European Commission

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FIDELIO 2: Overview and theoretical foundations of the second version of the Fully Interregional Dynamic Econometric Long-term Input-Output model for the EU-27

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2017

Joint Research Centre

EUR 28503 EN

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JRC Science Hub
https://ec.europa.eu/jrc

JRC105900

EUR 28503 EN

PDF	ISBN 978-92-79-66258-4	ISSN 1831-9424	doi:10.2760/313390

Luxembourg: Publications Office of the European Union, 2017

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How to cite this report: Kratena et al. (2017). *FIDELIO 2: Overview and theoretical foundations of the second version of the Fully Interregional Dynamic Econometric Long-term Input-Output model for the EU-27*, JRC Technical Report JRC105900, EUR 28503 EN, doi:10.2760/313390

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Preface

This report serves as an update of "FIDELIO 1: Fully Interregional Dynamic Econometric Long-term Input-Output model for the EU27" by Kratena et al. (2013), i.e. the manual of the first version of the FIDELIO model. FIDELIO fits into the generation of macroeconomic multi-sectoral input-output (IO) models whose earliest contributions include the Cambridge MDM (Barker, 1976) and the INFORUM (Almon et al., 1974) models for the UK and the US, respectively. Such econometric IO models have grown over time in terms of complexity and scope and are used for macroeconomic modelling purposes alongside other types of general equilibrium models (including DSGE ones).

This report explores the theoretical foundations of the latest version of the model, FIDELIO 2 (which has being developed between 2014 and 2016), and contains a description of its main features. With respect to its initial version, the model has been extended in a number of ways. For instance, and without entering into detail at this stage, seven non-EU countries are now included in the model (Brazil, China, India, Japan, Russia, Turkey and the USA) in addition to the 27 EU countries already included in the first version; both trade and household final demand are now modelled in a considerably more complex way than before; there is an environmental block dealing with greenhouse gas (GHG) emissions; and the base year is 2007 rather than 2005. Thus, it was deemed necessary to present all the new model characteristics in an organic way via the present technical report. The remainder of this report is organised as follows: Section 1 provides a concise macro-overview of FIDELIO 2 which relies very much on the first section of the FIDELIO 1 manual by Kratena et al. (2013). Section 2 presents the economic theories underlying the core blocks of FIDELIO 2.

This report serves two main purposes. First, it is an adequate resource for the readers who are interested in the model's main features. Second, it facilitates the process of understanding all the details of FIDELIO 2 for those who want to learn the logic and the theory behind its construction. Such readers are expected to grasp the general structure of the model by reading Section 1, helped by the overview of the model's main economic flows contained in Figure 1. Then, Section 2 goes through the theoretical foundations of the various model blocks.

1. Macro-overview of FIDELIO 2

Figure 1 illustrates the main economic flows of FIDELIO 2. The flows represented in the figure are expressed in nominal terms (monetary transactions), and not in real terms (quantities). Real quantities are derived by dividing those flows by the corresponding prices which will be discussed below. The discussion explains the main economic flows illustrated in Figure 1, to be used as a reference in order to understand the logic behind the explanations below.

1.1. The main economic flows in FIDELIO 2

FIDELIO 2 models a set of users constituted by the 59 industries and by all the final demand categories (private consumption, public consumption, non-profit sector servicing households -NPISH-, investment, inventories, and exports). This structure is taken from the use matrix of the input-output (IO) Supply and Use Tables[1] and the World Input-Output Database (WIOD) which constitute the modelling starting point as well as the sources of a large part of the data used in FIDELIO 2. There are 59 goods in FIDELIO 2 which do not completely overlap with the 59 industries, exactly as in IO tables: for example, industry 1 may produce good 1 as well as other goods.

Let us now illustrate the main economic flows modelled in FIDELIO 2. It is worth starting from the second rectangle in the top row of Figure 1, i.e. demand for domestic goods. It represents the demand by user *u* for good *g* which is domestically produced in region *r* expressed in basic prices (see Section 1.2 for more details on the prices' structure). The supply of goods, which is essentially gross output, is derived from that demand and it is defined by region *r* and by sector *s* (the first rectangle of the top row of Figure 1). By assuming constant market proportions, the shares of industries' outputs in the production of each good for all simulation years are assumed to remain fixed at their base-year levels.

The implication of the choice of what drives the production of goods makes FIDELIO 2 a demand-driven model. In fact, there is a reminder of this in the very name of the model: the IO label refers not only to the fact that a large part of the data employed in the model come from the IO Supply and Use Tables and the WIOD, but also to the standard IO quantity model

[1] Symmetrical IO tables (where the number of industries is equal to the number of products) are obtained from Supply and Use Tables using assumptions about the technology of industries and products for the secondary outputs of industries (Ten Raa and Rueda-Cantuche, 2007).

by Leontief (1936, 1941) which is inherently a demand-driven model.[2] FIDELIO 2 contains some of the features pertaining to that model (in fact, non-EU countries are modelled almost like in a standard IO model, without many of the features with which EU ones are modelled), but, as should become evident by the end of this report, FIDELIO 2 is a more powerful and flexible (hence, realistic) model for policy impact assessment purposes. This is because:

1. FIDELIO 2 features various flexible functions such as translog cost functions and an Almost Ideal Demand System (AIDS) which have well-established theoretical foundations;
2. FIDELIO 2 models theory-based direct and indirect links between prices and quantities, which are entirely separate within the traditional IO framework;
3. the reaction of prices and quantities is two-sided: not only quantities react to prices in the demand systems, but, for example, the factor price reacts to quantity changes in its demand;
4. while prices in the IO price model are identical for all intermediate and final users, in FIDELIO 2 prices are user-specific due to its proper accounting of margins, taxes and subsidies, and import shares which are different for each user;
5. final demand categories as well as value added components in FIDELIO 2 are endogenous, while in the IO quantity framework they are set exogenously.

As a result, FIDELIO 2 shows some similarities with CGE models. The main one is that, similarly to the equilibrating function of the price mechanism in CGE models, in the FIDELIO 2 labour market there is a feedback from quantities (such as the unemployment rate) to prices, so that the balance between supply and demand in the labour market in a certain sense determines its price. Another similarity lies in the balance of the public sector acting as a macroeconomic constraint on the economy (the so-called macroeconomic closure rules of CGE modelling). In FIDELIO 2, the public sector function is specified as a dynamic constraint in terms of public finance aggregate targets.

It is important to note, however, that FIDELIO 2 also deviates from specifications in CGE models in some important aspects. On the supply side there is a translog production function, and the price of output depends on the relative prices of the various inputs used to

[2] This is the reason why the IO quantity model is also called the demand-pull input-output quantity model. On the other hand, in the standard IO price model (which is independent from the IO quantity model) changes in prices are driven by changes in the value added per output (e.g. wage rates). Therefore, the price model is often referred to as the cost-push input-output price model (for details, see Miller and Blair, 2009).

produce the goods and services featured in the model. Output is supplied by firms with constant returns to scale. Although supplied quantities are determined by the demand side, supply-side aspects come into play since cost factors and prices are related to the level of demand.

As stated by Kratena and Streicher (2009), the differences between econometric IO and CGE modelling have often been exaggerated and can in many cases be reduced to differences in the macroeconomic closure rules of the models. This view can be upheld when it comes to the differentiation of FIDELIO 2 from a dynamic CGE model, like the Intertemporal General Equilibrium Model (IGEM) for the US economy (Goettle et al., 2009).[3] In CGE models, the changes in prices ensure that there is equilibrium in all markets. In FIDELIO 2, there it is no guarantee that equilibrium will be reached in all markets, as markets are modelled according to behavioural functions and relationships based on empirical regularities. For instance, in the labour market, the price of labour has an equilibrating function in FIDELIO 2, though not in the strict sense of the competitive labour market used in many standard CGE models. Rather, the wage rate reacts to the unemployment rate according to the 'wage curve' concept, thereby introducing feedback mechanisms of the price to the balance of supply and demand (Blanchflower and Oswald, 1994).

In general, the base-year data used in CGE and other modelling frameworks do not necessarily represent the state of equilibrium of the economy. That is why in FIDELIO 2 there is nothing ensuring that, for instance, the unemployment rate of the base year is the equilibrium unemployment rate towards which the economy tends to move. Rather, equilibrium in FIDELIO 2 results from the interaction between the demand functions at all levels of users and all types of goods and factor inputs with the corresponding supply that is determined under the restrictions applying to the factor markets. The latter are mainly represented by an exogenous benchmark interest rate and by the presence of liquidity constraints (affecting the input of capital), and by the institution of union-wage bargaining at the industry level (affecting labour). The current account balances are not fixed, as household savings are determined within the buffer-stock model of consumption which also takes into account household wealth. Both current and lagged household income affects private consumption, and there are lending constraints in the capital market according to which the purchase of consumer durables cannot be fully financed by borrowing.

[3] The complete description and applications of the IGEM are available at: http://www.igem.insightworks.com/.

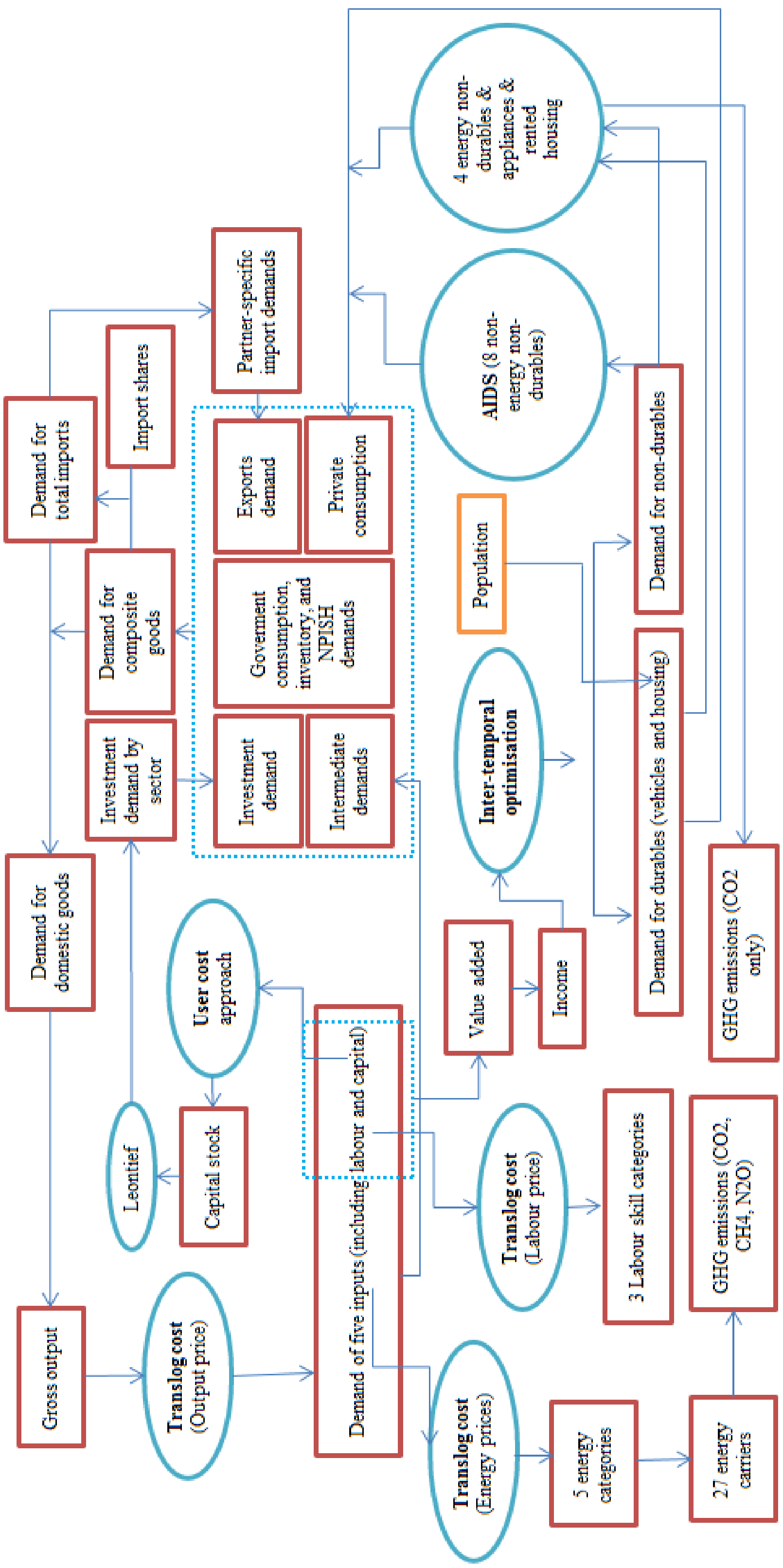

Figure 1. Overview of the main economic flows in FIDELIO 2

The public sector in FIDELIO 2 is specified with endogenous public consumption taking into account a net lending constraint for the public sector. The net lending constraint imposed in FIDELIO 2 for all European countries (there is no public sector for the seven non-

EU countries of the model) is in line with their mid-term fiscal stabilisation targets in terms of their public debt/GDP ratio and defines the target path of net lending as a percentage of GDP.

Given the gross output and the input prices, firms are assumed to minimise their total costs. Markets are perfectly competitive, so input prices are taken as given at this stage. As constant returns to scale are also assumed, the cost function can be substituted by an output price equation which in fact is the average (and marginal) cost function as well. As can be seen from the part of Figure 1 under gross output in the top row, translog cost functions (see Section 2.2.1 for more details on this) are used to derive the demands for the five aggregate inputs, which are the following:

1. capital *K*;
2. labour *L*;
3. total energy inputs *E*;[4]
4. total domestic non-energy inputs *D*;
5. total imported non-energy inputs *M*.

The derived input demand (and supply) of labour and capital makes up the total value added by sector (basically, $VA = L + K$).[5] As Figure 1 shows, aggregate labour is further disaggregated into demands for three different skill types: high-, medium-, and low-skilled labour. The translog cost approach is employed again here, whereby the cost function defines the wage earned per hour which in turn determines the labour price. With regards to capital, sectoral capital stocks are obtained assuming that their total user cost value is equal to the sectoral capital compensation (cash flow). A static concept of the user cost of capital is employed (Hall and Jorgenson 1967) and the rate of return is assumed to be the same for all assets to calculate ex-post user costs (this is equivalent to assuming that the capital market is in equilibrium in each period) according to the method proposed by Christensen and Jorgenson (1969). The user cost of capital depends on the price of investments, the interest rate for capital costs of firms' purchases and the industry-specific depreciation rate. Using Leontief technology based on the base-year investment-to-capital stock proportions, the investment demand by sector in purchasers' prices is obtained (see the "Investment demand by

[4] Production-related greenhouse gas (GHG) emissions are linked to the use of energy here. As explained in more detail in Section 2.2.3 below, five aggregate energy categories exist which are then disaggregated into 27 energy carriers, each responsible for CO_2, CH_4 and N_2O emissions. The model also takes into account CO_2 emissions related to households' consumption of energy (both heating fuel and fuel for private transport).

[5] In fact, there are other value added components such as social security contributions, taxes and subsidies, and depreciation that we do not discuss here (and that do not appear in Figure 1) for the sake of simplicity.

sector" rectangle in Figure 1). This is finally transformed into the investment demand for products at purchasers' prices using the base-year product structure of investments. The demand for investment just explained enters Figure 1 in the set of intermediate and final demands highlighted by a dotted light blue line (the set includes investment demand, intermediate demand private consumption, exports demand, and government consumption, inventory, and NPISH demands in purchasers' prices).

It is worth explaining how to obtain the various demand categories. The demand for intermediate goods by sector at purchasers' prices ("Intermediate demands") are computed by allocating the intermediate energy and (both domestic and imported) non-energy inputs over all goods using the corresponding product use structure, i.e. the proportions, of the base year.[6]

Private consumption is the largest component of aggregate demand and, as such, its modelling is given very careful consideration within FIDELIO 2. There are three steps needed to obtain private consumption demand at purchasers' prices (which in Figure 1 lies to the right of intermediate demands). The first stage is based on the theory of intertemporal optimisation of households with buffer-stock saving as proposed by Luengo-Prado (2006) and also discussed in Section 2.1.1. This theory takes into account that households cannot optimise according to the permanent income hypothesis due to credit market restrictions (liquidity constraints) and down payments linked to the purchase of durables. From the optimality conditions of the intertemporal problem, we derive demand functions of durable (own houses and vehicles) and nondurable goods and services that depend on households' wealth, down payment requirements (needed for purchasing durable goods), and the user cost of durables. The latter in turn depends on durables' prices, depreciation rates, and the interest rate relevant to households' durables purchases.

In the second stage, the energy components of private consumption are modelled with single equations taking into account the energy-relevant stock of durables (houses, vehicles, and appliances), its energy efficiency, energy prices, and other socio-demographic variables. Finally, in the third stage, the derived aggregate non-energy nondurables demand is split up into its different components using the AIDS proposed by Deaton and Muellbauer (1980), whose exact specification is discussed in Section 2.1.3 below. Finally, the demands obtained for durable and nondurable commodities consistent with the COICOP[7] are transformed into

[6] This structure matrix can be changed if one expects that, for example, the energy input proportions are going to be different in the future compared to those in the base year. Also, this can be the object of simulations revolving around the use of factors of production.

[7] COICOP stands for "Classification of Individual Consumption According to Purpose". For more details on this, please see the COICOP information provided by the United Nations Statistics Division at: http://unstats.un.org/unsd/cr/registry/regcst.asp?Cl=5.

private consumption demands for products that are consistent with the 2002 CPA[8] using the corresponding region-specific bridge matrices between the two systems. This step is needed because final consumption by purpose (COICOP) must be converted into final consumption by product (CPA) and vice versa.

Exports are obtained from the endogenous trade flows among the model regions (which in Figure 1 is denoted as "Partner-specific import demands" and indicates each region's demands for imports from its trade partners[9]), plus the exogenous exports to the rest of the world. Inventory and NPISH demand are also exogenous. The first is assumed to rapidly shrink towards zero over time from its base-year value for all products, while the second one is assumed to remain fixed at its base-year values. Government consumption, on the other hand, can be set to be exogenous, but depending on its closure rule it can be linked to a specific public budget balance. At the moment, in FIDELIO 2 all EU-27 countries are assumed to follow the fiscal rules set by the strengthened Stability and Growth Pact according to which public debts in excess of 60% of GDP must be adjusted so that the gap between a country's debt level and the 60% reference needs to be reduced by one twentieth annually.

This ends the brief description of all demands for both domestically produced and imported goods (composite goods, for short) expressed in purchasers' prices. Taking into account trade and transport margins (as in Streicher and Stehrer, 2015), as well as taxes (less subsidies) on products, these purchasers' price demands are translated into the demands for composite goods at basic prices (see the "Demand for composite goods" rectangle above the set of demands just discussed).

The total import shares are multiplied by the corresponding basic price demands for composite goods in order to obtain the final users' demand in each region for each total imported good valued at CIF prices (which means prices including cost, insurance and freight). That is the rectangle labelled "Demand for total imports". Sectoral demand for intermediate imports of energy goods is derived similarly, but the demand for non-energy goods is obtained differently, namely by multiplying the total demand for non-energy imported inputs (coming from the translog model of factor demands) with the use-structure matrix for imported non-energy intermediates. This use-structure matrix and the total import shares are assumed to be those of the base year for all users except for private consumers where Armington elasticities (Armington 1969) are applied. Thus, the total import shares of

[8] CPA stands for "Classification of Products by Activity". For more details on this, please see the RAMON (Reference And Management Of Nomenclatures) classification list published by Eurostat at: http://ec.europa.eu/eurostat/ramon/index.cfm?TargetUrl=DSP_PUB_WELC.

[9] The partner-specific import demands are computed from the total imports demand using the base-year trade shares in combination with the Armington approach (see Section 2.3).

consumers depend on both domestic and import prices.

Finally, deducting imports from the demands for composite goods at basic prices yields the demand for domestically produced goods in basic prices with which we have started this overview of the main economic flows illustrated in Figure 1. This closes the loop of the main economic flows and interactions of FIDELIO 2. Clearly, crucial details behind these dependencies have been skipped for the sake of clarity. The next section expands on the theory behind FIDELIO 2.

1.2. The main prices in FIDELIO 2

We now turn to the discussion of the main (for the sake of brevity) prices which directly and/or indirectly affect the variables discussed so far. The main idea behind the system of prices is that both basic prices and purchasers' prices are used in the model, following the logic of Supply and Use Tables. Supply Tables are valued at basic prices: these are the prices of products prior to their arrival to the markets (without trade and transport margins, and net product taxes). Use Tables are valued at purchasers' prices (the prices paid by either consumers or producers for final or intermediate consumption respectively).[10] Consistency between the two is guaranteed by the valuation matrices which account for distribution margins and taxes and subsidies, and a similar consistency is achieved within FIDELIO 2 where such margins and net taxes are modelled (data availability is problematic for this kind of data according to Mongelli et al., 2010).

The ensuing brief discussion is organised similarly to the one above on the main economic flows of the model following Figure 1. Let us start once again from the first rectangle of the top row in Figure 1: the gross output's basic prices are determined through the translog cost approach from the prices of the five factors of production. Such prices also play a role for the derivation of their own demands.

Basic prices of domestic products (related to the second rectangle of the top row of Figure 1, demand for domestic goods) are obtained as weighted averages of the sectoral gross output prices, where the base-year market shares of sectors are used as weights. Such prices are the same for all users, much like in the standard IO price model. However, given the fact that demand in purchasers' prices is essentially demand for a composite good made by the good itself plus trade and transport margins and taxes (less subsidies) on production, the

[10] According to the ESA-95 criteria, in final demand the following relationship holds: $y_{bp} = y_{pp} - trade - transport - (taxes - subsidies)$, where *bp* stands for basic prices, *pp* for purchasers' prices, *trade* and *transport* for trade and transport margins respectively, and the brackets contain taxes and subsidies on products.

purchasers' prices of domestically produced products are ultimately user-specific. The Free-On-Board (FOB) prices of exports in each exporter region are comparable to basic prices, and, once corrected for the exchange rates and augmented by international transport costs and tariffs, give the CIF prices at the border of each importing region. Since the rest of the world is exogenous to the model, the CIF prices for imports from the rest of the world are also exogenous. Next, the weighted average of the import prices of trading partners gives the total import CIF price at the border of each region r for each good g and each user u, where the endogenously determined trading-partner-specific import shares are taken as weights. Further accounting for domestic mark-ups turns such prices into the total import prices including domestic margins and taxes less subsidies on products.

Products' use prices for intermediate and final users (of the set of intermediate and final demands) are the weighted averages of the purchasers' prices of domestic products and import prices using the import shares as the corresponding weights. As for the factors of production, the aggregate price of energy inputs in the production function is determined using the base-year product structure of energy inputs and the corresponding sectoral use prices. Similarly, combining the purchasers' prices of domestic (imported) goods with the base-year product structure of domestic (imported) non-energy inputs results in the aggregate prices of domestic (imported) non-energy inputs. By the same principle, the prices of investments are determined from the products' use prices for investments and the base-year product structure of investments.

The regional use price for each user is the aggregate price of "inputs" for that user, and is obtained as the weighted average of the corresponding use prices with weights representing the product shares of endogenously derived demands for goods in purchasers' prices. For instance, the price for the private consumer is the regional consumer price. Using the COICOP-CPA bridge matrices of the base year, products' use prices for private consumption are translated into the prices of durable and nondurable consumption commodities. The prices of stocks of durable commodities are obtained by applying the concept of user cost of durable goods.

The wages per employee of the high-, medium- and low-skilled labour are determined by wage curves (see Section 2.4 for more details on this), which in FIDELIO 2 relate labour skill type wages per employee to labour productivity per hour, the consumer price, the aggregate unemployment rate, and the hours worked by the employee (the latter is exogenous). These wages are used within the translog cost framework to determine the average wage earned per hour, which in turn defines the price of labour. Finally, the price of sectoral capital stock is obtained from the investment prices using the notion of the user cost of capital.

1.3. A brief conclusion

This overview should have clarified the reasons behind the name of this version of the model. FIDELIO 2 is 'fully interregional' because it is an interregional economic model accounting for the most important features (at least from a policy analysis point of view) of consumption, production, the labour market, trade, and the environment. The quantity and price interactions between regions are taken into account by comprehensive modelling of interregional trade flows. The model is dynamic because, as mentioned above, the consumption block is based on an intertemporal optimisation approach, and recursive dynamics is used to solve the model for scenario-based analysis. The model is econometric because the crucial parameters' values in the functions characterising the economic agents' behaviour in the consumption, production and labour blocks are estimated from appropriate time series data employing econometric techniques. The model is a long-term one because the durables and nondurables demands are expressed in the form of long-run equilibrium relationships with error correction model specifications. This allows for the existence not only of short-run effects, but also of long-run equilibrium relationships, as well as various adjustment speeds of the short-run deviations toward such long-run equilibrium. Finally, the IO part lies in the model being demand-driven, and also in the fact that IO Supply and Use Tables constitute the main source of the data employed in FIDELIO 2. This provides data which are at the root of the commodity-industry approach used in IO analysis.

2. Theoretical foundations of FIDELIO 2

2.1. The consumption block

Private consumption is the largest demand category in most economies, as demonstrated by both national accounts data and IO tables where it usually accounts for most of final private demand. In order to take into account the importance of private consumption, the consumption block of FIDELIO 2 is considerably more developed with respect to traditional IO analysis which was initially imagined by Leontief (1966) in order to describe the inter-industry relationships through a combination of economic theory and data. Traditionally, IO tables have been extended in order to include consumption blocks through the use of Social Accounting Matrices (SAM - see for example Pyatt and Round, 1985). SAM are social in the sense that households are at the heart of the framework, which includes distributional features of the household sector and details about the circular flow of income and the interactions among the various agents of the economy (Round, 2003). SAM have been used extensively to study the multiplier effects of income injections both in socio-economic and distributional terms. Similar analyses have been done by adding Keynesian income-expenditure loops to the IO framework, i.e. basic consumption functions relating consumption with current disposable income (Miyazawa, 1976). However, such a simple treatment of aggregate consumption in an IO modelling context neglects various significant developments in consumption theory over the last decades.[11]

Immediately after World War II, influential contributions by Modigliani and Brumberg (1954) and Friedman (1956) put forward the life-cycle hypothesis of saving and consumption, according to which consumption depends on permanent income rather than current income. This led to the development of life-cycle/permanent income models featuring dynamically optimising consumers who smooth out consumption over their life span according to their permanent income, i.e. the expected long-term average income. However, the permanent income hypothesis has also been challenged due to several well-known empirical puzzles such as the so-called excess sensitivity and excess smoothing. The former refers to the fact that consumption reacts to changes in current income far more than the permanent income hypothesis would suggest (Hall and Mishkin, 1982). On the other hand, excess smoothing

[11] One notable exception in this respect is Chen et al. (2010) who extend the standard IO model by including a more realistic treatment of private consumption.

characterises the behaviour of consumers who show little reaction to permanent income shocks (Campbell and Mankiw, 1989). A number of reasons have been put forward by the literature to explain these failures of the life-cycle/permanent income hypothesis, including the existence of precautionary motives for saving, irrational behaviour (such as myopia, habit formation and inertia) and the presence of liquidity constraints (Zeldes, 1989).

An alternative model that can be used to describe the typical household's saving and consumption is the buffer-stock model (Carroll, 1997) which is thought to be more capable of reflecting the empirical regularities which go against the permanent income hypothesis (Ludvigson and Michaelides, 2001). Buffer-stock behaviour emerges when consumers both face income uncertainty, and thus have a precautionary saving motive, and are sufficiently impatient, so that if future income were known with certainty they would choose to consume more than their current income. A buffer-stock model is the starting point of the modelling of the consumption block in FIDELIO 2, which builds upon the model by Mongelli et al. (2010) for its links with the IO core structure of FIDELIO 2. Households maximise the present discounted value of the expected utility obtained from consuming nondurable commodities and from the services provided by the stock of durables subject to a budget constraint.

Within FIDELIO 2, households consume both durable and nondurable goods. Households can save for the purchase of durables (real estate property and vehicles), which can be bought by taking out loans. The model features a down payment requirement parameter representing the fraction of durables that a household is not allowed to finance via borrowing (Luengo-Prado, 2006; Luengo-Prado and Sørensen, 2004). The demand for durables is modelled with a dynamic specification (Caballero, 1993; Eberly, 1994) for which parameters are based on an econometric analysis carried out with data for 14 EU countries for the period 1995–2011.[12]

Second, energy consumption is modelled as a demand for services linked to the utilisation of the capital stock. There are four categories: 1) heating fuel and 2) electricity, which are both related to the stock of owned and rented houses; 3) fuel for private transport, which is related to the stock of vehicles; and 4) public transport. This implies modelling the stock of own houses and vehicles (from the durables' part of the consumption block), as well as accounting for rented houses and appliances. The total stock of housing is a function of population, while appliances are treated differently from the set of durables simply because they cannot be used as collateral for loans. The number of vehicles is related to the capital

[12] The countries are the following: Austria, Belgium, Cyprus, Czech Republic, Denmark, Finland, Germany, France, Italy, Lithuania, Poland, Portugal, Romania and Slovakia.

stock of households. The inclusion of the fourth nondurable energy category, public transport, allows for substitution between public transport services and private car transport.[13] The econometric estimates for this second set of the consumption block rely on data for a panel formed by all EU-27 countries from 1995 to 2011, as well as on cross-sectional survey data for the following countries: Austria, France, Italy, Slovakia, Spain and the UK.

Third, non-energy nondurables consumption is modelled within a demand system. This is split into two nests: (i) an aggregate level of eight categories (food, clothing, furniture and equipment, health, communication, recreation and accommodation, financial services, and other) featured in an AIDS model, and (ii) a detailed model of 47 COICOP categories, explained by constant sub-shares of the eight aggregate categories. The relevant parameters have been estimated econometrically – as in the case of energy consumption – both with time series data for a panel of EU-27 countries (1995–2009), and with cross-sectional survey data.

It is important to note that households are divided into quintiles, so that for most of the variables discussed above the distributional properties can also be analysed. Data from the EU-SILC (Statistics on Income and Living Conditions) dataset are used to derive the data by quintiles.

2.1.1. Private consumption of durables and total nondurables

The starting point for determining total private consumption is a specification of the buffer-stock model (Deaton, 1991; Carroll, 1997) proposed by Luengo-Prado (2006) whereby, in addition to uncertainty over future income, there is an extra motive for saving: the existence of collateralised constraints and down payments for the purchase of durables.[14] Consumers maximise the present discounted value of the expected utility from consumption of nondurable commodities (*C*) and from the services provided by the stock of durables (*K*) as follows:

$$\max_{(C_t, K_t)} V = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t, K_t) \right\} \tag{1}$$

The utility function used in the model is the following Constant Relative Risk Aversion (CRRA) function:

[13] A particularly interesting variable from a scenario analysis viewpoint is the average energy efficiency of the durables stock (heating for dwellings, vehicles for fuels for transport, and appliances for electricity), which allows for simulations focused on technological progress increasing the efficiency of durable goods.

[14] A similar approach, but focused on nondurable consumption only, can be found in Chah et al. (1995).

$$U(C_t, K_t) = \frac{C_t^{1-\rho}}{1-\rho} + \varphi \frac{K_t^{1-\rho}}{1-\rho} \quad , \tag{2}$$

where φ is a preference parameter and $\rho > 0$ implies that consumers are risk-averse. The budget constraint in this model dictates that assets at time t, A_t, must be equal to past assets plus disposable income minus what has been consumed in period t:

$$A_t = (1+r)(1-t_r)A_{t-1} + YD_t - \left[C_t + \left(K_t - (1-\delta)K_{t-1}\right)\right] \tag{3}$$

In (3), $\left[C_t + \left(K_t - (1-\delta)K_{t-1}\right)\right]$ represents total consumption, i.e. the sum of nondurables and durables expenditure (the latter includes the depreciation of the durables stock according to a constant depreciation rate δ). Profits can be extracted from the assets held in the previous period, A_{t-1}, at the profit rate r (gross profit income, rA_{t-1}, is taxed with the tax rate t_r). Finally, YD_t is disposable household income which excludes profit income. Disposable income is defined as follows:

$$YD_t = (1 - t_S - t_Y)w_t H_t + (1 - t_Y)\Pi_{h,t} + Tr_t \,, \tag{4}$$

thus is the sum of three components: a) net wages $(1 - t_S - t_Y)w_t H_t$; b) net operating surplus accruing to households $(1 - t_Y)\Pi_{h,t}$; c) transfers Tr_t. t_S stands for social security contributions (further decomposed into employees' and employers' contributions), and t_Y is the income tax rate. Those rates are applied to the gross wage which results from the multiplication of the hourly wage rate w_t with the total hours demanded by firms H_t. Wage bargaining between firms and unions takes place over the employee’s gross wage as explained in Section 2.4.2.

All incomes are modelled at the level of quintiles q, thus a more precise representation of the income modelling choices within FIDELIO 2 is as follows:

$$YD_t = \sum_q \left(1 - t_{S,q} - t_{Y,q}\right) w_{t,q} H_{t,q} + \left(1 - t_{Y,q}\right) \Pi_{t,q} + Tr_{t,q} \tag{5}$$

In fact, there is a further layer of complexity regarding wages: they are determined at three skill levels (*s*), low, medium and high. Therefore, net wages depend not only on social security contributions and income taxes, but they are also characterised according to different quintiles and skill levels of households. These two dimensions are linked via a bridge matrix: the column vector of wages by quintiles $\mathbf{y}^{\mathrm{w}}_{\mathrm{q,t}}$ (containing the wages by quintiles $w_{t,q}H_{t,q}$) can be obtained by multiplying the matrix $\mathbf{S}_{\mathrm{q,s}}$ of the shares $w_{q,s}$ with the column vector of wages by skill levels, $\mathbf{y}^{\mathrm{w}}_{\mathrm{s,t}}$: $\mathbf{y}^{\mathrm{w}}_{\mathrm{q,t}} = \mathbf{S}_{\mathrm{q,s}}\mathbf{y}^{\mathrm{w}}_{\mathrm{s,t}}$.

Going back to equation (3), it is worth explaining how assets are modelled. The financial assets of households are built up by saving after durables purchases have been financed. Such purchases are characterised by the following lending constraint:

$$A_t + (1-\theta)K_t \geq 0, \tag{6}$$

where θK_t is the down payment required to acquire durables, i.e. the part of the durables' cost that cannot be financed via debt. We can define voluntary equity holding as follows: $Q_t = A_t + (1-\theta)K_t$. This is basically the gross wealth of households ($A_t + K_t$) including the required down payments. The characterisation of collateralised constraints lies in the down payment requirement's parameter θ, which represents the fraction of durables purchases that households are not allowed to finance. Let us define another important variable in the buffer-stock model of consumption, cash on hand, which measures the total resources of households (it is the sum of assets held, durables stock, and disposable income):

$$X_t = (1+r)(1-t_r)A_{t-1} + (1-\delta)K_{t-1} + YD_t \tag{7}$$

Total private consumption is then defined as:

$$CP_t = C_t + K_t - (1-\delta)K_{t-1} = r(1-t_r)A_{t-1} + YD_t - \left(A_t - (1-t_r)A_{t-1}\right) \tag{8}$$

where net lending equals the difference between disposable income and total consumption. The model solution works via the maximisation of the utility function (2) with respect to consumption of durables and nondurables. Following Luengo Prado (2006), one possible solution is to find an intra-temporal equilibrium relationship between C_t and K_t (see also Chah et al., 1995) where the constraint is not binding, i.e. the down payment share θ equals the user cost of the durables $\frac{r+\delta}{1+r}$ (which is the rental equivalent cost of one durable unit). In all the other cases, and more relevantly for the purposes of our model, the collateral constraint is binding. Thus, functions for C_t and K_t can be derived as functions of the difference between cash on hand and voluntary equity holding Q_t that the consumer wants to hold in the next period.

The FIDELIO 2 demand functions are formulated so as to be consistent with the properties of the equations presented above. With regard to durables consumption (the general notation K is used, standing either for own houses or for vehicles), the following function represents the starting point to derive the function used in the model (see equation (10) below), similar to that used for nondurables by Luengo-Prado and Sørensen (2004):

$$\log C_{dur,t} = \beta_K + \beta_{K,1}\log X_t + \beta_{K,2}\theta_{Ct} + \beta_{K,3}\log\left(p_{dur,t}\left(r_t+\delta\right)\right) + \beta_{K,4}\log X_t \log\left(\frac{K_{t-1}}{h_{t-1}}\right) \tag{9}$$

Equation (9) can be seen as the long-run relationship between $C_{dur,t}$ (which is equal to $K_t-\left(1-\delta\right)K_{t-1}$ in equation (8)) and cash on hand X_t and the user costs of durables $p_{dur,t}$.[15] The down payment parameter θ in Luengo-Prado (2006) is a long-term constraint between the liabilities stock and the durables stock of households and is specified here by imposing limits on the down payment for durables' purchases. Changes in the value of θ_{Ct} can be used to simulate changes in financial markets affecting the down payments for durable purchases.

The long-run marginal propensity of durable demand with respect to cash on hand depends on the accumulated stock $\frac{K}{h}$ (where h stands for the number of households) and is calculated as follows: $\beta_{K,1}+\beta_{K,4}\log\left(\frac{K_{t-1}}{h_{t-1}}\right)$. In the long run, as income rises, the relationship between durables and income does not remain constant, but the relationship between voluntary equity holding and income does. That corresponds to the long-run solution of the buffer-stock model without durables, where all equity accumulation is voluntary because no collateral constraint is active. Usually, in the buffer-stock model, non-stationarity of

[15] It is based on the concave consumption functions in Luengo-Prado (2006), whereby, for high levels of cash on hand, proportionally more voluntary equity is accumulated. Therefore, for high levels of durables consumption, the marginal propensity of investment in durables C_{Kt} with respect to X_t decreases ($\beta_{K,4}<0$).

consumption, income, and wealth is dealt with by normalising the variables (by dividing by permanent income). In FIDELIO 2, the non-stationarity is instead taken into account through an error correction model (ECM) as in Caballero (1993) and Eberly (1994):

$$\begin{aligned}&\Delta\log C_{dur,t}=\gamma_K+\gamma_{K,1}\log\Delta\log X_t+\gamma_{K,2}\Delta\theta_{Ct}+\gamma_{K,3}\Delta\log\left(p_{dur,t}\left(r_t+\delta\right)\right)+\\&\gamma_{K,ECM}\left[\log C_{dur,t-1}-\left(\beta_K+\beta_{K,1}\log X_{t-1}+\beta_{K,2}\theta_{C,t-1}+\right.\right.\\&\left.\left.+\beta_{K,3}\log\left(p_{dur,t-1}\left(r_{t-1}+\delta\right)\right)+\beta_{K,4}\log X_{t-1}\log\left(\frac{K_{t-2}}{h_{t-2}}\right)\right)\right]\end{aligned}\tag{10}$$

In equation (10), β_K and γ_K are constant terms (corresponding to cross-sectional fixed effects in a panel data econometric setting), and $\gamma_{K,ECM}$ is the error correction parameter (the following holds: $\gamma_{K,ECM}<0$). Equation (10) is specified both for own houses (dwelling investment) and for vehicles ($C_{hous,t}$ and $C_{veh,t}$, respectively), which are the only two categories of durables in FIDELIO 2. The capital stocks for both categories ($K_{hous,t}$ and $K_{veh,t}$) change over time according to the following equation: $K_t=K_{t-1}(1-\delta)+C_{dur,t}$, with category-specific depreciation rates δ.[16] It is worth highlighting that demand for vehicles is basically described by an investment function as in equation (8): ($K_t-(1-\delta)K_{t-1}$). As for own houses, the consumption data do not contain dwelling investment for own houses, but imputed rents. This is due to the definitions used in national accounting, which treat housing differently from other durables. Imputed rents are calculated as the following simple static user cost: $C_{rent,t}=p_{dur,t}(r_t+\delta)K_t$, where P_{dur} and K_t in this case refer to houses only.

The demand function for total nondurables consumption is modelled as a function of cash on hand and the product of the down payment parameter with durable consumption. The starting point to get to the error correction representation used in FIDELIO 2 is the following:

$$\log C_t=\beta_C+\beta_{C,1}\log X_t+\beta_{C,2}\theta_{Ct}\log C_{dur,t}\tag{11}$$

According to our estimates, $\beta_{C,1}>0$ and $\beta_{C,2}<0$. Equation (11) takes into account the households' need to finance the sum of $C_t+\theta_{Ct}C_{dur,t}$. Down payments will not be fully financed by savings in the same period and consumers smooth nondurables consumption over time

[16] This requires an estimated initial durables stock at time $t=0$.

according to the parameter $\beta_{C,2}$. The long-run marginal propensity with respect to cash on hand is given by $\beta_{C,1}$, plus the indirect impact via the term $\theta_{Ct} \log C_{dur,t}$. The latter again depends on $\beta_{K,1} + \beta_{K,4} \log\left(\frac{K_{t-1}}{h_{t-1}}\right)$, so the total marginal propensity of nondurables demand with respect to cash on hand is as follows: $\beta_{C,1} + \theta_{Ct}\beta_{C,2}\beta_{K,1} + \theta_{Ct}\beta_{C,2}\beta_{K,4} \log\left(\frac{K_{t-1}}{h_{t-1}}\right)$. The second term in this relationship measures the higher savings for down payments due to an increase in durable demand triggered by a marginal increase in cash on hand. The third and last term measures the impact of the non-linearity in the reaction of durables demand to both cash on hand on savings and nondurables demand. Note that as durable demand reacts to the price of durables and nondurables demand is linked to durable demand in (10), there is also an implicit price elasticity for nondurables. As with equation (9), an ECM representation is also used for equation (11).[17] The parameters for that equation and for equation (10) are estimated mainly using data taken from Eurostat's National Accounts. Those accounts include expenditure data as well as all income components and asset data, which are part of cash on hand.[18]

As a first step, the capital stock of housing property was estimated for a starting year based on the Household Financial and Consumption Survey (HFCS) of the ECB. By using property prices taken from the Bank of International Settlement (BIS) and Eurostat population data, a time series for own houses was constructed for the 14 EU countries for which sectoral accounts were available from 1995 to 2011.[19] The estimate of the base year stock of houses, the time series of their physical stock, and housing prices were used (together with information on the price of dwelling investment) to derive investment in own houses by inverting equation (13) below. A simpler procedure was applied for vehicles, as the expenditure data are available, and no revaluation of the existing stock had to be taken into account. Thus, the two capital stocks K_{veh} and K_{hous} in current prices change over time according to the following equations:

$$K_{veh,t} = K_{veh,t-1}(1 - \delta_{veh}) + C_{veh,t} \tag{12}$$

[17] The choice of the ECM specifications is supported by the results of unit root tests and cointegration tests (not reported for the sake of brevity).

[18] Due to the specific treatment of housing spending in national accounting, investment in own houses is pooled together with investment in other dwellings to derive total dwelling investment.

[19] The countries (whose choice was dictated by the availability of sectoral accounts' data with no gaps from 1995 to 2011) are the following: Austria, Belgium, Cyprus, Czech Republic, Denmark, Finland, France, Germany, Italy, Lithuania, Poland, Portugal, Romania and Slovakia.

$$K_{hous,t} = \frac{p_{hous,t}}{p_{hous,t-1}}\left[K_{hous,t-1}\left(1-\delta_{hous,t}\right)+CF_{hous,t} / p_{CF,t}\right] \quad (13)$$

The estimation of equation (12) is straightforward, once an estimate of the capital stock for the first year of the sample is obtained and δ_{veh} is assumed from information on the average age of vehicles. Equation (13) is different from (12) because a yearly revaluation of the stock of houses is modelled, and assumed to be driven by the yearly change in house prices. p_{hous} stands for the price of the housing stock and measures increases in construction prices (p_{CF}) as well as changes in land prices. The estimated investment in housing, CF_{hous}, adds up to total gross capital formation by households.[20] Given the demand and the accumulated stock of owner-occupied houses, imputed rents are calculated by applying the following user cost formulation: $C_{rent,t} = p_{dur,t}\left(r_t+\delta\right)K_t$.

The expenditure for imputed rents, vehicles and total nondurables adds up to total private consumption in FIDELIO 2.[21] The main result of the estimation of equations (10) and (11), the latter in its error correction version, is the estimate of the non-linear relationship between cash on hand and both durables and nondurables consumption. Table 1 reports both the short- and long-run parameters for the two durable categories, as well as for total nondurables.

First of all, most of the numbers contained in Table 1 can be interpreted as elasticities, as models are log-linear. Looking at the long-term parameters, there is unsurprisingly a positive relationship between cash on hand and consumption of all goods. There is a negative relationship between the down payment parameter θ_t and durables consumption: the harder it is for households to finance via borrowing, the less they will consume. As for vehicle purchases, the price elasticity is equal to the coefficient minus one since the consumption variable on the left-hand side is in nominal terms (if the price elasticity were zero, nominal consumption would increase along with the price, therefore the elasticity of real demand is the

[20] We do not know how gross capital formation by households in National Accounts is calculated/estimated, but it is very close to our estimates of the sum of vehicles and own houses capital formation. We bridge the difference with a country-specific factor whose value is always close to 1.

[21] Let us return to the down payment parameter $\theta_{C,t}$ for the purchase of durables in equation (11). It is calculated by relating the change in liabilities to the durables demand (the sum of C_{veh} and CF_{hous}) that gives $(1-\theta_{C,t})$. The original θ_t in the article by Luengo-Prado (2006) is measured in this model as (1 – liabilities/durables stock) and can only be controlled by fixing certain values of $\theta_{C,t}$ and solving the model to derive the path of θ_t. An iterative procedure ensures dynamic convergence towards target values of θ_t (i.e. if you want the model to converge to a certain θ_t, a 'trial and error' process is needed for $\theta_{C,t}$).

parameter minus one so that, if the parameter is equal to one, the elasticity is zero). Thus, the higher the prices, the less households will spend on vehicles (both in the short and in the long run).

Table 1. Short- and long-run parameters of durables and nondurables demand

	Own houses: $log(C_{dur,t})$	**Vehicles: $log(C_{dur,t})$**	**Nondurables: $log(C_t)$**
Long-run parameters			
$log(X_t)$	1.417	0.812	0.819
θ_t	-0.120	-0.020	
$log(p_{dur,t})$		0.855	
$log(X_t)log(K_{t-1}/h_{t-1})$	-0.040	-0.017	
$\theta_t log(C_{dur,t})$			-0.009
Short-run parameters			
$\Delta log(X_t)$	0.900 (0.12)	0.431 (0.09)	0.232 (0.05)
$\Delta\theta_t$	-0.035 (0.01)	-0.017 (0.01)	
$\Delta log(p_{dur,t})$		0.911 (0.14)	
ECM	-0.181 (0.03)	-0.356 (0.05)	-0.144 (0.03)
No. of observations	224	224	224
R^2	0.44	0.40	0.38

Note: Panel estimates for durables and nondurables demand with country fixed effects (14 EU countries, 1995–2011). Standard errors are in brackets.

Looking at the nondurables consumption equation, the negative impact of $\theta_{Ct} log(C_{dur,t})$ representing the need to finance the down payment, is an indication that the consumer is smoothing consumption without fully reacting to liquidity shocks (represented by $\theta_{C,t}$). Also, this negative coefficient has to be taken into account in order to calculate the long-run marginal propensity with respect to cash on hand, which is calculated as follows, as explained above: $\beta_{K,1} + \beta_{K,4} \log\left(\frac{K_{t-1}}{h_{t-1}}\right)$. As for the short-term parameters, the signs of the various coefficients are in line with those of the long-term parameters illustrated above. Also, the error correction terms are all negative, ensuring that the ECM is working correctly.

The econometric results reported in Table 1 have been used in the FIDELIO 2 model and calibrated for the income quintiles - see equation (5) as well as equations (12) and (13) - in all the countries of the EU-27, based on income data from EU-SILC and wealth data from the HFCS survey. For instance, by using country-specific values for $\frac{K_{t-1}}{h_{t-1}}$, country-specific (and quintile-specific) values for the long-term marginal propensity of durable consumption with respect to cash on hand can be obtained. Thus, a unique elasticity to cash on hand and different ratios of durables stock per household across quintiles will ultimately yield different marginal propensities for consumption.

Some econometric estimates (following Luengo-Prado, 2006, who did it with US data) performed on the model's output data have been carried out to check whether FIDELIO 2 can reproduce the first of the two empirical puzzles of the permanent income hypothesis ("excess sensitivity" and "excess smoothness") that led to the development of the buffer-stock model. Excess sensitivity describes the reaction of consumption (growth) to lagged income growth, an empirical phenomenon first found by Hall (1978). Excess sensitivity has been tested by regressing the growth rate of total consumption (durable plus nondurable) on the growth of lagged disposable income (excluding profit income, as this income source is a result of intertemporal optimisation of consumers and therefore not affected by transitory income shocks), both extracted from the FIDELIO baseline scenario from 2007 to 2050. The estimated coefficients are larger for the first quintile, and are progressively smaller for the other quintiles, revealing significant differences in the marginal propensity for consumption across quintiles, thus suggesting evidence of excess sensitivity in the behaviour of the FIDELIO 2 households.

2.1.2. Households' energy nondurables demand

The energy demand of households includes electricity, fuel for heating, fuel for private transport, and public transportation services. These demands are part of total nondurables consumption and are modelled separately from the eight non-energy goods and services. The durables stock of households (vehicles and houses, as well as appliances) is characterised by an efficiency factor (η_{ES}) measuring the durables' efficiency in converting energy demand for a certain fuel (E) into demand for services: $S = \eta_{ES} \times E$. For a given conversion efficiency factor and a service price (p_E), the marginal cost of a service (p_S) can be derived as in equation (14) below. Any increase in efficiency leads to a decrease in the service price and thereby to an increase in service demand (according to the rebound effect described in Khazzoom, 1989). This is true for electricity, heating, and for private transport demand.

$$p_S = \frac{p_E}{\eta_{ES}} \tag{14}$$

FIDELIO 2 takes into account the possible substitution between the demand for fuel for private transport (CP_{fuel}) and that for public transport services (CP_{pub}). In order to do so, the price (pc_{tr}) of the aggregate transport demand (CP_{tr}) is constructed as follows:

$$pc_{tr} = \exp\left[\frac{CP_{fuel}}{CP_{tr}}\log pc_{S,fuel} + \frac{CP_{pub}}{CP_{tr}}\log pc_{pub}\right], \tag{15}$$

where $pc_{S,fuel}$ stands for the price for fuel for private transport, and pc_{pub} stands for the price for public transport. The total transport demand of households depends on this aggregate price as well as on total nondurables expenditure according to the following log-linear specification in FIDELIO 2 which resembles its econometric counterpart estimated with panel data:

$$\log CP_{tr} = \mu_{tr} + \beta_{tr,0}\log CP_{tr,t-1} + \beta_{tr,1}\log pc_{tr} + \beta_{tr,2}\log C_t \tag{16}$$

The parameters $\beta_{tr,1}$ and $\beta_{tr,2}$ in equation (16) stand for the price and the expenditure elasticity, respectively, and are estimated via OLS (μ_{tr} is a constant which again corresponds to a cross-sectional fixed effect in a panel data framework). The demand for transport fuel is linked to the vehicles stock and also depends on the service price of fuel ($pc_{S,fuel}$) as well as on the average endowment of vehicles of the whole population. The latter term controls for the fact that the second car of a household is usually used less in terms of distance covered (and therefore in terms of fuel consumption) than the first. Equation (17) below is the equation featured in FIDELIO 2 which once again is closely related to an econometric equation estimated with panel data in order to retrieve the values of the parameters of interest:

$$\log\left(\frac{CP_{fuel,t}}{K_{veh,t}}\right) = \mu_{fuel} + \alpha_{fuel}\log\left(\frac{CP_{fuel,t-1}}{K_{veh,t-1}}\right) + \gamma_{fuel}\log\left(\frac{p_{fuel,t}}{\eta_{fuel,t}}\right) + \xi_{fuel}\log\left(\frac{K_{veh,t}}{h_t}\right) \tag{17}$$

In (17), μ_{fuel} is a constant, i.e. a fixed effect in a panel framework, and γ_{fuel} is the price elasticity under the condition that there is a unitary elasticity of fuel demand to the vehicle stock. Public transport demand is also a function of the consumption of fuel, as the more households spend on private transportation, the less they will spend on public transport services.

The panel data models for equations (16) and (17) are based on a sample of all EU-27 countries for the period 1995–2009. The energy expenditure of households is based on consumption expenditure data from Eurostat, but the Energy Accounts from the WIOD and the IEA Energy Prices have also been used to calculate the consumption of the following: electricity, heating, and private transport (all of them both in terajoules - TJ - and in millions of euros). The physical and nominal data have then been used to calculate the price indices for the three energy categories. Energy consumption modelling includes a variable measuring the efficiency with which the durables stock of households converts energy flows into services. Thus, in order to calculate the service prices, energy efficiency data had to be used.

Table 2. Panel estimates for transport demand of households (EU, 1995–2009)

	Total transport $log(CP_{tr})$	**Private transport** $log(CP_{fuel}/K_{veh})$
$log(pc_{tr})$	0.188*** (0.032)	
$log(C)$	0.614*** (0.040)	
$log(p_{fuel}/\eta_{fuel})$		0.229*** (0.048)
$log(K_{veh}/h)$		-0.340*** (0.058)
No. of observations	224	224
R^2	0.99	0.96

Note: Standard errors in brackets.

Table 2 reports the estimates of the econometric counterparts of equations (16) and (17) for total transport and for private transport consumption respectively. It appears that the implicit price elasticities of transport demand are relatively high. Note that, due to the fact that the consumption variables for energy on the left-hand side are in current prices, the price elasticity is given by one minus the corresponding parameter (γ_{fuel}, γ_{heat}, or γ_{el}). The expenditure elasticity of the total transport demand is considerably below unity and the density of vehicle endowment is an important factor dampening the demand for transport.

The equations for heating and electricity demand are the following:

$$\begin{cases} \log\left(\frac{CP_{heat,t}}{K_{hous,t}}\right) = \mu_{heat} + \alpha_{heat}\log\left(\frac{CP_{heat,t-1}}{K_{hous,t-1}}\right) + \gamma_{heat}\log\left(\frac{p_{heat,t}}{\eta_{heat,t}}\right) + \xi_{heat}\log\left(dd_{heatdays}\right) \\ \log\left(\frac{CP_{el,t}}{K_{app,t}}\right) = \mu_{el} + \alpha_{el}\log\left(\frac{CP_{el,t-1}}{K_{hous,t-1}}\right) + \gamma_{el}\log\left(\frac{p_{el,t}}{\eta_{el,t}}\right) + \xi_{el}\log\left(dd_{heatdays}\right) \end{cases} \quad (18)$$

The energy efficiency for electricity is calculated as a weighted average of the efficiency of electrical appliances taken from the ODYSSEE database. The heating efficiency indicator of the ODYSSEE database is used for heating in FIDELIO 2. γ_{heat} and γ_{el} represent the price elasticity of heating and electricity demand, respectively. Equation (18) differs from equation (17) because it includes a new variable, $dd_{heatdays}$, measuring the number of days in which the outside temperature goes above the base temperature at which buildings need heating.[22] This variable is meant to capture an important factor driving the demand for energy needed to heat a building, either in the form of electricity of in the form of heating fuel.

Table 3. Panel estimates for the electricity and heating fuel demand of households

	Log(CP_{el})/log(K_{hous})	***Log(CP_{heat})/log(K_{hous})***
γ_{heat}		0.132*** (0.035)
ξ	0.104** (0.044)	0.509*** (0.086)
γ_{el}	0.186*** (0.018)	
No. of observations	405	405
R^2	0.98	>0.99

Note: Panel estimates for transport demand (27 EU countries, 1995–2009). Standard errors in brackets.

The stock variables used in equation (18) are the total housing stock ($K_{hous,t}$) and the appliance stock ($K_{app,t}$). Consumption of the latter is derived as follows:

$$\log CP_{app} = \mu_{app} + \beta_{app,1} \log pc_{app} + \beta_{app,2} \log C_t , \qquad (19)$$

where CP_{app} is consumption of appliances.[23] $K_{hous,t}$ stands for total housing stock and is the sum of the stock of own and rented houses. The latter is exogenously driven by population dynamics depending on a coefficient of rented houses to population.

The existing literature offers a number of studies dealing with the estimation of the price elasticity for these four categories of energy-related goods and services. The studies differ in terms of data used, country and time coverage, and econometric techniques employed. Measuring gasoline/diesel price elasticities has always proven to be a popular exercise in the economics literature due to the related interesting policy implications. As for

[22] Eurostat defines this variable as follows: (18°C - T_m)*d, if $T_m \leq$ 15°C, and nil if T_m is > 15°C. T_m is the mean ((T_{min} + T_{max})/2) outdoor temperature over d days (over one year).
[23] The log-linear form of equation (20) is similar to that of transport demand, i.e. equation (17).

the case of electricity and heating fuel consumption, systems of equations have been estimated to account for public transport spending as well. As pointed out by Kratena and Wüger (2010), the price elasticity of electricity and for heating fuel are normally estimated to be around -0.1 and -0.5 respectively. For transport fuels (gasoline and diesel), the literature suggests an elasticity between -0.3 and -0.4. The fact that our estimated elasticities appear to be high in absolute terms might be due to the fact that the elasticities calculated here are conditional on the stock of durables, thus implicitly assuming a unitary elasticity of the energy demand to the durables stock as a strong driving force of demand.

One of the main parameters in the nondurables demand functions of FIDELIO 2 is the elasticity to total consumption for which appropriate values have been estimated using household-level survey data. The cross-sectional model used for electricity consumption and for heating fuel consumption of the *k* households is the following:

$$\log CP_{el,k} = \mu_{elec} + \beta_{el} \log C_k + \sum \boldsymbol{\xi}_{\mathrm{el,j}} \mathbf{V}_{\mathrm{j,k}} + \varepsilon_{el} \tag{20}$$

$$\log CP_{heat,k} = \mu_{heat} + \beta_{heat} \log C_k + \sum \boldsymbol{\xi}_{\mathrm{heat,j}} \mathbf{V}_{\mathrm{j,k}} + \varepsilon_{heat} \tag{21}$$

The dependent variable is the logarithm of expenditure in either electricity (*el*) or heating fuel (*heat*), and β_{heat} and β_{el} are the parameters used in FIDELIO 2 to be interpreted as income (i.e. total expenditure for nondurable goods and services) elasticity. In order to obtain sensible estimates of such parameters, the econometric model includes the following socio-demographic controls included in the vector $\mathbf{V}_{\mathrm{j,k}}$: a) dummies for the age group of the household head; b) one dummy taking the value one if the household head is retired; c) one dummy taking the value one if the household head is unemployed; d) one dummy taking the value one if the household head is the owner of the house he/she lives in; e) the logarithm of the household size; f) up to seven dummies indicating the age of the house; g) the logarithm of the number of rooms the house has; h) dummies indicating the type of house (detached, semi-detached, apartments, etc.); i) one dummy taking the value one for rural households; l) the population density of the area where the house is located; and m) regional dummies (usually referred to the NUTS2 regions of the European Union). All the controls are meant to capture factors that can have non-negligible effects on the consumption of electricity and heating fuel in order to obtain an accurate estimate of the total expenditure elasticity, which is the objective of the analysis. As data from six different surveys (countries) have been used to estimate these econometric models, the exact model specification in terms of included controls depends on the survey design (for more details on those estimates, see Salotti et al., 2015). μ_{elec} and μ_{heat} are constant terms, and ε_{elec} and ε_{heat} are error terms.

The cross-sectional model used to estimate the total expenditure elasticity for private and public transport spending is similar to models (20) and (21) but, rather than controlling for the housing stock, in this case it is more appropriate to control for the vehicle stock:

$$\log\left(CP_{fuel,k}\right) = \mu_{fuel} + \beta_{fuel}\log C_k + \sum \xi_{\mathrm{fuel,j}}\mathbf{N}_{\mathrm{j,k}} + \eta_{fuel} \tag{22}$$

$$\log\left(CP_{transport,k}\right) = \mu_{transport} + \beta_{transport}\log C_k + \sum \xi_{\mathrm{transport,j}}\mathbf{N}_{\mathrm{j,k}} + \eta_{transport} \tag{23}$$

The dependent variables are expenditure for fuel for private vehicles and for public transport in equations (22) and (23) respectively. Note that public transport stands for train, bus and coach transportation and does not include expenditure for air transport. The idea behind this definition is to capture journeys for which a private car could be a viable alternative. The $\mathbf{N}_{\mathbf{j}}$ vector contains the following variables: a) age group dummies for the household head; b) one dummy taking the value one if the household head is retired (*retired*); c) one dummy taking the value one if the household head is unemployed (*unemployed*); d) one dummy taking the value one if the household head is the owner of the house he/she lives in (*owner*); e) the logarithm of the household size (*log(hhsize)*); f) a dummy taking the value one when the household owns 1 car (*one car*); g) a dummy taking the value one if the household owns two or more cars (*more than one car*)[24]; h) one dummy taking the value one for rural households (*rural*); i) the population density of the area where the house is located (*pop_density*); and l) regional dummies. η is the error term.

Models (22) and (23) are estimated with household-level survey data referring to the 2004/2006 period for six EU countries (Austria, France, Italy, Slovakia, Spain and the UK). Such datasets,[25] used by the national institutes of statistics for the construction of the national Consumer Price Index (CPI), allow for the estimation of the income elasticities for the various commodities featured in the FIDELIO 2 model. Ultimately, and due to the lack of detailed country-specific data, a weighted average of those elasticities (with GDP per capita as a weighting factor) is chosen as the relevant parameter in the FIDELIO 2 model.

[24] Note that results are not affected when the number of vehicles is used instead: only a tiny minority of households own more than two vehicles.
[25] See Salotti et al. (2015) for more details on the data used.

Table 4 and Table 5 contain the estimated coefficients of models (20) and (21) for electricity consumption and for heating fuel consumption, respectively. Since both the dependent variable and the total expenditure variable C_k are in logarithmic form, the expenditure coefficients can be directly interpreted as elasticities. Results from the six surveys appear to be broadly consistent with each other. The estimated income elasticities are always significant and within the range of the existing literature estimates (see, among others, Hondroyiannis, 2004; Tiezzi, 2005; Labanderia et al., 2006; Rehdanz, 2007; and Druckman and Jackson, 2008). The income elasticity for electricity consumption ranges from 0.05 (UK) to 0.33 (Spain), while the heating fuel income elasticity lies between 0.12 (France) and 0.47 (Spain). Since both the dependent variable and the total expenditure variable are in logarithmic form, those numbers are to be directly interpreted as elasticities.

Results for the some of the controls are both consistent and interesting. For example, older households spend more on both electricity and heating fuel than younger ones. The larger the household, the higher the expenditure for both commodities. Finally, houses with more rooms call for higher electricity and heating fuel consumption.

Table 4. Cross-sectional estimates for the electricity demand of households

	Austria	**France**	**Italy**	**Slovakia**	**Spain**	**UK**
logC_k	0.18***	0.30***	0.18***	0.17***	0.33***	0.05***
age(35-49)	0.13***	0.03	0.10***	0.04**	0.03	0.04*
age(50-64)	0.23***	0.18***		0.06***	0.06**	0.10***
age(65+)	0.15***	0.18***	0.11***	-0.01	0.09**	0.03
retired	-0.03	-0.06		-0.02	-0.03*	0.01
unemployed	0.07*	0.01		0.04	-0.02	0.06
owner		-0.52***		0.03	0.04	
hhsize	0.46***	0.19***	0.50***	0.28***	0.17***	0.33***
house age dummies	YES	YES	YES	YES	YES	YES
house type dummies	YES	YES	YES	YES	YES	YES
rooms		0.21***	0.36***	0.32***	0.20***	0.17***
rural		-0.09***			-0.06***	
pop_dens	-0.08***			-0.08***	-0.02**	
Reg dummies	YES	YES	YES	NO	YES	YES
Constant	4.10***	3.30***	0.77***	4.48***	2.02***	0.96***
No. of observations	6336	8977	24657	4058	7545	2844
R^2	0.32	0.17	0.27	0.31	0.33	0.20

Note: Standard errors in brackets. ***, **, * indicate significance at 1%, 5%, and 10% respectively. Survey weights are used in all cases. Period dummies are included in the Italian and UK estimates due to the non-yearly frequency of the data. The missing (reference) age group is *age(18-34)*. The age group dummies in the Italian data are different from the rest and the two categories for the older households are merged into one.

Table 5. Cross-sectional estimates for the heating fuel demand of households

	Austria	**France**	**Italy**	**Slovakia**	**Spain**	**UK**
logC_k	033***	0.12***	0.33***	0.23***	0.47***	0.14***
age(35-49)	0.06*	0.06	0.11***	0.00	0.00	0.08*
age(50-64)	0.16**	0.11***		0.03	0.05	0.14***
age(65+)	0.22***	0.32***	0.26***	0.04	0.12**	0.18***
retired	0.02	-0.14**		0.02	0.02	0.02
unemployed	0.13***	-0.05		-0.02	0.05	-0.10
hhsize	0.31***	0.08***	0.12***	0.04	0.08**	0.08*
house age dummies	YES	YES	YES	YES	YES	YES
house type dummies	YES	YES	YES	YES	YES	YES
rooms		0.76***	0.59***	0.67***	0.36***	0.68***
rural		0.20***			-0.06*	
pop_dens	-0.03			0.10***	-0.05***	
Reg dummies	YES	YES	YES	NO	YES	YES
Constant	3.05***	3.73***	0.89***	3.56***	-0.38**	-0.46***
No. of observations	6272	6587	22752	3875	7075	2413
R^2	0.18	0.17	0.30	0.18	0.37	0.25

Note: Standard errors in brackets. ***, **, * indicate significance at 1%, 5%, and 10% respectively. Survey weights are used in all cases. Period dummies are included in the Italian and UK estimates due to the non-yearly frequency of the data. The missing (reference) age group is *age(18-34)*. The age group dummies in the Italian data are different from the rest and the two categories for the older households are merged into one.

Table 6 and Table 7 contain the estimated coefficients of models (22) and (23) for fuel used for private transport and for public transportation, respectively. Results are consistent across the six surveys. The estimated income elasticities are always significant and within the range of the available literature estimates in the case of fuel for private transport (see, among others, Graham and Glaister, 2002; Wadud et al., 2009; and Dahl, 2012), ranging from 0.33 (UK) to 0.94 (Austria). The income elasticities for public transport consumption lie between 0.29 (UK) and 0.58 (Austria).[26] Results for the controls are also mostly consistent across countries. For example, households owning one car obviously consume more fuel than those without one, and having more than one car also impacts positively on fuel consumption. The

[26] In this case it is harder to perform a direct comparison with the existing literature since studies on public transport normally concentrate on variables such as the number of trips using public transport, rather than expenditure (see for example Johansson-Stenman, 2002).

other side of this is reflected in the negative coefficients of the car dummies in the public transport model: owning one or more cars lowers the expenditure for public transport.

Table 6. Cross-sectional estimates for the private transport demand of households

	Austria	**France**	**Italy**	**Slovakia**	**Spain**	**UK**
logC_k	0.94***	0.34***	0.47***	0.68***	0.83***	0.33***
age(35-49)	-0.02	-0.02	-0.04	-0.15	-0.08	0.03
age(50-64)	-0.11**	-0.01		-0.23***	-0.07	0.02
age(65+)	-0.24***	-0.14***	-0.10***	-0.19**	-0.08	-0.11***
retired	-0.08	-0.08		-0.10*	-0.01	-0.10***
unemployed	0.12*	-0.15***		-0.14	-0.01	-0.07
hhsize	-0.26***	-0.06*	-0.04	-0.10*	0.02**	-0.01
one car	0.26***	0.49***	0.38***	0.40***	0.21***	0.14***
more cars	0.59***	0.73***	0.54***	0.59***	0.29***	0.45***
rural		0.08***			0.12***	
pop_dens	-0.06***			0.02	-0.03*	
Reg dummies	YES	YES	YES	NO	YES	YES
Constant	-1.84***	3.21***	1.11***	0.22	-1.49***	0.85***
No. of observations	6128	6250	17811	2157	5649	4091
R^2	0.26	0.20	0.27	0.25	0.29	0.26

Note: Standard errors in brackets. ***, **, * indicate significance at 1%, 5%, and 10% respectively. Survey weights are used in all cases. Period dummies are included in the Italian and UK estimates due to the non-yearly frequency of the data. The missing (reference) age group is *age(18-34)*. The age group dummies in the Italian data are different from the rest and the two categories for the older households are merged into one.

Table 7. Cross-sectional estimates for the public transport demand of households

	Austria	**France**	**Italy**	**Slovakia**	**Spain**	**UK**
logC_k	0.58***	0.49***	0.50***	0.56***	0.45***	0.29***
age(35-49)	-0.18**	-0.22***	0.02	0.15**	-0.05	-0.19***
age(50-64)	0.03	-0.21***		0.12*	-0.07	-0.04
age(65+)	-0.11	-0.38***	-0.14*	-0.42***	-0.27**	-0.21***
retired	-0.17	-0.01		-0.48***	0.06	-0.12**
unemployed	0.15	-0.01		-0.17	-0.15	0.09
hhsize	-0.37***	0.14***	0.10	0.43***	-0.00	0.11
one car	-0.35***	0.34***	-0.24***	-0.31***	-0.45***	-0.23***
more cars	-0.55***	0.64***	-0.21***	-0.50***	-0.55***	-0.27***
rural		-0.14***			-0.14*	
pop_dens	0.17***			0.02	0.41***	
Reg dummies	YES	YES	YES	NO	YES	YES
Constant	-0.04	1.39***	-0.24	0.04	-0.33***	-0.11
No. of observations	2854	9166	5990	2737	4852	1934
R^2	0.13	0.33	0.11	0.28	0.15	0.12

Note: Standard errors in brackets. ***, **, * indicate significance at 1%, 5%, and 10% respectively. Survey weights are used in all cases. Period dummies are included in the Italian and UK estimates due to the non-yearly frequency of the data. The missing (reference) age group is *age(18-34)*. The age group dummies in the Italian data are different from the rest and the two categories for the older households are merged into one.

The ideal way to proceed for the production of the income elasticities to be used in the FIDELIO 2 model would have been to estimate them using household-level panel data: such a procedure would have also permitted the simultaneous estimation of the price and income elasticities with a unique model. Unfortunately such data simply do not exist, therefore cross-sectional data have to be used, implying that the income elasticities need to be estimated separately from the price elasticities (the latter have been presented above). Moreover, since cross-sectional data on the nondurables consumption of households are not available for all the EU-27 countries, the existing data for six countries need to be used in order to produce credible estimates of the income elasticities of the four energy-related categories of consumption. In principle, one option would have been to calculate the simple average of the elasticities arising from the six surveys. We adopted a slightly different approach by using a weighted average based on GDP per capita in order to have values as representative as possible of the EU countries in terms of their incomes. As Table 8 below shows, the differences between the simple averages and the weighted averages are minimal.

Table 8. EU-level income elasticities for the four energy-related nondurables

	Austria	France	Italy	Slovakia	Spain	UK	Average	Weighted average
Electricity	0.18	0.30	0.18	0.17	0.33	0.05	0.20	0.20
Heating fuel	0.33	0.12	0.33	0.23	0.47	0.14	0.27	0.26
Private transport	0.94	0.34	0.47	0.68	0.83	0.33	0.60	0.58
Public transport	0.58	0.49	0.50	0.56	0.45	0.29	0.48	0.47

2.1.3. Households' non-energy nondurables demand

In the first version of FIDELIO, all nondurables consumption was treated in one block without any specific treatment of energy consumption, using a quadratic AIDS. In FIDELIO 2, the consumption of the eight non-energy-related nondurables mentioned in Section 2.1 (food, clothing, furniture and equipment, health, communication, recreation and accommodation, financial services, and other) is modelled within a simplified version of an AIDS. Deaton and Muellbauer (1980) provide the seminal contribution for demand system analysis with the AIDS which rapidly gained popularity and eventually became a workhorse model (Buse, 1994). The AIDS starts from the cost function for $C(u,\ p_i)$, describing expenditure (for C) as a function of a given level of utility u and prices of consumer goods, p_i. The AIDS is represented by the budget share equations for the i nondurable goods in each period:

$$w_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log\left(\frac{C}{P}\right); \quad i = 1 \ldots n, 1 \ldots k \tag{24}$$

with the price index P defined by $\log P = \alpha_0 + \sum_i \alpha_i \log p_i + 0.5 \sum_i \sum_j \gamma_{ij} \log p_i \log p_j$,

often represented by the Stone price index: $\log P^* = \sum_k w_i \log p_i$.

The expressions for the compensated price elasticities (ε_{ij}^C) and for the expenditure elasticity (η_i) within the AIDS model for the quantity of each consumption category C_i can be written as follows (the details of the derivation can be found in Green and Alston, 1990):

$$\varepsilon_{ij}^C = \frac{\partial \log C_i}{\partial \log p_j} = \frac{\gamma_{ij} - \beta_i w_j}{w_i} - \delta_{ij} + \eta_i w_j \tag{25}$$

$$\eta_i = \frac{\partial \log C_i}{\partial \log C} = \frac{\beta_i}{w_i} + 1 \tag{26}$$

In (25), δ_{ij} is the Kronecker delta with $\delta_{ij} = 0$ for $i \neq j$ and $\delta_{ij} = 1$ for $i = j$. The time series model has been estimated using National Accounts data for a panel of the EU-27 countries from 1995 to 2009. The estimated own price elasticity (25) and the expenditure elasticity (26) are reported in

Table 9. While the expenditure elasticities are closely distributed around unity, the price elasticity shows more heterogeneity across categories. The estimated expenditure elasticities are all very close to one, possibly due to the nature of the data used for the estimation. The literature suggests that household-level data should be used in order to get more accurate estimates of such elasticities, and that is why we supplement the panel estimates reported in Table 9 with survey-based cross-sectional estimates, as explained below. The survey data mentioned above for the energy-related goods and services have been used for this purpose.

Table 9. Estimated own price and expenditure elasticity of the non-energy nondurables demand of households

	Own price elasticity	Expenditure elasticity
Food	-0.142	0.882
Clothing	-0.638	1.010
Furniture and equipment	-1.057	1.061
Health	-0.827	0.977
Communication	-0.886	1.031
Recreation and accommodation	-0.504	1.060
Financial services	-0.937	1.253
Other	-0.684	1.071

Note: Panel estimates for durable and nondurables demand with country fixed effects (27 EU countries, 1995–2009).

As before, due to the cross-sectional nature of the data, the system employed in our analysis is the following simplified (due to the absence of prices) AIDS:

$$w_i = \mu_i + \beta_i \log C_k + \sum \boldsymbol{\theta}_{\mathrm{ji}} \mathbf{M}_{\mathrm{ji}} \tag{27}$$

The vector $\mathbf{M}_{\mathbf{ji}}$ contains all the variables of vector $\mathbf{V}_{\mathbf{j}}$ above - see equations (20) and (21) - plus those of the vector $\mathbf{N}_{\mathbf{j}}$ - see equations (22) and (23) - not included in $\mathbf{V}_{\mathbf{j}}$. The idea is to explain the expenditure shares of the eight nondurable commodities with total expenditure and a number of socio-demographic controls. In terms of expected results, it is hard to find examples in the literature using the same commodities as our system, therefore it is almost impossible to compare our numerical results with existing ones. However, theoretical and practice-based priors suggest that food, communication and health-related goods could be necessary goods (elasticity below one); clothing, furniture and equipment, recreation and accommodation, and financial services could be superior goods (elasticity above one); the 'other' category is used as a residual and its construction makes it hard to state a clear prior.

Table 10 presents the estimated expenditure elasticities derived from the simplified AIDS model explained above. The figures in Table 10 mostly depict a consistent picture: food and communication are necessary goods (with income elasticities below one, apart from the case of communication with Austrian data), while clothing, furniture and equipment, health, and recreation and accommodation seem to be superior/luxury goods (with the exception of an elasticity below one for health with the Slovakian data). Financial services and the residual category (other) are the only two commodities for which our estimates do not permit neat conclusions to be drawn. The latter case is at least understandable on the grounds that a residual category contains a wide variety of expenditures that may differ across the six surveys.

Table 10. Cross-sectional estimated expenditure elasticity of the eight non-energy nondurables

	Austria	France	Italy	Slovakia	Spain	UK
Food	0.48	0.67	0.63	0.67	0.71	0.59
Clothing	1.44	1.20	1.33	1.55	1.08	1.19
Furniture and equipment	1.48	1.40	1.67	1.87	1.31	1.32
Health	1.43	1.26	1.23	0.94	1.14	1.00
Communication	1.11	0.60	0.50	0.83	0.65	0.45
Recreation and accommodation	1.26	1.26	1.44	1.31	1.35	1.09
Financial	n.a.	0.82	0.53	1.40	0.73	1.45
Other	0.93	1.19	1.27	1.00	0.91	1.00

Note: The elasticities above are derived from cross-sectional estimates fully reported in Salotti et al. (2015).

As for the energy nondurables above, the ideal way to proceed for the production of the expenditure elasticities to be used in FIDELIO 2 would have been to estimate them using household-level panel data. Such a procedure would have also permitted the simultaneous estimation of the price elasticities with a unique model. Given the lack of such data, we estimated the expenditure elasticities separately from the price elasticities. As before, we combined the results for the six EU countries for which data are available in order to produce EU-level estimates.

Table 11. Own price elasticity (panel estimates) and expenditure elasticity (cross-sectional estimates)

	Own price elasticity	Expenditure elasticity
Food	-0.142	0.612
Clothing	-0.638	1.275
Furniture and equipment	-1.057	1.463
Health	-0.827	1.196
Communication	-0.886	0.678
Recreation and accommodation	-0.504	1.271
Financial services	-0.937	1.000
Other	-0.684	1.000

Table 11 reports the final values that have been used for the calibration of the parameters in the third stage of the consumption model (non-energy nondurables) in FIDELIO 2, namely the price elasticity from the panel estimation (simple sample average) and the weighted average – as for the energy nondurables case – of the expenditure elasticity from the cross-sectional estimation.

There is an additional step to be performed in order to use the estimated elasticities above in FIDELIO 2, both for the energy nondurable goods and services and for the non-energy nondurable goods and services. It consists of calibration involving a mathematical procedure (besides using the consumption data by quintile). This is due to the property of the AIDS model whereby the price or expenditure elasticity is a linear combination of fixed parameter values and changing budget shares. In FIDELIO 2 we are interested in the elasticity, as this is the most informative measure of behavioural reactions of individuals and firms to economic signals. Therefore we start from the estimated elasticity values reported above, and, by inverting equation (26), we can calculate the country-specific β_i:

$$\beta_i = (\eta_i - 1) w_i \tag{28}$$

Then, these β_i are used together with both price and expenditure elasticities to calculate the country-specific γ_i by inverting equation (25):

$$\gamma_{ij} = \left[\varepsilon_{ij}^{C} + \delta_{ij} - \eta_i w_j\right] w_i + \beta_i w_j \tag{29}$$

2.2. The production block

In comparison with FIDELIO 1, the main part of the production block – i.e. the equations for output prices and factor demand for K, L, E, M_m, M_d – has not undergone major changes. The approach chosen is based on cost minimisation using a translog cost function.

In applied econometrics, flexible functional forms are used in order to model second-order effects (like elasticities of substitution) that are functions of the second derivatives of cost/utility/production functions. The translog function can be seen as a second-order Taylor series approximation of an arbitrary cost function and is flexible enough to embody many standard assumptions and results of microeconomic theory concerning cost functions. It is also highly analytically tractable. For these reasons it has been extensively used in empirical works. Given the importance of the translog function not only in the production block, but also in other parts of the FIDELIO 2 model (i.e. labour market, and energy demand), we first provide a brief overview of its derivation, and then discuss how it has been used for the production block of the model.

2.2.1. The translog function

The translog function is one of the most popular functional forms used in the empirical studies of production. This is possibly due to its better performance compared to other similar flexible functional forms (Guilkey and Lovell, 1980; Guilkey et al., 1983; Byron and Bera, 1983).[27] This function is usually interpreted as a second-order approximation of an unknown function of interest. Suppose the function is $y = g(x_1, x_2, ..., x_n)$, which can be taken as $\ln y = \ln g(x_1, x_2, ..., x_n)$. Since $x_i = \exp(\ln x_i)$, we can interpret the function of interest as a function of the logarithms of x_i's, that is: $\ln y = f(\ln x_1, \ln x_2, ..., \ln x_n)$. Next, expand the last function as a second-order Taylor series around the point $\mathbf{x} = (1,1,...,1)'$ so that the expansion point is conveniently taken as zero (that is, $\ln 1 = 0$). This gives the following:

[27] Here we provide a brief discussion of this function. For further details, the reader is referred to Christensen et al. (1973, 1975), Bernt and Christensen (1973), Bernt and Wood (1975), Christensen and Greene (1976), and Greene (2012).

$$\ln y = f(\mathbf{0}) + \sum_i \left[\frac{\delta f(\bullet)}{\delta \ln x_i}\right]_{\ln \mathbf{x}=\mathbf{0}} \ln x_i + \frac{1}{2}\sum_i \sum_j \left[\frac{\delta^2 f(\bullet)}{\delta \ln x_i \, \delta \ln x_j}\right]_{\ln \mathbf{x}=\mathbf{0}} \ln x_i \ln x_j + \varepsilon \tag{30}$$

where ε is the approximation error. Since the function and its derivatives evaluated at the fixed value $\mathbf{0}$ are constants, these can be seen as the coefficients in a regression setting and thus one can write (30) as follows:

$$\ln y = \beta_0 + \sum_i \beta_i \cdot \ln x_i + \frac{1}{2}\sum_i \sum_j \gamma_{ij} \ln x_i \ln x_j + \varepsilon \tag{31}$$

Although (31) is a linear regression model, in its role of approximating another function it actually captures a significant amount of curvature. If the unknown function is assumed to be continuous and twice continuously differentiable, then by Young's theorem it must be the case that $\gamma_{ij} = \gamma_{ji} \;\; \forall i, j$. Notice also that the other widely used Cobb-Douglas function (log-linear model) is a special case of the translog function when $\gamma_{ij} = 0 \;\; \forall i, j$.

2.2.2. Sectoral output prices and derived input demands

Suppose that production is characterised by a production function $Q = f(\mathbf{x})$ and firms are minimising their costs subject to a fixed level of production. Assuming perfect competition in the input markets, the input prices $\mathbf{p}$ are taken as given by the firms. According to that framework, firms decide their optimal input (or factor) demands $x_i = x_i(Q, \mathbf{p})$ and the total cost of production is given by the cost function:

$$C = \sum_i p_i x_i(Q, \mathbf{p}) = C(Q, \mathbf{p}) \tag{32}$$

Assuming constant returns to scale, the cost function can be shown to take the form $C(Q, \mathbf{p}) = Q \cdot c(\mathbf{p})$, where $c(\mathbf{p})$ is the unit or average cost function. Hence, $\ln C(Q, \mathbf{p}) = \ln Q + \ln c(\mathbf{p})$. The optimal (cost-minimising) input demands x_i assuming perfect competition are derived using Shepard's lemma as:

$$x_i = \frac{\delta C(Q,\mathbf{p})}{\delta p_i} = Q \cdot \frac{\delta c(\mathbf{p})}{\delta p_i} \tag{33}$$

Using (33), we obtain the cost-minimising cost share of input i as follows:

$$s_i = \frac{p_i \cdot x_i(Q,\mathbf{p})}{C(Q,\mathbf{p})} = \frac{p_i}{C(Q,\mathbf{p})} \cdot x_i(Q,\mathbf{p}) = \frac{p_i}{Q \cdot c(\mathbf{p})} \cdot \frac{Q \cdot \delta c(\mathbf{p})}{\delta p_i} = \frac{p_i}{c(\mathbf{p})} \cdot \frac{\delta c(\mathbf{p})}{\delta p_i} = \frac{\delta \ln c(\mathbf{p})}{\delta \ln p_i} \tag{34}$$

In FIDELIO 2, sectoral outputs are produced using five types of factor inputs: capital (k), labour (l), total energy inputs (e), imported non-energy inputs (m) and domestic non-energy inputs (d). The corresponding output and input prices are denoted, respectively, by p_q, p_k, p_l, p_e, p_m and p_d. The unit cost or, equivalently, the output price function (i.e. $\ln c(\mathbf{p}) \equiv \tilde{p}_q$) can be written as:

$$\tilde{p}_q = \beta_0 + \sum_{i \in \{k,l,e,m,d\}} \beta_i \cdot \tilde{p}_i + \frac{1}{2} \sum_{i,j \in \{k,l,e,m,d\}} \gamma_{ij} \cdot \tilde{p}_i \cdot \tilde{p}_j \tag{35}$$

where the ~ symbol indicates the logarithm of the variable (for example, $\tilde{p}_d \equiv \ln p_d$). Deriving (35) with respect to $\ln p_i$ and imposing the symmetry condition $\gamma_{ij} = \gamma_{ji} \quad \forall i, j$, the cost shares (34) take the following form:

$$\begin{aligned}
s_k &= \beta_k + \gamma_{kk}\tilde{p}_k + \gamma_{kl}\tilde{p}_l + \gamma_{ke}\tilde{p}_e + \gamma_{km}\tilde{p}_m + \gamma_{kd}\tilde{p}_d \\
s_l &= \beta_l + \gamma_{kl}\tilde{p}_k + \gamma_{ll}\tilde{p}_l + \gamma_{le}\tilde{p}_e + \gamma_{lm}\tilde{p}_m + \gamma_{ld}\tilde{p}_d \\
s_e &= \beta_e + \gamma_{ke}\tilde{p}_k + \gamma_{le}\tilde{p}_l + \gamma_{ee}\tilde{p}_e + \gamma_{em}\tilde{p}_m + \gamma_{ed}\tilde{p}_d \\
s_m &= \beta_m + \gamma_{km}\tilde{p}_k + \gamma_{lm}\tilde{p}_l + \gamma_{em}\tilde{p}_e + \gamma_{mm}\tilde{p}_m + \gamma_{md}\tilde{p}_d \\
s_d &= \beta_d + \gamma_{kd}\tilde{p}_k + \gamma_{ld}\tilde{p}_l + \gamma_{ed}\tilde{p}_e + \gamma_{md}\tilde{p}_m + \gamma_{dd}\tilde{p}_d
\end{aligned} \tag{36}$$

The cost shares in (36) must sum to one, which implies that the following extra conditions must be imposed:

$$\sum_i \beta_i = 1 \text{ and } \sum_i \gamma_{ij} = \sum_j \gamma_{ij} = 0, \tag{37}$$

where all the summations are taken over all factors, i.e. $i, j \in \{k, l, e, m, d\}$. The conditions (37) imply that the cost (or output price) function (35) is homogeneous of degree one in input prices. This implies that the total cost (price) increases proportionally when all input prices increase proportionally.

When conditions (37) derived from the fact that the cost shares add up to one are imposed, expressions (35) and (36) can be simplified, reducing the number of parameters to be estimated. In the price function (35) this is done by means of taking prices relative to the price of domestic non-energy materials, p_d, and imposing the conditions in (37). Let $\tilde{p}_{id}$ stand for $\ln\left(p_i / p_d \right)$ for $i \in \{k, l, e, m\}$, then the final prices of sectoral outputs are computed as follows:

$$
\begin{aligned}
\tilde{p}_{qd} = \beta_0 + \sum_{i\in\{k,l,e,\mathrm{m}\}} \beta_i \cdot \tilde{p}_{id} + \frac{1}{2} \sum_{i\in\{k,l,e,\mathrm{m}\}} \gamma_{ii} \cdot \left(\tilde{p}_{id} \right)^2 + \sum_{j\in\{l,e,\mathrm{m}\}} \gamma_{kj} \cdot \tilde{p}_{kd} \cdot \tilde{p}_{jd} \\
+ \sum_{j\in\{e,\mathrm{m}\}} \gamma_{lj} \cdot \tilde{p}_{ld} \cdot \tilde{p}_{jd} + \gamma_{em} \cdot \tilde{p}_{ed} \cdot \tilde{p}_{md}
\end{aligned}
\tag{38}
$$

The system of equation (36) is simplified in a similar way. Again defining $\tilde{p}_{id}$ as $\ln\left(p_i / p_d \right)$ for $i \in \{k, l, e, m\}$, the system of equations is reduced to:

$$
\begin{aligned}
s_k &= \beta_k + \gamma_{kk} \tilde{p}_{kd} + \gamma_{kl} \tilde{p}_{ld} + \gamma_{ke} \tilde{p}_{ed} + \gamma_{km} \tilde{p}_{md} \\
s_l &= \beta_l + \gamma_{kl} \tilde{p}_{kd} + \gamma_{ll} \tilde{p}_{ld} + \gamma_{le} \tilde{p}_{ed} + \gamma_{lm} \tilde{p}_{md} \\
s_e &= \beta_e + \gamma_{ke} \tilde{p}_{kd} + \gamma_{le} \tilde{p}_{ld} + \gamma_{ee} \tilde{p}_{ed} + \gamma_{em} \tilde{p}_{md} \\
s_m &= \beta_m + \gamma_{km} \tilde{p}_{kd} + \gamma_{lm} \tilde{p}_{ld} + \gamma_{em} \tilde{p}_{ed} + \gamma_{mm} \tilde{p}_{md} \\
s_d &= \beta_d - \sum_{i\in\{k,l,e,m\}} \left(\sum_{j\in\{k,l,e,m\}} \gamma_{kj} \right) \cdot \tilde{p}_{id}
\end{aligned}
\tag{39}
$$

The estimation of the 14 parameters in (38) and (39) is done with the Seemingly Unrelated Regressions (SUR) method on pooled data of EU-27 countries for each industry with the inclusion of country-specific fixed effects for the 35 industries breakdown. Since the cost shares add up to one, the last equation in (39) is dropped and it is computed as a residual (otherwise the disturbance covariance matrix in the SUR system would be singular). Equation

(38) is also included in the SUR system in order to estimate β_0. The data used for the system estimation come mostly from the WIOD, but combined with other sources for the energy and capital prices. The factor cost shares are computed according to (34), i.e. by dividing the nominal values of every input ($p_i x_i$) by the nominal gross output. The WIOD Socio-Economic Accounts (SEA) series are used for nominal gross output ($p_q Q$), capital compensation ($p_k x_k$), and labour ($p_l x_l$). In order to derive the nominal value of energy intermediates ($p_e x_e$), data on gross energy use by energy commodity and industry available in the Environmental Accounts of the WIOD are used. Energy price information by commodity is taken from the International Energy Agency's "Energy Prices and Taxes Statistics" database. From the SEA, we also get the total of intermediate inputs $\left(p_d \cdot x_d + p_m \cdot x_m + p_e \cdot x_e\right)$; by subtracting $p_e x_e$ from this total, we derive the total non-energy intermediates $\left(p_d \cdot x_d + p_m \cdot x_m\right)$. This subtotal had to be split into domestic and imported intermediates according to the information available in the International Supply and Use Tables of the WIOD (WIOT).

The price level of the gross output (p_q) is taken from the SEA, while the price for labour p_l has been calculated by combining the nominal values for labour compensation and the SEA employment data. A similar approach is followed with regards to energy, as the energy inputs in current values ($p_e x_e$) are combined with the physical information available (energy in TJ) contained in the SEA. The WIOT are available at current as well as previous years' prices, which allows us to calculate the prices for domestic and imported inputs. The price of capital is computed as the user cost of capital according to $p_k = p_I\left(r + \delta\right)$, where p_I is the price level of gross fixed capital formation from the SEA, r is the real rate of return on capital that can be obtained as the interest rate of a risk-free asset such as the rate on treasury bills in the secondary market deflated by the gross output price level, and δ is the depreciation rate of the capital stock.

The pooled data estimation results in a common parameter (γ_{ij} for $i, j \in \{k, l, e, m, d\}$) for each industry of the whole EU-27. From the parameters' estimates of the unit cost function, we can easily derive the elasticities of substitution between the factors of production and the price elasticities of demand at a national level. For the translog cost function, the Allen-Uzawa elasticity of substitution between inputs *i* and *j* are defined as (Uzawa, 1962):

$$\sigma_{ij} = \frac{c(\mathbf{p}) \cdot \left[\delta^2 c(\mathbf{p}) / \left(\delta p_i \delta p_j \right) \right]}{\left[\delta c(\mathbf{p}) / \delta p_i \right] \cdot \left[\delta c(\mathbf{p}) / \delta p_j \right]} = \begin{cases} \left(\gamma_{ij} + s_i \cdot s_j \right) / s_i \cdot s_j & \text{if } i \neq j \\ \left(\gamma_{ij} + s_i^2 - s_i \right) / s_i & \text{if } i = j \end{cases} \tag{40}$$

The price elasticity of demand for factor *i* with respect to input price *j*, given the output quantity and all other input prices, is calculated as follows:

$$\varepsilon_{ij} = \frac{\delta \ln x_i}{\delta \ln p_j} = s_j \sigma_{ij} \tag{41}$$

Observe from (40) and (41) that although $\sigma_{ij} = \sigma_{ji}$ for $i \neq j$, in general, the price elasticities are not symmetric (i.e., $\varepsilon_{ij} \neq \varepsilon_{ji}$) because the corresponding factor shares are different. A positive (negative) value of σ_{ij} or ε_{ij} implies that factors *i* and *j* are substitutes (complements), which is an important piece of information for the purposes of economic analysis. However, it should be kept in mind that these are partial equilibrium concepts and miss various crucial and complex feedback mechanisms that are captured in models like FIDELIO 2. Therefore, the most likely effects on factor demand are not directly visible in the computed price elasticities.

The parameters' estimates for a few selected sectors of Spain (as an example) are presented in Table 12, where the intercepts and the error terms of the cost shares equations are combined. This implies that the reported estimates of $\beta_i + \varepsilon_i$ in (39) for $i \in \{k, l, e, m\}$ are nothing but the *base-year observed shares* of, respectively, k, l, e and m because for the base year all the log-prices and time terms are zero. The base-year observed share of the domestic non-energy input can be derived as the residual, if needed. For example, for 'sec01' it is as follows: $\overline{\beta_d} = 1 - \sum_{i \in \{k,l,e,m\}} \overline{\beta_i} = 1 - 0.311 - 0.243 - 0.027 - 0.059 = 0.360$.

Table 12. Estimates of the translog parameters in equation (38) of selected Spanish industries

Sector	$\overline{\beta_k}$	$\overline{\beta_l}$	$\overline{\beta_e}$	$\overline{\beta_m}$	γ_{kk}	γ_{ll}	γ_{ee}	γ_{mm}	γ_{kl}	γ_{ke}	γ_{km}	γ_{le}	γ_{lm}	γ_{em}
sec01	0.311	0.243	0.027	0.059	-0.154	0.006	0.026	-0.028	0.056	-0.016	0.060	0.007	0.015	0.002
sec02	0.486	0.394	0.006	0.008	-0.325	-0.049	0.006	-0.003	0.045	-0.003	0.008	0.002	0.002	0.000
sec05	0.141	0.488	0.037	0.065	-0.046	-0.107	0.034	-0.031	0.115	-0.022	0.066	0.009	0.015	0.002
sec10	0.047	0.382	0.139	0.108	0.008	0.122	-0.004	0.077	0.027	0.066	-0.012	0.002	0.019	0.010
sec11	0.195	0.150	0.097	0.248	0.003	0.083	0.001	0.142	0.009	0.045	-0.037	0.005	0.044	0.021
sec12	0	0	0	0	0	0	0	0	0	0	0	0	0	0
sec13	0.180	0.098	0.032	0.116	0.006	0.059	0.002	0.082	0.007	0.015	-0.014	0.002	0.024	0.012
sec14	0.199	0.189	0.117	0.079	0.003	0.097	-0.001	0.058	0.010	0.054	-0.009	0.005	0.016	0.008
sec75	0.154	0.531	0.033	0.047	-0.035	-0.099	-0.123	-0.124	0.073	0.054	0.064	-0.083	-0.051	0.001
sec80	0.100	0.762	0.019	0.014	0.001	0.028	0.010	0.000	0.003	0.001	0.001	0.002	0.010	0.010
sec85	0.084	0.557	0.017	0.080	0.002	0.050	0.000	0.000	0.010	0.001	0.007	0.009	0.041	0.019
sec90	0.141	0.295	0.030	0.025	0.005	0.134	0.022	0.009	-0.001	-0.013	0.003	0.028	0.010	0.006
sec91	0.034	0.547	0.014	0.048	0.005	0.111	0.011	0.016	0.011	-0.006	0.006	0.014	0.018	0.012
sec92	0.161	0.403	0.011	0.063	0.002	0.140	0.008	0.020	-0.011	-0.005	0.008	0.010	0.024	0.015
sec93	0.177	0.405	0.030	0.041	0.000	0.140	0.022	0.014	-0.014	-0.013	0.005	0.028	0.016	0.010
sec95	0	1	0	0	0	0	0	0	0	0	0	0	0	0

Note: The cost shares errors and the corresponding intercepts are combined, i.e. $\overline{\beta_i} = \beta_i + \varepsilon_i$ for $i \in \{k, l, e, m\}$. Since for the base year all other log-prices and time terms of the factor shares equations are zero, $\overline{\beta_i}$ is the base-year observed share of input *i*.

Using (41) and Table 12 we can show how to compute the price elasticities of demand. For example, from Table 12 we see that for 'sec01' (Agriculture, Hunting and related services) in the base year the share of its capital is $s_k = 0.311$ and $\gamma_{kk} = -0.154$. Substituting these values in (41) we have that $\varepsilon_{kk} = -1.183$, which is negative, as expected. Similarly, together with $\gamma_{kl} = \gamma_{lk} = 0.056$, $s_k = 0.311$ and $s_l = 0.243$, we get $\varepsilon_{kl} = 0.422$ and $\varepsilon_{lk} = 0.540$. Hence, capital and labour in 'sec01' are substitutes and demand for labour is more sensitive to changes in capital prices than the reaction of capital demand to changes in labour price.

2.2.3. Energy demand by fuels and GHG emissions

In a second nest of the production block, the energy factor is split up into aggregate categories of energy (coal, oil, gas, renewable, electricity/heat) in another translog model. The unit cost function of this model determines the bundle price of energy, p_E, and the cost shares of the five aggregate energy types:

$$\log p_E = \alpha_0 + \sum_i \alpha_{E,i} \log(p_{E,i}) + \frac{1}{2} \sum_i \gamma_{E,ii} \left(\log(p_{E,i})\right)^2 + \sum_{i,j} \gamma_{E,ij} \log(p_i) \log(p_j) \tag{42}$$

$$v_{E,i} = \left[\alpha_{E,i} + \sum_{i,j} \gamma_{E,ij} \log(p_{E,i}) \right] \tag{43}$$

The estimated own and cross price elasticities of this model of inter-fuel substitution are in some cases very close to zero, but in most industries assume values around -0.5. The cross price elasticities also show negative signs in a large number of industries, indicating complementarity between fuels. The set of five energy categories of the model of inter-fuel substitution needs to be directly linked in FIDELIO 2 to two parts of the model:

(i) the energy accounts by industry and detailed fuel category (26 energy categories) in physical units which are the basisfor calculating CO_2 emissions in FIDELIO 2;

(ii) the energy commodities and industries of the use table (NACE/CPA 10, 11, 23, 40) in monetary units.

Link (i) is carried out by deflating the five energy inputs in nominal terms by an adequate price per unit of physical input (TJ) and by applying sub-shares in physical terms to the resulting physical energy inputs. Link (ii) is carried out by applying the changes in the structure of the five energy inputs to the use structure matrix of the factor *E*. The energy prices $p_{E,i}$ used in this translog model and derived from the exogenous world market prices are based on prices per physical unit (TJ). The part of primary energy prices in $p_{E,i}$, i.e. the prices for coal, oil, gas, and biomass, are directly derived from exogenous world market prices and in turn determine import prices and part of the domestic output price vector of the four energy commodities in the use matrix. The domestic price of the electricity- and heat-producing sector is determined by the corresponding output price equation of this sector and drives the electricity/heat price in $p_{E,i}$. Finally, the composite price of each energy commodity is the energy price in the *K, L, E, M, D* translog model.

A third nest for the energy factor starts from the cost shares in each industry j, $v_{E,ij}$ for given energy prices $p_{E,i}$. The physical energy inputs are dealt with at the level of 26 energy categories e (EQ_{eij}), applying fixed sub-shares s_{eij} by industry and energy category. The physical inputs are the link to GHG emissions by industry, applying a fixed emission factor $em_{GHG,e}$ per energy unit.

$$EQ_{eij} = s_{eij} \frac{v_{E,ij} E}{p_{E,i}} \; ; \; EM_{GHG,j} = em_{GHG,e} \left[s_{eij} \frac{v_{E,ij} E_j}{p_{E,i}} \right] \qquad (44)$$

The emission factors at the detailed level of energy categories e are the link to introduce prices of GHG emissions in the FIDELIO 2 model which are then aggregated to the level of fuel prices, $p_{E,i}$, and added to the price as a tax component.

2.3. Trade

In FIDELIO 2, imports are modelled in a two-stage procedure. The first stage determines (starting from the base-year structures) the share of imports in the total demand in basic prices for each commodity and user. Final and intermediate demands are treated separately, and energy and non-energy goods are differentiated in intermediate demand. For the private consumption part of final demand, previous year imports' structures for each commodity evolve over time according to the ratio between domestic and import prices and the Armington elasticities. On the other hand, import structures for public consumption, investments and re-exports are kept constant. Within intermediate demand, due to the normally limited domestic resources for energy production, fixed import shares are used for the energy commodities. For non-energy commodities, the shares of imports by sector are determined by multiplying the IO commodity structure of the non-energy commodities by the demand shares for imports obtained in the translog production block. Then, these import shares are used to calculate total imports by commodity.

In the second stage, this vector is used to distribute imports to the country of origin. As in standard multi-regional IO modelling, the imports of each country determine the exports of the other countries. This leads to the calculation of trade among countries.

2.3.1. First stage: towards a vector of total imports by commodities

Let $\mathbf{M}_E^i$ and $\mathbf{M}_E^f$ stand for the import shares matrices for intermediate goods and for final energy demand (*i* stands for intermediate commodities, *f* for final demand, and *E* for energy). As stated above, $\mathbf{M}_E^i$ is fixed and is the share of imports in the use matrix of the base year for each energy good and user. $\mathbf{M}_E^f$ is the share of imports for each energy commodity and final demand user. The final demand shares of energy for each final demand user are equal to their respective share in the base year except for private consumption. In this case, private consumption imports shares for each commodity change according to the ratio between domestic and import prices and the Armington elasticities.

These shares matrices are multiplied element-wise by the matrices of total final demand $\mathbf{F}_E$ and total intermediate demand $\mathbf{U}_E$ to obtain the energy imports by sector. As a result of this, the total imports of energy are calculated by adding, by sector, the imports of energy (the symbol $\odot$ represents the element-by-element multiplication of two matrices and $\boldsymbol{\iota}$ is a vector of ones with the appropriate dimension): $\mathbf{IM}_E = \left(\mathbf{M}_E^{\mathrm{m}} \odot \mathbf{U}_E\right)\cdot\boldsymbol{\iota} + \left(\mathbf{M}_E^{\mathrm{f}} \odot \mathbf{F}_E\right)\cdot\boldsymbol{\iota}$.

The matrix of the domestic final demand for energy commodities $\mathbf{F}_E^{\mathbf{d}}$ is then derived from the matrix of total final demand $\mathbf{F}_E$ as follows: $\mathbf{F}_E^{\mathbf{d}} = \mathbf{F}_E - \mathbf{M}_E^{\mathbf{f}} \odot \mathbf{F}_E$. For the final demand for non-energy commodities, a similar treatment of imports for energy commodities is used. Let $\mathbf{M}_{NE}^{\mathrm{f}}$ stand for the import share of non-energy goods and final demand users. We multiply this structure matrix element-wise by the final demand matrix for non-energy goods: $\mathbf{M}_{NE}^{\mathrm{f}} \odot \mathbf{F}_{NE}$. Again, the structure of $\mathbf{M}_{NE}^{\mathrm{f}}$ is fixed for all the final demand users except for private consumption, where these shares change according to the evolution of domestic prices compared to import prices and their respective Armington elasticities.

The specification of imports for non-energy commodities is different. Imported intermediate inputs for a non-energy commodity are determined by multiplying the use structure matrix of every sector (normalised only for non-energy commodities) by the diagonal matrix of the factor shares of imported goods ($\hat{\mathbf{v}}_M$) derived in the translog model described in Section 2.2, and with the column vector of output $\mathbf{q}$ at basic prices: $\mathbf{S}^{\mathbf{m}}_{NE} \cdot \hat{\mathbf{v}}_M \cdot \mathbf{q}$.

Finally, the vector of total imports by commodities (**IM**) is given by joining the total imports for both energy and non-energy commodities:

$$\mathbf{IM} = \begin{bmatrix} \mathbf{IM}_{NE} \\ \mathbf{IM}_E \end{bmatrix} = \begin{bmatrix} \mathbf{S}^{\mathbf{m}}_{NE} \cdot \hat{\mathbf{v}}_M \cdot \mathbf{q} + \left(\mathbf{M}^{\mathrm{f}}_{NE} \odot \mathbf{F}_{FE} \right) \cdot \boldsymbol{\iota} \\ \left(\mathbf{M}^{\mathrm{m}}_E \odot \mathbf{U}_E \right) \cdot \boldsymbol{\iota} + \left(\mathbf{M}^{\mathrm{f}}_E \odot \mathbf{F}_E \right) \cdot \boldsymbol{\iota} \end{bmatrix} \tag{45}$$

2.3.2. Second stage: splitting total imports by country of origin

In the second stage, the vector of total imports by commodity is divided across sourcing countries. As in standard multi-regional IO modelling, the imports of each country determine the exports of the other countries. Compared to FIDELIO 1, this part of trade remains relatively unchanged, so many details are available in the FIDELIO 1 manual by Kratena et al. (2013). The basic facts are given below as a reminder.

Once a vector of total imports for each commodity is calculated in the first stage, the amounts in this vector have to be allocated to their specific origins (i.e. export regions). This is done by means of a matrix of trading partners (**TMSH**) where, for every region, the trading partners' import shares are accounted for each good. In FIDELIO 2 this matrix refers to a single user, hence shares by importing region are kept constant for all users. Imports from the rest of the world to one region are equal to the proportion of exports not accounted for by the rest of the countries included in the model. The calculation of imports by country is done separately for the services used in international trade and transport (TIR services) and the rest of the goods (non-TIR goods and services). Starting from the vector of total calculated in the first stage, **IM**, and **TMSH** (and without taking into consideration currency exchange rates), the matrix of imports by country for non-TIR goods and services ($\mathbf{TM}_{nTIR}$) is calculated as:

$$\mathbf{TM}_{nTIR} = \left(\mathbf{IM}_{nTIR} \cdot \mathbf{TMSH}_{nTIR} \right) \left(\odot^{-1} \right) \left(\mathbf{TNCS} \right), \tag{46}$$

where these calculation are done only for the non-TIR goods and services. Columns in $\mathbf{TM}_{nTIR}$ and in $\mathbf{TMSH}_{nTIR}$ are trading regions, rows are non-TIR goods and services, $\odot^{-1}$ stands for element-wise division, and **TNCS** is a transit cost matrix accounting for the transit costs of a third country which is not the destination region. Transit costs are zero in the case of services, hence the corresponding **TNCS** elements are set to unity.

TIR services are obtained with a two-step procedure. First, we determine the TIR services included in transit costs for non-TIR goods. These services are added to TIR services not associated to transit costs for non-TIR goods (directly available in the **IM** vector) and the total sum is distributed across countries of origin in a similar procedure to $\mathbf{TM}_{nTIR}$.

The TIR services included in transit costs for non-TIR goods:

$$\mathbf{TIRS} = \mathbf{TM}_{nTIR} \odot \left(\mathbf{TNCS} - \boldsymbol{\iota}\cdot\boldsymbol{\iota}'\right) \tag{47}$$

These transit cost services associated to non-TIR goods are aggregated for every trading partner and afterwards distributed among TIR services according to a region-specific share matrix. These shares are trading partner-specific:

$$\mathbf{TC}_{TIR} = \mathbf{TIRSH}\cdot\left(\boldsymbol{\iota}'\cdot\mathbf{TIRS}\right), \tag{48}$$

where **TIRSH** is a matrix whose columns describe the TIR services breakdown for every country of origin. These $\mathbf{TC}_{TIR}$ services are aggregated across trading countries in order to be distributed again afterwards in a similar approach to non-TIR goods and services:

$$\mathbf{TC}_{TIR,Agg} = \mathbf{TC}_{TIR}\cdot\boldsymbol{\iota} \tag{49}$$

The aggregation/disaggregation process described so far implies that TIR services associated to the transit costs of non-TIR goods coming from a specific country of origin are not necessarily supplied by transport companies from the same country of origin. On the contrary, it is more natural to assume that these services are distributed according to the **TMSH** matrix deduced for TIR services. Finally, (50) gives the trade matrix for TIR services by country of origin:

$$\mathbf{TM}_{TIR} = \left(\mathbf{IM}_{TIR} + \mathbf{TC}_{TIR,Agg}\right)\cdot\mathbf{TMSH}_{TIR} \tag{50}$$

The **TMSH** matrix changes over time. Shares evolve according to trade elasticities and to the evolution of the specific prices of imports of every trade partner. Basically, the procedure is as follows:

$$\mathbf{TMSH}_{t+1}\left(R,TR,g\right)=\mathbf{TMSH}_{t}\left(R,TR,g\right)\cdot\left[\mathbf{I}\left(TR,g\right)_{t+1/t}\right]^{Trade\ elasticity\left(R,g\right)}, \tag{51}$$

where R stands for the destination region, TR stands for the trading partners (the rest of the regions included in the FIDELIO 2 model). Also, $\mathbf{I}\left(TR,g\right)_{t+1/t}$ is a price index for the commodity g coming from the specific trading region TR. This index price is constructed endogenously in FIDELIO 2 using the domestic price index of that specific good in the trading region and it is marked up according to the rate of change of transit costs, tariffs and currency exchange rates between the country of origin and the destination country. Finally, the trade elasticities are the Armington elasticities for imported goods.

2.3.3. Armington elasticities sets in FIDELIO 2

There are three different sets of Armington elasticities that can be used within FIDELIO 2. Two sets are taken from the literature (the set used in the GTAP model, and another set derived from the estimates by Hertel et al., 2007 and Németh et al., 2011) and one has been estimated with WIOD data using a panel fixed effects model. Table 13 below contains the three sets of elasticities that can be used in FIDELIO 2 (the default is the one derived from the panel estimates with WIOD data). It emerges that the Armington elasticities from the literature show considerably higher values than those estimated from the WIOD data.

Table 13. Armington elasticities of FIDELIO 2 (not country-specific)

Commodities	Own estimates: final demand	Own estimates: intermediate	Hertel et al. (2007), Németh et al. (2011)	GTAP
01. Agriculture, hunting and related...	1.11	0.97	3.60	3.08
02. Forestry, logging and related...	1.11	0.97	2.50	2.50
05. Fishing, operating of fish hatcheries...	1.11	0.97	1.25	1.25
10. Mining of coal and lignite...	0.79	0.64	3.05	3.05
11. Extraction of crude petroleum and...	0.79	0.64	17.20	11.20
12. Mining of uranium and thorium ores	0.79	0.64	0.90	0.90
13. Mining of metal ores	0.79	0.64	0.90	0.90
14. Other mining and quarrying	0.79	0.64	0.90	0.90
15. Manufacture of food products and...	1.21	0.80	2.00	3.21
16. Manufacture of tobacco products	1.21	0.80	1.15	1.15

17. Manufacture of textiles	1.39	1.53	3.75	3.75
18. Manufacture of wearing apparel...	1.39	1.53	3.70	3.70
19. Tanning and dressing of leather...	0.00	0.81	4.05	4.05
20. Manufacture of wood and...	1.22	0.78	3.40	3.40
21. Manufacture of pulp, paper...	0.74	0.67	2.95	2.95
22. Publishing, printing, and...	0.74	0.67	2.95	2.95
23. Manufacture of coke, refined...	0.00	0.36	2.10	2.10
24. Manufacture of chemicals and...	0.76	0.70	3.30	3.30
25. Manufacture of rubber and plastic...	0.28	0.41	3.30	3.30
26. Manufacture of other non-metallic...	0.96	0.66	2.90	2.90
27. Manufacture of basic metals	0.67	0.81	2.95	3.58
28. Manufacture of fabricated metal...	0.67	0.81	4.20	3.75
29. Manufacture of machinery and...	0.00	0.00	3.75	4.23
30. Manufacture of office machinery...	0.75	0.57	4.05	4.23
31. Manufacture of electrical machinery...	0.75	0.57	4.05	4.23
32. Manufacture of radio, television...	0.75	0.57	4.40	4.40
33. Manufacture of medical, precision...	0.75	0.57	4.40	4.05
34. Manufacture of motor vehicles...	0.99	1.30	2.80	2.80
35. Manufacture of other transport...	0.99	1.30	4.30	4.30
36. Manufacture of furniture...	0.00	0.00	3.75	3.75
37. Recycling	0.00	0.00	3.75	3.75
40. Electricity, gas, steam and hot water...	0.45	0.60	2.80	2.80
41. Collection, purification and...	0.45	0.60	2.80	2.80
45. Construction	0.00	0.27	1.90	1.90
50. Sale, maintenance and repair...	0.43	0.68	1.90	1.90
51. Wholesale trade and commission...	0.84	0.30	1.90	1.90
52. Retail trade, except of motor...	0.56	0.41	1.90	1.90
55. Hotels and restaurants	0.93	1.02	0.00	1.90
60. Land transport, transport via pipelines	0.31	0.28	1.90	1.90
61. Water transport	0.69	0.75	1.90	1.90
62. Air transport	0.79	0.65	1.90	1.90
63. Supporting and auxiliary transport...	0.86	0.00	1.90	1.90
64. Post and telecommunication	0.67	0.68	1.90	1.90
65. Financial intermediation except...	0.38	0.26	1.90	1.90
66. Insurance and pension funding...	0.38	0.26	1.90	1.90
67. Activities auxiliary to financial...	0.38	0.26	1.90	1.90
70. Real estate activities	0.70	1.12	1.90	1.90
71. Renting of machinery and...	0.80	0.25	1.90	1.90
72. Computer and related activities	0.80	0.25	1.90	1.90
73. Research and development	0.80	0.25	1.90	1.90
74. Other business activities	0.80	0.25	1.90	1.90
75. Public administration and defence...	1.02	1.16	1.90	1.90
80. Education	0.57	0.65	1.90	1.90
85. Health and social work	0.52	0.71	1.90	1.90
90. Sewage and refuse disposal...	1.03	0.87	1.90	1.90
91. Activities of membership organisation	1.03	0.87	1.90	1.90
92. Recreational, cultural and...	1.03	0.87	1.90	1.90
93. Other service activities	1.03	0.87	1.90	1.90
95. Private households with employed...	0.00	0.00	1.90	1.90

For the second stage, trade elasticities for imports are basically the first stage's Armington elasticities multiplied by two. The logic behind this is the following: in the first stage, substitution between domestic and imported products is less likely to occur since substitution is not as strongly based on relative prices; hence, domestic and imported goods behave more like imperfect substitutes (Lächler, 1985). However, in the second stage, substituting imports among third countries is more sensitive to changes in relative prices. Thus, imports among different countries behave more like perfect substitutes, with larger elasticities.

2.4. The labour market

Some of the characteristics of the labour market modelled in FIDELIO 2 have not changed with respect to its first version, while others have been updated. The subsections below illustrate how the labour market is modelled within the current version of FIDELIO.

2.4.1. Demands for labour skill types

Three labour skills are modelled: high-, medium- and low-skilled. Labour demand by skill type is modelled with a translog model and can be seen as a second nest of the modelling of the demands of the factors of production. The unit cost in the labour market is the wage per hour which then defines the labour price (index). The hourly wage (*W*) is defined as follows (the parameters have been estimated with the SURE estimator with pooled data for the EU-27 countries separately for each industry):

$$\ln W = \beta_0 + \sum_{i \in \{l,h\}} \beta_i \tilde{w}_{im} + \ln w_m + \frac{1}{2} \sum_{i \in \{l,h\}} \gamma_{ii} \left(\tilde{w}_{im} \right)^2 + \gamma_{lh} \tilde{w}_{lm} \tilde{w}_{hm} , \tag{52}$$

where *h*, *m* and *l* refer to, respectively, high-, medium- and low-skilled labour, and the hourly wages of high-skilled and low-skilled labour are defined relative to the hourly wages of medium-skilled labour, i.e.: $\tilde{w}_{im} = \ln\left({w_i}/{w_m} \right)$ for $i = \{l, h\}$. Applying Shepard's lemma to equation (52) gives the following equations for the different labour type shares:

$$\begin{aligned} v_l &= \beta_l + \gamma_{ll} \tilde{w}_{lm} + \gamma_{lh} \tilde{w}_{hm} \\ v_h &= \beta_h + \gamma_{lh} \tilde{w}_{lm} + \gamma_{hh} \tilde{w}_{hm} \end{aligned} \tag{53}$$

2.4.2. Wage curves

The labour price of the translog model is defined by adding the employers' social security contribution to the employee's gross hourly wage. The combination of the *meta-analysis* of Folmer (2009) of the empirical wage curve literature with a basic wage bargaining model from Boeters and Savard (2013) gives a specification for the industry-specific hourly wages. These functions describe the responsiveness of hourly wages to labour productivity, to consumer prices, to the rate of unemployment, and to the hours worked per employee. The inclusion of the latter variable corresponds to assuming a bargaining model where firms and workers bargain over wages and hours worked simultaneously (Busl and Seymen, 2013). The basic idea is that the gains in labour productivity can be used for cutting hours worked and to increase wages simultaneously. While unions formally bargain over an hourly wage rate, they also take the annual (or monthly) wage income per head into account.

The wage function is specified so that the hours can be determined in a first step, and then the hourly wage rate afterwards. A bargaining process over hours that leads to less hours worked would *ceteris paribus* lower the annual wage income per head, leading workers to aim for an increase in the hourly wage rate (and avoid a fall in yearly income). In a search model, firms and workers bargain over the distribution of the value of a successful match and the wage rate can be derived from the optimality conditions of the problem (Boeters and Savard, 2013):

$$w = \frac{\lambda\left(\rho + \frac{s}{ur}\right)}{\pi_V\left(1 - br\right)}\gamma \tag{54}$$

In this wage equation, λ is the parameter measuring the bargaining power of workers, ρ is the discount rate, s is the (exogenous) separation rate, π_V the probability of filling a vacancy, and ur is the rate of unemployment. The cost of an open vacancy for the firm is measured by γ and br is the wage replacement rate of the unemployment benefit. One important property of the wage function is the reaction of the wage rate to the unemployment rate, which according to the empirical literature is about -0.1. Taking these considerations into account, we derive the following log-linear wage curve by industry:

$$\log w_{j,t}\left(1-t_{L,t}\right)=\alpha_{w,j}+\sum_{t=t-n}^{t}\beta_{1,wj}\log pc_t+\sum_{t=t-n}^{t}\beta_{2,wj}\log\left(Q_{j,t}/H_{j,t}\right)+$$
$$+\sum_{t=t-n}^{t}\beta_{3,wj}\log\left(Q_t/H_t\right)+\sum_{t=t-n}^{t}\beta_{4,wj}\log\left(ur^*/ur_t\right)-\sum_{t=t-n}^{t}\beta_{5,wj}\log\left(H_{j,t}/L_{j,t}\right) \tag{55}$$

The above specification includes various lags of the explanatory variables, including the consumer price. The term ur^*/ur_t considers the unemployment elasticity of the wage rate in terms of the difference to the equilibrium rate ur^* (proxied with its minimum value within the sample used for the estimates). The estimation of the parameter $\beta_{4,wj}$ yields the same result (only with $\beta_{4,wj} > 0$) as the parameter of the unemployment rate elasticity in the traditional wage curve, because all the variance in the term ur^*/ur_t stems from changes in the denominator. Such a specification of the unemployment term implies that wages increase when approaching full employment. Due to the non-stationarity of the variables, an autoregressive term is also included.[28]

Labour supply is given by age- and gender- (g) specific participation rates of the k age groups of the population at working age (16–65) and evolves over time according to demographic changes and logistic trends in participation rates. Therefore, labour supply does not react endogenously to policy shocks. Unemployed persons are the difference between labour supply and employment as follows:

$$UN_t=\sum_{k,g}\pi_{k,g,t}\,pop_{k,g,t}-w_tH_t\left(\frac{L_t}{H_t}\frac{1}{w_t}\right) \tag{56}$$

Total wages are found using the same method as the other factor inputs (E, M^m, and M^d) by multiplying the factor shares production block model with the column vector of output in current prices. Wage data including hours worked are taken from WIOD sectoral accounts and are complemented by labour force data from Eurostat. The wage equations have been estimated for the full EU-27 panel. Table 1 only shows the short-run coefficients, the long-run elasticities are considerably larger and for the unemployment elasticity (ur^*/ur_t) the unweighted average across industries is about 0.09 (these estimates are the same as those of

[28] The separation rate and the probability of filling a vacancy have not been included in equation (55) due to data availability problems. The income replacement rate of the unemployment benefit did not yield significant results in the panel data estimation across European countries. The latter finding suggests that there is no clear correlation between the generosity of the unemployment benefit regulation and the unemployment rate.

the first version of the FIDELIO model). The long-run productivity elasticity of wages is also correspondingly higher and close to unity. Not all industries show a significant impact of hours worked on the hourly wage rate. In industries without this coefficient in the wage curve, a reduction of hours worked *ceteris paribus* leads to a proportional income loss for workers.

Table 14. Estimation results of industry wage equations (EU-27, 1995–2009)

	$\log(PC_{t-1})$	$\log\left(Q_{i,t-1}/H_{i,t-1}\right)$	$\log\left(ur^*/ur_t\right)$	$\log\left(H_{i,t}/L_{i,t}\right)$
Agriculture	0.289***	0.133**	0.019*	
Mining, quarrying		0.438*	0.054***	
Food, beverages	0.268***	0.100*	0.015***	
Textiles	0.019*	0.036**	0.048***	
Leather, footwear	0.442***		0.001***	-0.334***
Wood, cork	0.315***	0.040	0.024***	-0.013***
Pulp, paper	0.181***	0.055***	0.021**	-0.007***
Coke, refinery	0.043*		0.007	
Chemicals	0.175***	0.031*	0.024**	-0.046***
Rubber, plastics	0.291***	0.114**	0.011***	
Non-metal materials	0.323***	0.223**		-0.224***
Basic metals	0.267*	0.083**	0.017***	-0.051***
Machinery	0.231***	0.026*	0.026***	
Electrical equipment		0.156*	0.003***	
Transport equipment	0.170***	0.090***	0.009	-0.155***
Other manufacturing	0.138***	0.020*	0.018*	-0.029***
Electricity, gas, water	0.181***	0.233	0.023**	-0.121***
Construction	0.232***	0.052**	0.032***	-0.145***
Sale of motor vehicles	0.265***	0.027	0.020***	-0.080***
Wholesale trade	0.342***	0.057***	0.009***	-0.213
Retail trade	0.138***	0.213***	0.015***	-0.244***
Hotels, restaurants	0.088	0.158***	0.033***	
Inland transport	0.169***	0.050**	0.028***	-0.150***
Water transport	0.223***	0.036***	0.008	-0.145***
Air transport	0.113***	0.027	0.022*	-0.197***
Other transport activities	0.330***	0.290***	0.010***	-0.202***
Post, telecommunications	0.232***	0.150**	0.021	-0.112***
Financial intermediation	0.125***	0.125***	0.026***	
Real estate activities	0.235***		0.017***	-0.071***
Other business activities	0.428***	0.059*	0.030***	-0.242***
Public administration	0.223*	0.210**	0.021***	
Education	0.136***	0.124***	0.020***	
Health	0.187**	0.180***	0.031***	-0.182***
Social, personal services	0.000	0.365***	0.005***	

2.5. Government and model closure

The public sector balances close the model and summarise the main interactions among households, firms and the government. Taxes paid by households and firms are endogenous (and behave according to certain tax rates). In order to be able use FIDELIO 2 to study labour market policies, unemployment benefits are separated from the other social expenditure categories. Government consumption is set to behave so as to make the path of the deficit/GDP move according to the EU stability programmes in EU countries. Alternatively, government consumption can be set to move according to a simpler path, for instance with a fixed growth rate or with a rate equal to the growth rate of the economy.

The wage income of households is taxed with social security contributions (the tax rates are the following: t_{wL} and t_L) and the wage income plus operating surplus accruing to households is taxed with income taxes (the tax rate is t_Y). Additionally, households' gross profit income is taxed with tax rate t_r. Taxes less subsidies are not only levied on private consumption, but also on the other final demand components in purchasers' prices ($\mathbf{f}_{\mathbf{pp}}$, including capital formation, changes in stocks, exports, and public consumption) as well as on gross output. Total tax revenues of government, T_t, are obtained as follows:

$$T_t = (t_{wL} + t_L) w_t H_t + t_Y (w_t H_t + \Pi_{h,t}) + t_r r_t A_{t-1} + \hat{\mathbf{T}}_{\mathrm{N}} \left[\mathbf{c}_{\mathrm{pp},t} + \mathbf{f}_{\mathrm{pp},t} + \mathbf{p}_{\mathrm{Q},t} \mathbf{Q}_{\mathbf{t}} \right] \tag{57}$$

Taxes less subsidies and profit income also include the economic activity of the public sector itself. The expenditure side of government is made up of unemployment transfers (calculated as follows: $brw_t (1 - t_S - t_Y) UN_t$) and other transfers to households (Tr), public investment (cf_{gov}) and public consumption (cg). Additionally, the government pays interests on public debt D_{gov} with interest rate r_{gov}. The change in public debt is equal to negative government net lending, which is then given by:

$$\begin{aligned} \Delta D_{gov,t} &= brw_t (1 - t_S - t_Y) UN_t + Tr_t + cf_{gov,t} + cg_t + r_{gov,t} D_{gov,t-1} + \\ &- (t_{wL} + t_L) w_t H_t - t_Y (w_t H_t + \Pi_{h,t}) - t_r r_t A_{t-1} - \hat{\mathbf{T}}_{\mathrm{N}} \left[\mathbf{c}_{\mathrm{pp,t}} + \mathbf{f}_{\mathrm{pp,t}} + \mathbf{p}_{\mathrm{Q,t}} \mathbf{Q}_{\mathrm{t}} \right] \end{aligned} \tag{58}$$

The model is closed by further introducing a public budget constraint, specified according to the stability programme for public finances that defines the future path of government net lending to GDP ($p_y Y$). The latter can be defined as the difference between total output ($p_Q Q$) and intermediate demand ($p_E E$, $p_{Mm}M^m$, $p_{Md}M^d$). Linking public investment with a fixed ratio (w_{cf}) to public consumption and introducing the net lending to GDP constraint, public consumption is then derived as the endogenous variable that closes the model:

$$
\begin{aligned}
cg\left(1+w_{cf}\right) = {}& \Delta D_{gov,t} \big/ p_y Y - r_{gov,t} D_{gov,t-1} - brw_t\left(1-t_S-t_Y\right)UN_t - Tr + \\
& \left(t_{wL}+t_L\right)w_t H_t + t_Y\left(w_t H_t + \Pi_{h,t}\right) + t_r r_t A_{t-1} + \hat{\mathbf{T}}_{\mathrm{N}}\left[\mathbf{c}_{\mathrm{pp,t}} + \mathbf{f}_{\mathrm{pp,t}} + \mathbf{p}_{\mathrm{Q,t}}\mathbf{Q}_{\mathrm{t}}\right]
\end{aligned}
\tag{59}
$$

Therefore, transfers and tax rates are treated like fiscal policy variables, whereas public consumption and investment adjust according to the net lending to GDP constraint. Euro area countries in FIDELIO 2 are bound to behave according to the Stability and Growth Pact as modified by the Six Pack and to the Fiscal Compact (Directive 2011/85/EU) introduced in 2013 and 2014 which force countries to reduce their debt/GDP when over 60%.

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doi:10.2760/313390

ISBN 978-92-79-66258-4