of "adaptivity" and the graphical production of an animation in the presence of adaptivity is also tested here. The geometrical part of the adaptivity algorithm behaves as expected. The graphical production of an animation in the presence of adaptivity is also tested here.

BEGIN DESCRIPTION
FIN

END DESCRIPTION

test10triv.epx

TEST10TRIV
ECNO
REV ALIC 'test10triv.ali' GARD PICT
OPTI PRINT
SORT VDD REF0 1

PLAY
CAME 1 EYE 5.00000E+00 5.00000E+00 2.82843E+01

ENDPLAY

trado2.epx

TRADO2
ECNO
CONV win
LAGE DPLA
DME
ADAP NP0I 5 PL23 8 ENHA
TERM
opti dump dpea
GEOM LINK PICT 4 PL23 2 TERM

ENDPLAY

trado3.epx

TRADO3
ECNO
CONV win
LAGE DPLA
DME
ADAP NP0I 20 TRIA 20 CL2D 10 ENDA
TERM
opti dump dpea
GEOM LINK PICT 4 TRIA 2 CL2D 1 TERM

ENDPLAY

trado4.epx

TRADO4
ECNO
**tube21.epx**

**TRAC 3 4 AXES 1.0 'PRES. [PA]'**

**TRAC 1 2 AXES 1.0 'VEL. [M/S]'**

**COUR 4 'p_5' ECRO COMP 1 LECT 5 TERM**

**COUR 3 'p_4' ECRO COMP 1 LECT 4 TERM**

**COUR 2 'vx_6' VITE COMP 1 LECT 6 TERM**

**AXTE 1000.0 'Time [ms]'**

**SORT GRAP**

**RESU ALIC TEMP GARD PSCR**

**ECHO**

**Post-treatment (bande alice temps)**

**SUIT**

**Post-treatment (time curves of surface temps)**

**FIN**

**tube42.epx**

**TIME = 43**

**END**

**CONV win**

**TUBE R11**

**DIME**

**PTIL 24 PLIS 4 PLIS 2 PLIS 2 CLQ 3 ZONE 4**

**NALE 1 BLOC 48 ELCV 50**

**TERM**

**GEOM LIBR POIN 3 TRIA 1 CL2D 10**

**FIN**

**ENDPLAY**

**TRAC REND**

**GO**

**GO**

**TRAC REND**

**GO**

**ADAP SPLI 1 TERM**

**PLAY**

**CALC TINI 0. TEND 50.0E-3 NMAX 3**

**ECRI COOR DEPL VITE ACCE FINT FEXT CONT ECRO FREQ 1**

**LNKS STAT DUMP**

**FICH ALIC TEMP FREQ 1**

**PMIN 0 NUM 1 VXFF 1.0**

**ALF0 1 BET0 1 KINT 0 AHGF 0 CL 0.5 CQ 2.56**

**LIST 4 5 6 AXES 1.0 'VELOC. [M/S]'**

**LIST 1 2 3 AXES 1.0 'DISPL. [M]'**

**FIN**

**tube421.epx**

**TRAC 1 1.0**

**FICH AVI CONT NOCL**

**FIN**

**ENDPLAY**

**GO**

**GOTR LOOP 227 OFFS FICH AVI CONT NOCL REND**

**SCEN GEOM NAVI FREE**

**Q 1.00000E+00 0.00000E+00 0.00000E+00 0.00000E+00**

**CAME 1 EYE 5.00000E-01 5.00000E-02 2.51247E+00**

**PLAY**

**ADAP CHEC**

**OPTI NOTE STEP LIBR**

**FIN**

**LOG 1**

**ECRO**

**RESU TEMP GARD BISCH**

**SORT GRAP**

**ASTE 4.0 'Time [s]'**

**COU 10 'dx_g1' DSEF COMP 1 POIN LECT p1 TERM**

**COU 10 'dx_g2' DSEF COMP 1 POIN LECT pmid TERM**

**COU 10 'dx_g3' DSEF COMP 1 POIN LECT p0 TERM**

**COU 4 'vx_p0' VITE COMP 1 POIN LECT p0 TERM**

**COU 5 'vx_pmid' VITE COMP 1 POIN LECT pmid TERM**

**COU 6 'vx_p1' VITE COMP 1 POIN LECT p1 TERM**

**COU 7 'vx_p0p' VITE COMP 1 POIN LECT p0p TERM**

**KICOU 11 'dx_g0_p' FICH 'twod01.pun' RENA 'dx_g0_p_01'**

**KICOU 12 'dx_g0_pmid' FICH 'twod01.pun' RENA 'dx_g0_pmid_01'**

**KICOU 13 'dx_g0_p1' FICH 'twod01.pun' RENA 'dx_g0_p1_01'**

**KICOU 14 'vx_g0_p' FICH 'twod01.pun' RENA 'vx_g0_p_01'**

**KICOU 15 'vx_g0_pmid' FICH 'twod01.pun' RENA 'vx_g0_pmid_01'**

**KICOU 16 'vx_g0_p1' FICH 'twod01.pun' RENA 'vx_g0_p1_01'**

**KICOU 41 'dx_g0_p' FICH 'twod04.pun' RENA 'dx_g0_p_04'**

**KICOU 42 'dx_g0_pmid' FICH 'twod04.pun' RENA 'dx_g0_pmid_04'**

**KICOU 43 'dx_g0_p1' FICH 'twod04.pun' RENA 'dx_g0_p1_04'**

**KICOU 44 'vx_g0_p' FICH 'twod04.pun' RENA 'vx_g0_p_04'**

**KICOU 45 'vx_g0_pmid' FICH 'twod04.pun' RENA 'vx_g0_pmid_04'**

**KICOU 46 'vx_g0_p1' FICH 'twod04.pun' RENA 'vx_g0_p1_04'**

**TRAC 1 2 AXES 1.0 'DISPL. [M]'**

**TRAC 4 5 AXES 1.0 'VELOC. [M/S]'**

**LIST 1 2 AXES 1.0 'DISPL. [M]'**

**LIST 4 5 AXES 1.0 'VELOC. [M/S]'**

**FIN**
COLO NOIR ROUG

*=================================================================
ECRI COOR DEPL VITE ACCE FINT FEXT CONT ECRO
!COMP EPAI 1.E-2 LECT bar _tria TERM
WAVE 1 PLAN X 0 Y 0 NX 1 NY 0 T0 0 C 5000 MAXL 4 H1 0.15 H2 0.5
TERM
DIME
!CONV win
ECHO
TWAD22
twad22.epx
elim tol (bar et p0p et p1p et pmid et clpr);
fin loop1;
*
bar4 = c1 tran 1 (0 0.1);
c 1=p 0dnp 1 ;
n=1 0 ;
tol = 1.E-5;
p1 = 1 0;
p0p = p0 plus (0 0.1);
p0 = 0 0;
opti trac psc ftra 'twad22_mesh.ps';
opti sauv form 'twad22.msh';
opti echo 1;
FIN
TRAC 1 2 3 11 12 13 AXES 1.0 'DISPL. [M]'
TRAC 4 5 6 14 15 16 AXES 1.0 'VELOC. [M/S]'
TRAC 1 2 3 11 12 13 AXES 1.0 'DISPL. [M]'
TRAC 4 5 6 14 15 16 AXES 1.0 'VELOC. [M/S]'
TRAC 1 2 3 11 12 13 AXES 1.0 'DISPL. [M]'
RCOU 16 'vx_p1' FICH 'onad01.pun' RENA 'vx_p1_1D'
RCOU 15 'vx_pmid' FICH 'onad01.pun' RENA 'vx_pmid_1D'
RCOU 12 'dx_pmid' FICH 'onad01.pun' RENA 'dx_pmid_1D'
RCOU 11 'dx_p0' FICH 'onad01.pun' RENA 'dx_p0_1D'
COUR 5 'vx_pmid' VITE COMP 1 NOEU LECT pmid TERM
COUR 4 'vx_p0' VITE COMP 1 NOEU LECT p0 TERM
COUR 3 'dx_p1' DEPL COMP 1 NOEU LECT p1 TERM
COUR 2 'dx_pmid' DEPL COMP 1 NOEU LECT pmid TERM
COUR 1 'dx_p0' DEPL COMP 1 NOEU LECT p0 TERM
SORT GRAP
ECHO
TWAD11B
twad22b.epx
ENDPLAY
TRAC OFFS FICH AVI CONT REND
GO
GOTR LOOP 322 OFFS FICH AVI CONT NOCL REND
FREQ 1
SLER CAM1 1 NFRA 1
PLAY
*=================================================================
CALC TINI 0. TEND 1.E-3
!ADAP CHEX
CALC THI 0. TEND 1.E-3
*=================================================================
CAME 1 KEY 5.0000E-03 5.0000E-03 2.51247E+00
| 0 1.0000E-03 0.0000E+00 0.0000E+00 0.0000E+00
VIEW 0.0000E-00 0.0000E+00 0.0000E+00 0.0000E+00
RUSHE 0.1000E+00 0.0000E+00 0.0000E+00 0.0000E+00
PVU 2.49999E+01
SCHE GREN NAVI PERS
FACE WWRO
COLOR RARI
SIDE CFO 1 REP 1
TRAC OFFS FICH AVI CONT MOCL REND
GO
SUIT
Post-treatment (time curves from alice temps file)
END
COLO ALIC TEMP GAND SCHR
SORT GRAP
AXTE 1.0 'Time [s]'
```plaintext
FIN
*=================================================================
ECRI COOR DEPL VITE ACCE FINT FEXT CONT ECRO
MATE VM23 RO 8000. YOUNG 2.E11 NU 0.0 ELAS 2.E11
COMP COUL VERT LECT bar TERM
GEOM TETR bar CL3I clpr TERM
CAST mesh
!CONV win
ECHO
TWAD30
twad30.epx
fin;
trac qual clpr;
trac qual cach mesh;
sauv form mesh;
mesh = bar et clpr;
cl4 = manu tri3 p0p pc p0;
cl3 = manu tri3 p2p pc p0p;
cl2 = manu tri3 p2 pc p2p;
cl1 = manu tri3 p0 pc p2;
pc = 0 0.05 0.05;
*
fin loop1;
repe loop1 (nbel bar8);
i=0
*
elim tol (bar8 et p0p et p1p et p2 et p2p et p3 et p3p);
base = c1 tran 1 (0 0.1 0);
n=1
tol = 1.E-5;
p3p = p3 plus (0 0.1 0);
p3 = 1 0 0.1;
p2p = p2 plus (0 0.1 0);
p2 = 0 0 0.1;
p1p = p1 plus (0 0.1 0);
p1 = 1 0 0;
pmid = 0.5 0 0;
p0p = p0 plus (0 0.1 0);
p 0 = 0 0 0;
opti trac psc ftra 'twad30_mesh.ps';
opti sauv form 'twad30.msh';
opti dime 3 elem cub8;
opti donn 'pxhex2t2.proc';
opti echo 1;
twad30.dgibi

! Q 1.00000E+00 0.00000E+00 0.00000E+00 0.00000E+00
PLAY
*=================================================================
CALC TINI 0. TEND 1.E-3
OPTI NOTE STEP LIBR
COUR 9 'vx_p2p' VITE COMP 1 NOEU LECT p2p TERM
COUR 8 'vx_p2' VITE COMP 1 NOEU LECT p2 TERM
COUR 6 'vx_p1' VITE COMP 1 NOEU LECT p1 TERM
COUR 5 'vx_pmid' VITE COMP 1 NOEU LECT pmid TERM
COUR 4 'vx_p0' VITE COMP 1 NOEU LECT p0 TERM
COUR 3 'dx_p1' DEPL COMP 1 NOEU LECT p1 TERM
COUR 2 'dx_pmid' DEPL COMP 1 NOEU LECT pmid TERM
SORT GRAP
RESU ALIC TEMP GARD PSCR
ECHO
Post-treatment (time curves from alice temps file)
SUIT
*=================================================================
TRAC OFFS FICH AVI CONT REND
GO
GOTR LOOP 176 OFFS FICH AVI CONT NOCL REND
FREQ 1
TRAC OFFS FICH AVI NOCL NFTO 178 FPS 25 KFRE 10 COMP -1 REND
SLER CAM1 1 NFRA 1
SCEN GEOM NAVI FREE
ADAP DUMP CHEC
LOG 1 DPMA LNKS STAT
COLO noir noir noir roug roug roug
XMIN 0.E0 XMAX 6.E-4 NX 10
finsi;
sinon;
si (ega i 1);
tetr = pxhex2t2 hi;
hi = bar8 elem i;
i=i+1
bar = bar et tetr;
bar = tetr;
twad31.epx
fin;
trac qual cach mesh;
trac qual clpr;
fin;
```

**Post-treatment (time curves from alice temps file)**

**SUITE**

*=================================================================

**ENDPLAY**
### List of input files

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<thead>
<tr>
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Abstract

This report is a sequel to reports and publications on mesh adaptivity in fast transient dynamics and presents the formulation and implementation of mesh adaptivity for simplex elements (triangles in 2D, tetrahedra in 3D) in fast transient dynamics. The algorithms are implemented in the EUROPLEXUS code.

The present work extends mesh adaptivity to simplex element shapes, i.e. the 3-node triangle (TRI3) in 2D and the 4-node tetrahedron (TET4) in 3D. These elements are useful in fully general unstructured meshing of complex geometries.
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