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**Abstract**
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The Global Multi-country Model (GM): an Estimated DSGE Model for Euro Area Countries

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Abstract

This paper presents the European Commission’s Global Multi-country model (the GM model). The GM model is an estimated multi-country DSGE model that can be used for spillover analysis, forecasting and medium term projections. Its development is jointly performed by the Joint Research Centre and DG ECFIN. Since the GM model is developed to be flexible under different country configurations, we present the GM model in its configuration designed for EMU-countries (GM3-EMU), which has been estimated for the four largest European economies (Germany, France, Italy and Spain). We analyse business cycle properties, present the model fit and provide a quantitative assessment of the relative importance that supply, demand, and international shocks, as well as, discretionary policy interventions had in explaining the cyclical patterns observed in each country since the establishment of the EMU. We also discuss the main issues related to fitting observed data, and the associated modelling and estimation approaches that we applied so far and the main lines of development that are in place to further tackle such issues.

Keywords: DSGE, Bayesian estimation, EMU, Business cycle, Model fit, Cross-country comparison

JEL classification: C51, E31

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1. Introduction

The European Commission’s Global Multi-country model (GM model) belongs to the class of Dynamic Stochastic General Equilibrium (DSGE) models and it has been jointly developed by the Joint Research Centre and DG ECFIN. It is a fully estimated structural macroeconomic model, which can be used for spillover analysis, forecasting and medium term projections and it has been developed to be flexible to allow for different country configurations. The first estimated three-region version of the GM model (Euro Area, US and Rest of the World) analyses different patterns of the post-crisis slump in the Euro Area and the US (see Kollmann et al., 2016). Since Autumn 2015, this version has been used to perform decomposition of GDP growth forecasts by the European Commission (e.g. European Commission, 2017).

In this paper, we present the GM model in its configuration designed for EMU-countries (GM3-EMU), which has been estimated for the four largest EMU Member State countries (Germany, France, Italy and Spain). The GM3-EMU model consists of three countries, an EMU member state, the Rest of the Euro Area (REA) and the Rest of the World (RoW). The single EMU country economy features rich dynamics, while REA and RoW economies are defined in a rather stylized form.

The model builds on estimated versions of the QUEST III model (Ratto et al., 2009), from which it inherits most of its structure. The main differences with respect to Ratto et al. (2009) are: (i) the EA is split into two subregions (EMU-country and REA), which are linked by trade and flow of assets plus a common monetary policy for the Euro Area; (ii) the trade sector incorporates oil imports that are used for the production of domestic total output. It seems useful to underline here that, in the last decade, there has been a series of estimated versions of the QUEST III model, designed to address specific research questions at Euro Area (in’t Veld et al. (2011), Kollmann et al. (2013)) or EMU level (in’t Veld et al. (2014), in’t Veld et al. (2015) for Spain, Kollmann et al. (2015) for Germany). The QUEST versions have a larger degree of complexity, e.g. featuring housing and credit constrained households (borrowing constraint on mortgage loans) and, for Spain, some more detailed financial cross-holdings. Since it is intended to be regularly estimated for each forecast round, for all different country settings, the GM3-EMU is kept at a relatively smaller degree of complexity to lower the computational burden. In the context of this empirical focus, we also show specific modelling extensions to key components of the model (namely labor demand, hybrid Phillips curve, trade) that can be applied to better fit country specific patterns in the data, which would be otherwise captured with more difficulty by the plain core specifications. Such extensions are also designed to nest the core model specifications.

In terms of model complexity and specification, the GM model lies within the existing
literature on medium and large-scale DSGE models used at policy institutions, e.g. the ECB’s New Area Wide Model (NAWM) (see Christoffel et al., 2008; Karadi et al., 2017) or the SIGMA model from the Federal Reserve Board (Erceg et al., 2006). Additionally, most of the National Central Banks of Eurosystem have a DSGE model tailored to capture country-specific business cycles fluctuations, e.g. MEDEA (Burriel et al., 2010) and FiMOD (Stähler and Thomas, 2012) for Spain, AINO 2.0 for Finland (Kilponen et al., 2016) or GEAR for Germany (Gadatsch et al., 2016). However, even if they belong to the same class of models, all of these country-specific structural models differ along some dimensions. More precisely, the models are not always fully estimated, and even then, the information set used to discipline the estimations vary substantially from one country model specification to another. Since the information set might change substantially the structural estimates and model conclusions (see Canova et al., 2014), care is needed when comparing estimates proceeding from different data sources. Therefore, it is crucial to be prudent when using the recent empirical literature to perform meaningful cross-country comparisons.

The GM model contributes to the existing literature by providing a common platform to compare the estimates for the four largest EA countries. The appeal of the GM model is that the core structure used to estimate country-specific dynamics is ex-ante identical. In other words, all country-specific models have the same equations, the same prior parameter distributions and the same shocks. Models are only allowed to differ by the specific extensions previously mentioned, which are only applied whenever the fit is significantly improved, i.e. only if they satisfy information criteria addressing the trade-off between fit and model parsimony. Moreover, when we bridge the EA country-specific models to the data, we use the same information set and the same time span. While times series are typically different (so their empirical moments), the selection of observable variables is identical. All these features offer an interesting benchmark for meaningful cross-country comparisons and for a direct measurement of heterogeneity in the nature (size and persistence) of the shocks and in the transmission mechanisms associated to them.

This paper presents the theoretical specification of the GM model, it evaluates its fitting properties, its ability to replicate key moments in the data, and it analyses how the internal transmission dynamics vary across countries. Moreover, it provides a quantitative assessment of the relative importance that supply and demand factors, international shocks and discretionary policy interventions had in explaining the cyclical patterns observed in each country since the establishment of EMU. More on the empirical side, the paper also discusses the main issues related to fitting observed data, and the associated modelling and estimation approaches that are applied. It is also important to underline that the GM model project is in constant development, and the results presented here freeze the first milestone.
of the project, i.e. before the start of the Autumn Forecast round 2017. Therefore, we will also summarize the main lines of development that are in place to tackle the open issues to better capture observed data.

The remainder of the paper is organised as follows: Section 2 presents the theoretical model environment. The model solution and the econometric approach are discussed in Section 3. Section 4 presents and discusses the estimation results. Section 5 concludes.

2. The model

The model features three regions: an EMU country, the REA and the RoW. The EMU domestic economy is composed of households, non-financial firms operating either in the domestic market or in the import-export sector, a Government and a Central Bank.

We distinguish between two types of infinitely-lived households: Ricardian households have access to financial markets, can smooth their consumption and own the firms, and liquidity-constrained households who consume their disposable wage and transfer income each period and do not own any financial wealth. Both types of households provide labour services to domestic firms, at the wage set by a labour union with monopoly power.

In the domestic production sector, monopolistically competitive firms produce a variety of differentiated intermediate goods, which are assembled by perfectly competitive firms to a domestic final output good (value added). In a final step, perfectly competitive firms produce total output by combining value added with energy input.

In the import sector, perfectly competitive firms (import retailers) buy economy-specific goods from the foreign country and assemble them to a final imported good. Final good packagers combine the final imported good with domestic output to final aggregate demand components goods.

The fiscal authority purchases domestic final goods and makes lump-sum transfers to households that are financed by issuing debt and levying both distortionary taxes on labor, capital, and consumption as well as non-distortionary lump-sum taxes. Given the monetary union setting, the Central Bank sets the nominal interest rate following a Taylor rule defined on EA aggregate inflation and output gap.

The REA and RoW economies are more stylized, featuring a standard three equations New Keynesian model, consisting of an Euler equation for consumption, a New Keynesian Phillips curve and a Taylor rule. The model is augmented by international trade dimensions.

2.1. EMU country households

There is a continuum of households, indexed by $j \in [0, 1]$, living in each $k$ region. A share $\omega_k^s$ of households - savers ($s$) - owns firms and trades assets in the financial market.
The remaining share is liquidity-constrained ($c$) and consumes its entire wage and transfer income each period. Both households preferences are defined over consumption and leisure. Additionally, Ricardians’s utility depends also on the beginning-of-period financial assets holdings.

2.1.1. Ricardian households

Ricardian preferences are specified by the following infinite horizon expected life-time utility:

$$U_{j,k}^s = \sum_{t=0}^{\infty} (\tilde{\beta}_{k,t})^t u_{j,k,t}(.),$$

where $\tilde{\beta}_{k,t} = \beta_k \exp(\varepsilon_{k,t-1}^c)$, $\beta_k$ is the (non-stochastic) discount factor, $\varepsilon_{k,t}^c$ captures a shock to the subjective rate of time preference (saving shock).

Ricardians have full access to financial markets, allowing them to accumulate wealth, $A_{j,k,t}$, which consists of domestic private risk-free bonds, $B_{j,k,t}^{rf}$, domestic government bonds, $B_{j,k,t}^G$, one internationally traded bond, $B_{j,k,t}^W$, and domestic shares, $P_{k,t}^S S_{j,k,t}$:

$$A_{j,k,t} = B_{j,k,t}^{rf} + B_{j,k,t}^G + e_{RoW,k,t,}B_{j,k,t}^W + P_{k,t}^S S_{j,k,t},$$

where $P_{k,t}^S$ is the nominal price of shares at time $t$. Since the international bond is issued in RoW currency, financial wealth depends on the nominal exchange rate $e_{RoW,k,t}$.

The instantaneous utility function of savers, $u_s(.,)$, is defined as:

$$u_{j,k,t}^s(C_{j,k,t}^s, N_{j,k,t}^s, U_{j,k,t-1}^{A,P_{C,vat}}) = \frac{1}{1 - \theta_k} (C_{j,k,t}^s - h_k C_{k,t-1}^s)^{1 - \theta_k} - \frac{\omega_N^{U} e_{RoW,k,t} C_{k,t}}{1 + \theta_k^N} (N_{j,k,t})^{1 + \theta_N^U}$$

$$- (C_{k,t}^s - h_k C_{k,t-1}^s)^{-\theta_k} \frac{U_{j,k,t-1}^{A,P_{C,vat}}}{P_{k,t}^{C,vat}},$$

where $C_{k,t}^s = \int_0^1 C_{j,k,t}^s dj$, $h_k$ measures the strength of external habits in consumption and $\omega_N$ the weight of the disutility of labor. $\varepsilon_{k,t}^U$ captures a labor supply shock. The disutility of holding financial assets, $U_{j,k,t-1}^{A}$, takes the following form:

$$U_{j,k,t-1}^{A} = \left( \alpha_k^{b_0} + \varepsilon_{k,t-1}^B \right) B_{j,k,t-1}^G + \left( \alpha_k^{b_0} + \varepsilon_{k,t-1}^b \right) e_{RoW,k,t} B_{j,k,t-1}^W + \frac{\alpha_k^{b_1}}{2} \frac{(e_{RoW,k,t-1} B_{k,t-1}^W)^2}{P_{k,t-1}^{C,vat} Y_{k,t-1}}$$

$$+ \left( \alpha_{k}^{S_0} + \varepsilon_{k,t-1}^S \right) P_{k,t}^S S_{j,k,t-1}.$$
depends on an asset specific exogenous shock $\varepsilon^x$, $x \in \{B, S, bw\}$, and an asset specific intercept $\alpha^x$, $x \in \{b_0, S_0, bw_0\}$. Similar to Fisher (2015), we aim at capturing the households preferences for the safe short term bonds,\(^1\) which generates endogenously a wedge between the return on risky assets and safe bonds. As in Benigno (2009) and Ratto et al. (2009), we assume that only the RoW bond is traded internationally.\(^2\) It follows that households in the Euro Area can invest in both national and foreign assets, while RoW households can only invest in domestic bonds.

The $j^{th}$ Ricardian household faces the following budget constraint, for $t = 0, \ldots, \infty$:

$$P_{k,t}^{C,\text{vat}} C_{j,k,t}^s + A_{j,k,t} = (1 - \tau^N_k) W_{k,t} N_{j,k,t}^s + (1 + i_{k,t-1}) B_{j,k,t-1}^{rf} + (1 + i^G_{k,t-1}) B_{j,k,t-1}^G + (P_{k,t}^S + P_{k,t}^Y \Pi_{k,t}^f) S_{j,k,t-1} + (1 + i^W_{k,t-1}) e_{RoW,k,t} B_{j,k,t-1}^W + T_{j,k,t}^s - tax_{j,k,t}^s,$$

where $P_{k,t}^{C,\text{vat}}$ defines the private consumption deflator\(^3\) in terms of input factors, $W_{k,t}$ denotes the nominal wage rate, $N_{j,k,t}^s$ is the employment in hours, $T_{j,k,t}^s$ are government transfers and $tax_{j,k,t}^s$ lump-sum taxes paid by savers. Moreover, $i_{k,t}$, $i^G_{k,t}$, and $i^W_{t}$ are returns on domestic private risk-free bonds, domestic government bonds, and internationally traded bonds, respectively. As Ricardian households own the firms, they receive nominal profits in form of dividends $\Pi_{k,t}^f$, that are distributed by differentiated goods producers according to the number of shares held by the households. We define the gross nominal return on shares $S_t$ as:

$$1 + i^s_{k,t} = \frac{P_{k,t}^S + P_{k,t}^Y \Pi_{k,t}^f}{P_{k,t-1}^S}.$$  

The Ricardian household maximizes the present value of the expected stream of future utility subject to equation 1, by choosing the amount of consumption, $C_{j,k,t}^s$, and next period holdings of assets: $B_{j,k,t}^{rf}$, $B_{j,k,t}^G$, $S_{j,k,t}$. The maximization problem results in the following first-order conditions (FOCs):

$$\lambda_{j,k,t}^s = (C_{k,t}^s - h_k C_{k,t-1}^s)^{-\theta_h},$$

\(^1\)The modification is along the lines of Sidrauski (1967), in which model agents derive utility from their holdings of money. More recently, Vitek (2014), Vitek (2017) and Krishnamurthy and Vissing-Jorgensen (2012) adopt a similar framework.

\(^2\)This assumption consists in assuming a reduced form of a global bank lending domestically and abroad in RoW. A similar formulation is adopted also by Kollmann et al. (2013).

\(^3\) $P_{k,t}^{C,\text{vat}}$ is the VAT adjusted private consumption deflator, $P_{k,t}^{C,\text{vat}} = (1 + \tau^C_k) P_{k,t}^C$, where $\tau^C$ is the tax rate on consumption (VAT).
The Euler equations incorporate asset-specific risk premia. This allows capturing both the international spillovers that occur via the financial market channel, and the financial frictions that are commonly believed to have contributed to the first phase of the financial crisis and may have contributed to its second phase.\footnote{Observationally, this approach is equivalent to assuming exogenous risk premia as well as endogenous risk premia derived, e.g., in the spirit of Bernanke et al. (1996).}

Given the monetary union setting, the nominal exchange rate between the $k$-th EMU country and EA, $e_{EA,k,t} = 1$, implying that $\Delta \ln e_{RoW,EA,t+1} = \Delta \ln e_{RoW,k,t+1}$. We assume that an uncovered interest rate parity condition links the interest rate of the EMU country, $i_{k,t}$, to the EA interest rate (set by the central bank):

$$(1 + i_{k,t}) = (1 + i_{EA,t}) - (\alpha_{bw}^{RoW,EA,t} B_{k,t}^W + \varepsilon_{FQ}^k),$$

where $\alpha_{bw}^{RoW,EA,t} B_{k,t}^W$ captures a debt-dependent country risk premium on net foreign asset holdings as external closure to ensure long-run stability (see, e.g., Schmitt-Grohe and Uribe (2003), Adolfson et al. (2008)). Following Smets and Wouters (2007) we also introduce an additional risk premium shock, $\varepsilon_{FQ}^k$ ('Flight to Safety'), which creates a wedge between the EA interest rate, $i_{EA,t}$, and the return on domestic risk-free assets, $i_{k,t}^{rf}$. Since a positive shock increases the required return on domestic assets and the cost of capital, it reduces current consumption and investment simultaneously and helps explaining the comovement of consumption and investment.

### 2.1.2. Liquidity-constrained households

Liquidity-constrained households have no access to financial markets. Hence, the instantaneous utility functions, $u^c(\cdot)$, is:

$$u^c(C_{j,k,t}^c, N_{j,k,t}^c) = \frac{1}{1 - \theta_k} (C_{j,k,t}^c - h_k C_{k,t-1}^c)^1 - \theta_k - (C_{k,t})^{1-\theta_k} \xi_{k,t}^U \varepsilon_{k,t}^{U} 1 + \theta_k (N_{j,k,t}^c)^1 + \theta_k N_{j,k,t}^c.$$
In each period, they consume their disposable net income, which consists of labor income and net lump-sum transfers from the government. The budget constraint is described by:

\[(1 + \tau_C^C)P_{k,t}^C C_{j,k,t}^C = (1 - \tau_N^N)W_{k,t} N_{j,k,t}^c + T_{j,k,t}^c - tax_{j,k,t}^c.\]

2.1.3. Wage setting

Households are providing differentiated labor services \(N_{j,k,t}\) in a monopolistically competitive market. We assume that there is a labor union that bundles labor hours provided by both types of domestic households into a homogeneous labor service and resells it to intermediate good producing firms. We assume that Ricardian and liquidity-constrained households’ labor are distributed proportionally to their respective population shares \((\omega_k^s)\). Since both households face the same labor demand schedule, each household works the same number of hours as the average of the economy. It follows that the individual union’s choice variable is a common nominal wage rate for both types of households.

The union maximizes the discounted future stream of the weighted average of lifetime utility of its members with respect to the wage and subject to the weighted sum of their budget constraints. Additionally, we allow for real wage rigidity as in Blanchard and Galí (2007) and Coenen and Straub (2005), where the slow adjustment of real wages occurs through distortions rather than workers’ preferences. From the first order condition, the wage rule is determined by equating the marginal utility of leisure \(U_{N,k,t}\) to the weighted average of the marginal utility of consumption \(\lambda_{k,t}\) times the real wage adjusted for a wage mark-up:\(^5\)

\[
W_{k,t} \left[ (1 - \tau_N^N) \frac{\partial adj_{j,k,t}^W}{\partial W_{k,t}} \right] - \tilde{\beta}_{k,t} E_t \left[ \frac{\lambda_{k,t+1} P_{k,t}^{C,\text{vat}}}{\lambda_{k,t} P_{k,t}^{C,\text{vat}}} \frac{1}{P_{k,t}} \frac{\partial adj_{j,k,t+1}^W}{\partial W_{k,t+1}} \right] = W_{k,t} \left[ \frac{\mu_k^w U_{N,k,t} P_{k,t}^{C,\text{vat}}}{\lambda_{k,t} P_{k,t}^{Y}} \right]^{1 - \gamma_{k,t}^{wr}} \left[ \frac{(1 - \tau_N^N)W_{k,t-1} P_{k,t-1}^{Y}}{P_{k,t-1}^{Y}} \right]^{\gamma_{k,t}^{wr}}
\]

where \(\mu_k^w\) is the gross wage mark-up, \(\gamma_{k,t}^{wr}\) represents the index of real rigidity, and \(\varepsilon_{k,t}^w\)

\(^5\)As the German government implemented an extensive labour market deregulation in 2003-05 (‘Hartz’ reforms) that included a reduction in unemployment benefits, we capture the effect of the ‘Hartz reforms’ by treating the unemployment benefit ratio (ratio of unemployment benefit to wage rate) as an autocorrelated exogenous variable. Following the approach by Kollmann et al. (2015), we observe the historical benefit ratio and estimate the labour market reform as an exogenous permanent reduction in the unemployment benefit ratio. Therefore, real unemployment benefits (paid to unemployed workers of the labour force) enters the budget constraints of the households and the government and enters also in the wage setting equation. Since it is only a German-specific labour market shock, we abstract from the inclusion into the general model equations.
captures a shock to the wage markup (labour supply shock). Note that we do not observe wage dispersion in equilibrium, thus \( N_{d,k,t} = N_{k,t} \).

2.2. EMU country production sector

2.2.1. Total output demand

Total output \( O_{k,t} \) is produced by perfectly competitive firms by combining value added, \( Y_{k,t} \), with energy input, \( Oil_{k,t} \), using the following CES production function:

\[
O_{k,t} = \left[ \left( 1 - s^O_{k,t} \right) \frac{1}{\sigma^o_k} (Y_{k,t})^{\frac{\sigma^o_k - 1}{\sigma^o_k}} + \left( s^O_{k,t} \right) \frac{1}{\sigma^o_k} (Oil_{k,t})^{\frac{\sigma^o_k - 1}{\sigma^o_k}} \right]^{-\frac{1}{\sigma^o_k}}
\]

where \( s^O_{k,t} \) is the energy input share\(^6\) and \( \sigma^o_k \) is the elasticity of substitution between factors. Each total domestic output firm maximizes its expected profits:

\[
\max_{Y_{k,t},Oil_{k,t}} P^O_{k,t} O_{k,t} - P^Y_{k,t} Y_{k,t} - P^{Oil}_{k,t} Oil_{k,t}
\]

subject to the production function (2). The respective first order conditions for the total domestic output and oil are given by:

\[
Y_{k,t} = (1 - s^O_{k,t}) \left( \frac{P^Y_{k,t}}{P^O_{k,t}} \right)^{-\sigma^o_k} O_{k,t},
\]

\[
Oil_{k,t} = s^O_{k,t} \left( \frac{P^{Oil}_{k,t}}{P^O_{k,t}} \right)^{-\sigma^o_k} O_{k,t}.
\]

Oil is assumed to be imported from the RoW. Hence, the oil price is taken as given:

\[
P^{Oil}_{k,t} = e_{Row,k,t} P^{Oil}_{Row,t} + \tau^{Oil} P^0_t,
\]

where \( e_{Row,k,t} \) is the exchange rate, measured as price of foreign currency in terms of domestic currency, \( \tau^{Oil} \) and \( P^0_t \) are the excise duty and (global) GDP deflator, respectively. The price index of the composite total output is:

\[
P^O_{k,t} = \left[ (1 - s^O_{k,t}) (P^Y_{k,t})^{\sigma^o_k - 1} + s^O_{k,t} (P^{Oil}_{k,t})^{\sigma^o_k - 1} \right]^{\frac{1}{1-\sigma^o_k}}.
\]

\(^6\)Note that \( s^O_{k,t} \) is perturbed by a trend shock to the degree of country openness, as specified below in equation (12).
2.2.2. Value added sector

Value added, $Y_{k,t}$, is produced by perfectly competitive firms by combining a large number of differentiated goods, $Y_{i,k,t}$, produced by monopolistically competitive firms, according to a Dixit and Stiglitz (1977) production technology:

$$Y_{k,t} = \left[ \int_{0}^{1} Y_{i,k,t}^{\frac{\sigma_Y - 1}{\sigma_Y}} \frac{\sigma_Y}{\sigma_Y - 1} \, di \right]^{\frac{1}{\sigma_Y - 1}},$$

where $\sigma_Y$ represents the inverse of the steady state gross price mark-up on differentiated goods. The demand for a differentiated good $i$ is then:

$$Y_{i,k,t} = \left( \frac{P_{i,k,t}}{P_{k,t}^Y} \right)^{-\sigma_Y} Y_{k,t}, \quad (3)$$

where $P_{i,k,t}$ is the price of intermediate inputs and the corresponding price index is:

$$P_{k,t}^Y = \left( \int_{0}^{1} (P_{i,k,t})^{1-\sigma_Y} \, di \right)^{1-\sigma_Y}.$$  

2.2.3. Intermediate goods producers

Each firm $i \in [0, 1]$ produces a variety of the domestic good which is an imperfect substitute for varieties produced by other firms. Given imperfect substitutability, firms are monopolistically competitive in the goods market and face a downward-sloping demand function for goods.

Differentiated goods are produced using total capital, $K_{i,k,t-1}^{tot}$, and labor, $N_{i,k,t}$, which are combined in a Cobb-Douglas production function:

$$Y_{i,k,t} = \left[ \left( \frac{A_{k,t}^Y (N_{i,k,t} - FN_{i,k,t})}{CU_{i,k,t} K_{i,k,t-1}^{tot}} \right)^{\alpha_k} (CU_{i,k,t} K_{i,k,t-1}^{tot})^{1-\alpha_k} - A_{k,t}^Y FC_{i,k} \right] A_{k,t}^Y, \quad (4)$$

where $\alpha_k$ is the steady-state labor share, $A_{k,t}^Y$ represents the labor-augmenting productivity common to all firms in the differentiated goods sector, $CU_{i,k,t}$ and $FN_{i,k,t}$ are firm-specific levels of capital utilization and labor hoarding\(^7\), respectively. $FC_{i,k}$ captures fixed costs in production. Total capital is a sum of private installed capital, $K_{i,k,t}$, and public capital, $K_{i,k,t}^G$:

$$K_{i,k,t}^{tot} = K_{i,k,t} + K_{i,k,t}^G.$$  

\(^7\)According to Burnside and Eichenbaum (1996), firms prefer not to layoff workers when the demand is temporarily low, because firing workers may be more costly than hoarding them. Note that the inclusion of labor hoarding, $FN_{i,k,t}$, allows to match the observed co-movement between output and working hours.
Since total factor productivity (TFP) is not a stationary process, we allow for two types of shocks. They are related to a non stationary process and its autoregressive component:

\[
\log(A^Y_{k,t}) - \log(A^Y_{k,t-1}) = g^Y_{k,t} + \varepsilon^L_{k,t},
\]

\[
g^Y_{k,t} = \rho^Y g^Y_{k,t-1} + (1 - \rho^Y) g^{Y_0} + \varepsilon^Y_{k,t},
\]

where \(g^Y_{k,t}\) and \(g^{Y_0}\) are the time-varying growth and the long-run growth of technology, and \(\varepsilon^L_{k,t}\) is a permanent technological shock.

Monopolistically competitive firms maximize the real value of the firm \(P_{i,k,t} S^Y_{tot,k,t}\), that is the discounted stream of expected future profits, subject to the output demand (3), the technology constraint (4) and a capital accumulation equation \(K_{i,k,t} = I_{i,k,t} + (1 - \delta_k) K_{i,k,t-1}\). Their problem can be written as:

\[
\max_{P_i,N_i,I,K,CU,FN} \sum_{s=t}^{\infty} D_k^s \Pi^f_{i,k,t},
\]

where the stochastic discount factor, \(D_k^s\), is:

\[
D_k^s = \frac{1 + r_{k,t}^S}{\prod_{r=t}^{s}(1 + r_{k,r}^S)},
\]

with \(1 + r_{k,t}^S = \frac{1 + g_{k,t+1}^Y}{1 + \pi_{k,t+1}^Y}\) being the real stock return. The period \(t\) profit of an intermediate goods firm \(i\) is given by:

\[
\Pi^f_{i,k,t} = (1 - \tau^K_k)(P^-_{i,k,t} Y_{i,k,t} - W_{k,t} N_{i,k,t}) + \tau^K_k \delta_k P^I_{i,k,t} K_{i,k,t-1} - P^I_{i,k,t} I_{i,k,t} - \text{adj}_{i,k,t},
\]

where \(I_{i,k,t}\) is the physical investment at price \(P^I_{i,k,t}\), \(\tau^K_k\) is the corporate tax and \(\delta_k\) the capital depreciation rate.

Following Rotemberg (1982), firms face quadratic adjustment costs, \(\text{adj}_{i,k,t}\), measured in terms of production input factors. Specifically, the adjustment costs are associated with the output price \(P^Y_{i,k,t}\), labor input \(N_{i,k,t}\), investment \(I_{i,k,t}\), as well as capacity utilization variation \(CU_{i,k,t}\) and labor hoarding \(FN_{i,k,t}\):

\[
\text{adj}_{i,k,t}^P = \sigma_k^Y \gamma_k^F \frac{Y_{k,t}}{2} \left[ \frac{P^Y_{i,k,t}}{P^Y_{i,k,t-1}} - \exp(\pi) \right]^2,
\]
where $\gamma$-s capture the degree of adjustment costs and $\delta_K \neq \delta_k$ is a function of the depreciation rate adjusted for the capital trend in order to have zero adjustment costs on the trend-path.\footnote{We specify $\delta_k = \exp(g_k^{\text{pop}}) - (1 - \delta_k)$ so that $\frac{1}{k} - \delta_k \neq 0$ along the trend path.}

Given the Lagrange multiplier associated with the technology constraint, $\mu^Y$, the FOCs with respect to labor, labor hoarding, capital, investments and capital utilization are given by:

\[
(1 - \tau_K) \frac{W_{k,t}}{P_{k,t}} = \alpha_k \left( \mu_{k,t}^Y - \varepsilon^{NP}_{k,t} \right) \frac{Y_{k,t}}{N_{k,t} - FN_{k,t}} - \frac{\partial adj^{N}_{i,k,t}}{\partial N_{k,t}} + E_t \left[ \frac{1 + \pi_{k,t+1}^Y}{1 + i_{k,t+1}^s} \frac{\partial adj^{N}_{i,k,t+1}}{\partial FN_{k,t}} \right], \quad (5)
\]

\[
\mu_{k,t}^Y \alpha_k \frac{Y_{k,t}}{N_{k,t} - FN_{k,t}} = - \frac{Y_{k,t}}{Actr_{k,t}Pop_{k,t}} \left( \gamma^{FN,1}_{k} \left( FN_{i,k,t} - FN_{k,t} \right) + \frac{\gamma^{FN,2}_{k}}{2} \left( \frac{FN_{i,k,t}}{Actr_{k,t}Pop_{k,t}} - FN \right) \right)
+ \frac{\partial adj^{N}_{i,k,t}}{\partial FN_{k,t}} - E_t \left[ \frac{1 + \pi_{k,t+1}^Y}{1 + i_{k,t+1}^s} \frac{\partial adj^{N}_{i,k,t+1}}{\partial FN_{k,t}} \right], \quad (6)
\]

\[
Q_{k,t} = E_t \left[ \frac{1 + \pi_{k,t+1}^Y}{1 + i_{k,t+1}^s} \frac{P_{i,t+1}^I}{P_{k,t+1}^I} \frac{P_{i,t+1}^Y}{P_{k,t+1}^Y} \right] \left( \tau_k \delta_k - \frac{\partial adj^{CU}_{i,k,t}}{\partial K_{k,t-1}} + Q_{k,t+1} (1 - \delta_k) + (1 - \alpha_k) \mu_{k,t+1}^Y \frac{P_{i,t+1}^Y}{P_{k,t+1}^I} \frac{Y_{k,t+1}}{K_{k,t+1}^I} \right), \quad (7)
\]
\[ Q_{k,t} = \left[ 1 + \gamma_{k,t} I_{k,t} - \delta_{k,t} \right] + \frac{I_{k,t} - I_{k,t-1} \exp(g^Y_k + g^I_p)}{K_{k,t-1}} \]
\[ - \frac{1}{1 + \gamma_{k,t+1}} \frac{P_{k,t+1} Y_{k,t+1}}{P_{k,t} Y_{k,t}} \exp(g^I_k + g^I_p) \frac{I_{k,t+1} - I_{k,t} \exp(g^Y_k + g^I_p)}{K_{k,t}} \].

where \( Q_{k,t} = \mu_{k,t} / P_{k,t} \) is Tobin’s Q and \( Actr_{k,t} \) \( Pop_{k,t} \) is the active labor force of the domestic country. Equations (5) and (6) characterize the optimal level of labor input, taking into account labor hoarding. While (5) equates the marginal cost of labor to its marginal productivity, equation (6) determines the optimal level of labor hoarding at the expense of the loss in the marginal productivity. Equation (7) and (8) define the Tobin’s Q, which is equal to the replacement cost of capital (the relative price of capital). Finally, (9) describes capital utilization, where the left-hand side represents the additional output produced and the right-hand side - the costs of higher utilization rate.

Note that the basic model specification that does not include an optimal choice of labor hoarding is nested in equation (6) for the limit \( \gamma_{FN}^{k,t} \to \infty \): under that limit, we simply have constant overhead labor \( \frac{\gamma_{FN}^{k,t}}{Actr_{k,t} \ Pop_{k,t}} = FN \). The latter specification is used for Germany, while for all other countries we allow for a more flexible, time varying labor hoarding. This implies that cross-country comparison vis à vis labor market rigidities on the firms side needs to account the ensemble of labor input adjustment cost parameters \( [\gamma_{k,t}, \gamma_{FN}^{k,t}] \), where one needs to consider the, implicitly, Germany has a very large degree of rigidity on the labor hoarding side \( (\gamma_{DE}^{k,t} \to \infty) \).

Given the Rotemberg set-up, the FOC with respect to \( P^Y_{i,k,t} \), imposing the price symmetry condition \( P^Y_{i,k,t} = P^Y_{k,t} \), yields the New Keynesian Phillips curve:

\[ \mu^Y_{k,t} (1 - \alpha_k) Y^Y_{k,t} \frac{P^Y_{k,t}}{C^U_{k,t}} P^I_{k,t} = K^\text{tot}_{k,t-1} \left[ \gamma^{w,1}_k + \gamma^{w,2}_k (C^U_{k,t-1}) \right], \]

where \( \epsilon^\mu_{k,t} = \epsilon^{\mu,Y}_{k,t} + \epsilon^{T\mu,Y}_{k,t} \) is the inverse of the markup shock that can be decomposed into a trend component \( \epsilon^{T\mu,Y}_{k,t} \) and a transitory component \( \epsilon^{\mu,Y}_{k,t} \). The former is defined as follows:

\[ \epsilon^{T\mu,Y}_{k,t} - \rho_k \epsilon^{T\mu,Y}_{k,t-1} = -(1 - \rho_k) \epsilon^{\mu,Y}_0 \left( Y_{k,t} \ Y_{pot} - 1 \right) \]
where $\tau^{\mu,Y,Y_0}_k$ captures an endogenous trend in the procyclical markup. Hence, whenever actual output is above potential, it generates an additional upward pressure to inflation.

In order to allow firms to be less forward looking in their price setting, we introduce a weighted backward looking term $\pi^*_{k,t} = \rho^*_k (\pi_{k,ss} - \bar{\pi}) + (1 - \rho^*_k) (\pi_{k,t-1} - \bar{\pi})$, where $\pi_{k,ss}$ is the inflation steady state. The final New Keynesian Phillips curve takes then the following form:\(^{10}\)

$$
\mu^Y_{k,t} \sigma^Y_k = (1 - \tau^K_k) (\sigma^Y_k - 1) + \sigma^Y_k \frac{P^Y_{k,t}}{P^Y_{k,t-1}} \left( \pi^Y_{k,t} - \bar{\pi} \right) - \sigma^Y_k \frac{1}{1 + \iota^Y_{k,t+1}} \frac{P^Y_{k,t+1}}{P^Y_{k,t}} Y^Y_{k,t+1} \left[ sf^p_k \left( \pi^Y_{k,t+1} - \bar{\pi} \right) + (1 - sf^p_k) \pi^*_k \right] (11)
$$

where $sf^p_k$ is the share of forward looking price setters.

2.3. Trade

2.3.1. Exchange rates and terms of trade

The nominal effective exchange rate $e_{k,t}$ measures the trade weighted average price of foreign currency in terms of domestic currency and is defined as:

$$
e_{k,t} = \prod_l (e_{l,k,t})^{w^T_{l,k,t}},
$$

where $e_{l,k,t}$ are bilateral exchange rates between domestic country $k$ and foreign country $l$. Similarly, the real effective exchange rate $rer_{k,t}$ measures the trade weighted average price of foreign output in terms of domestic output:

$$
rer_{k,t} = \prod_l (rer_{l,k,t})^{w^T_{l,k,t}},
$$

where $rer_{l,k,t}$ are bilateral real exchange rates between $k$ and $l$. $w^T_{l,k,t}$ is the trade weight of the foreign trade partner $l$ in the domestic trade and defined as:

$$
w^T_{l,k,t} = \frac{1}{2} \left( \frac{P^X_{l,k,t} X_{l,k,t}}{P^X_{k,t} X_{k,t}} + \frac{P^M_{l,k,t} size_{l,k,t} M_{l,k,t}}{P^M_{k,t} M_{k,t}} \right)
$$

\(^{10}\)The core model is nested in (10-11) when $\tau^{\mu,Y,Y_0}_k = 0$ and $\rho^* = 0$. When $\rho^* = 0$, equation (11) nests the core specification including only static expectations. We use and estimate this specification for Italy and Spain, as it improves significantly the annual fit of GDP inflation and reduces the previous over-prediction of inflation during the last years of our sample.
where $X_{l,k,t}$ and $M_{l,k,t}$ stand for domestic exports to and imports from country $l$, respectively, and $P^{X}_{l,k,t}$ and $P^{M}_{l,k,t}$ are the relevant price indices. $P^{Mtot}_{k,t}M^{tot}_{k,t}$ includes oil imports from RoW and is defined as $P^{Mtot}_{k,t}M^{tot}_{k,t} = P^{M}_{k,t}M_{k,t} + P^{Oil}_{k,t}OIL_{k,t}$. $P^{X}_{k,t}$ and $P^{M}_{k,t}$ are the respective price aggregates and are defined in the next section.

The terms of trade, $TOT_{k,t}$, the relative price of export over import goods, is defined as:

$$TOT_{k,t} = \frac{P^{X}_{k,t}}{P^{M}_{k,t}}.$$ 

### 2.3.2. Import sector

**Final good packagers (Aggregate import demand)**

The final aggregate demand component goods $C_{k,t}$ (private consumption good), $I_{k,t}$ (private investment good), $G_{k,t}$ (government consumption good) and $I^{G}_{k,t}$ (government investment good) are produced by perfectly competitive firms by combining domestic output, $O^{Z}_{k,t}$ with imported goods $M^{Z}_{k,t}$, where $Z = \{C, I, G, I^{G}\}$, using the following CES production function:

$$Z_{k,t} = A^{p_{k,t}} \left[ (1 - u^{M,Z}_{k,t} s^{M}_{k,t})^{\frac{1}{\sigma^{Z}_{k}}} (O^{Z}_{k,t})^{\frac{1}{\sigma^{Z}_{k}}} + (u^{M,Z}_{k,t} s^{M}_{k,t})^{\frac{1}{\sigma^{Z}_{k}}} (M^{Z}_{k,t})^{\frac{1}{\sigma^{Z}_{k}}} \right]^{\frac{1}{\sigma^{Z}_{k}}}$$

where $\sigma^{Z}_{k}$ is the elasticity of substitution of imports, $A^{p_{k,t}}$ is a shock to productivity in the sector producing goods, $Z$, and $u^{M,Z}_{k,t}$ is a shock to the share $s^{M}_{k,t}$ of import demand components. The shock to the country openness is given by:

$$u^{M,Z}_{k,t} = \exp(\varepsilon^{M}_{k,t}) \left[ 1 + \sum_{l} (1 - \tau^{MAY1}_{ll,t} \tau^{TM}_{ll,t} s^{M}_{lk}) \varepsilon^{TM}_{ll,t} \varepsilon^{M}_{k,t} \right].$$ 

The shock is partially endogenized and composed of individual shock, $\varepsilon^{M}_{k,t}$, and bilateral trend, $\varepsilon^{TM}_{ll,t}$, which depends on changes in the technology of trading partners. The latter is defined as:

$$\varepsilon^{TM}_{xx,t} = \rho^{TM}_{x} \varepsilon^{TM}_{xx,t-1} - \left( 1 - \rho^{TM}_{x} \right) \left[ \tau^{MAY2}_{xx} A^{Y}_{x,t-1} \exp(g^{AY0}_{x}) \right]$$

where $\rho^{TM}_{x}$ captures the persistence of the trade trend. $\tau^{MAY2}_{xx}$ measures the relative competitiveness of the domestic country and $\tau^{MAY1}_{xx}$ gauges its impact on the openness, i.e. an increase in relative productivity lowers the domestic degree of openness (proportionally to $\tau^{MAY1}_{xx}$) and increases the degree of openness of trading partners towards domestic exports.
(proportionally to \((1 - \tau_{xx}^{MAY})\)). The endogenous trend tries to capture the trend in import share and is estimated for Italy and Spain. The core specification without this endogenous component is nested in (12) by setting \(\tau_{xx}^{MAY} = 0\).

From profit maximization we obtain the following domestic, \(O^Z\), and foreign, \(M^Z\), demand aggregates:

\[
O^Z_{k,t} = (A^x_{k,t})^{\sigma^z_k - 1} \left(1 - u^M_{k,t} s^M_k \right) \left( \frac{P^O_{k,t}}{P^Z_{k,t}} \right)^{-\sigma^z_k} Z_{k,t},
\]

\[
M^Z_{k,t} = (A^x_{k,t})^{\sigma^z_k - 1} u^M_{k,t} s^M_k \left( \frac{P^M_{k,t}}{P^Z_{k,t}} \right)^{-\sigma^z_k} Z_{k,t},
\]

where \(P^Z_{k,t}\) is the price deflator associated to the demand components:

\[
P^Z_{k,t} = (A^x_{k,t})^{-1} \left[ \left(1 - u^M_{k,t} s^M_k \right) (P^O_{k,t})^{1-\sigma^z_k} + u^M_{k,t} s^M_k (P^M_{k,t})^{1-\sigma^z_k} \right]^{1 - \sigma^z_k}.
\]

We define total Non-oil imports as:

\[
M_{k,t} = M^C_{k,t} + M^I_{k,t} + M^G_{k,t} + M^{IG}_{k,t},
\]

whereas total imports are:

\[
P^{M_{k,t}}_{k,t} = P^M_{k,t} M_{k,t} + P^{oil}_{k,t} OIL_{k,t}.
\]

Import retailers (Economy-specific final import demand)

Final imported goods are produced by perfectly competitive firms combining economy-specific final imports, which maximize the following profit function:

\[
\max P^M_{k,t} M_{k,t} - \sum_l P^M_{l,k,t} M_{l,k,t} \frac{size_l}{size_k}
\]

subject to the following CES production function:

\[
M_{k,t} = \left[ \sum_l \left( s^M_{l,k,t} u^M_{l,k,t} \right)^{\frac{1}{\sigma^M_{l,k,t}}} \left( \frac{M_{l,k,t} \frac{size_l}{size_k}}{s^M_{l,k,t} \frac{size_l}{size_k}} \right)^{\frac{\sigma^M_{l,k,t} - 1}{\sigma^M_{l,k,t}}} \right]^{\frac{1}{\sigma^M_{k,t} - 1}},
\]

where \(\sigma^M_{k,t}\) is the price elasticity of demand for country \(l\)’s goods and \(\sum_l s^M_{l,k,t} = 1\) are the

\[11\] Note that the same trend affects the oil import demand in a similar way.
import shares. The demand for goods from country $l$ is given by:

$$M_{l,k,t} = s_{l,k}^M u_{l,k,t}^M \left( \frac{P_{l,k,t}^M}{P_{k,t}^M} \right)^{-\sigma_k^{FM}} M_{k,t} \frac{\text{size}_k}{\text{size}_t},$$

where $u_{l,k,t}^M$ captures an endogenous bilateral trend component:

$$u_{l,k,t}^M = \frac{1 - (1 - \tau_{l,l}^{MAY^1}) \varepsilon_{l,l,t}^T \text{size}_k}{1 - \sum_z (1 - \tau_{z,z}^{MAY^1}) \varepsilon_{z,z,t}^T \text{size}_k}, \tag{14}$$

which depends on the relative weight of REA and RoW GDP in the world economy and on the aggregate import competitiveness, $\varepsilon^{TM}$. This trend components complements (12) in the sense of competitiveness gain/loss deriving from an increase/fall in productivity levels.

The import prices are:

$$P_{l,k,t}^M = \left[ \sum_l s_{l,k}^M u_{l,k,t}^M \left( P_{l,k,t}^M \right)^{1-\sigma_k^{FM}} \right]^{\frac{1}{1-\sigma_k^{FM}}}$$

with $P_{l,k,t}^M$ being the economy-specific imports good prices. Since all products from country $l$ are initially purchased at export price $P_{l,t}^X$, the economy-specific import goods price can be also expressed as:

$$P_{l,k,t}^M = e_{l,k,t} P_{l,t}^X.$$

### 2.3.3. Export sector

The exporting firms are supposed to be competitive and the export price equals the output price, subject to a shock, $\varepsilon_{k,t}^X$:

$$P_{k,t}^X = \exp(\varepsilon_{k,t}^X) P_{k,t}^O.$$

### 2.4. Fiscal policy

The government collects taxes on labour, $\tau_l^N$, capital, $\tau^K$, consumption, $\tau^C$, and lump-sum taxes, $tax_{k,t}$, and issues one-period bonds, $B_{k,t}^G$, to finance government consumption, $G_{k,t}$, investment, $I_{k,t}^G$, transfers, $T_{k,t}$, and the servicing of the outstanding debt. The government budget constraint is:

$$B_{k,t}^G = (1 + i_{k,t-1}^G) B_{k,t-1}^G - R_{k,t}^G + P_{k,t}^G G_{k,t} + P_{k,t}^I I_{k,t}^G + T_{k,t} P_{k,t}^Y,$$
where nominal government revenues, \( R^G \), are defined as:

\[
R^G_{k,t} = \tau^K (P^Y Y_{k,t} - W_{k,t} N_{k,t} - P^I_t \delta_k K_{k,t-1}) + \tau^N W_{k,t} N_{k,t} + \tau^C P^C_{k,t} C_{k,t} + \tau^{Oil} P^Y_t Oil_t + tax_{k,t} Y_{k,t}.
\]

Taxes on oil imports from RoW, \( \tau^{Oil} \), are assumed to be exogenously determined. To close the government budget constraint, lump sum taxes, \( tax_{k,t} \), adjust residually as follows:

\[
tax_{k,t} = \rho_r tax_{k,t-1} + \eta_k^{deft} \left( \frac{\Delta B^G_{k,t-1}}{Y_{k,t-1} P^Y_{k,t-1}} - DEFTAR_k \right) + \eta_k^{BT} \left( \frac{B^G_{k,t-1}}{Y_{k,t-1} P^Y_{k,t-1}} - BTAR_k \right) + \varepsilon^{tax}_{k,t},
\]

where \( DEFTAR_k \) and \( BTAR_k \) are the targets on government deficit and government debt, and \( \varepsilon^{tax}_{k,t} \) is a shock. Hence, government uses lump-sum taxes as budget closure and increases (decreases) taxes when the level of government debt and government deficit is above (below) the debt and deficit target. The accumulation equation for government capital is:

\[
K^G_{k,t} = (1 - \delta^G_k) K^G_{k,t-1} + I^G_{k,t},
\]

where \( \delta^G_k \) is the depreciation rate of public capital.

The model uses a measure of discretionary fiscal effort (DFE) as defined by the European Commission (2013):

\[
DFE_{k,t} = \frac{R^G_{k,t}}{Y_{k,t}} - \frac{\Delta E^G_{k,t} - \left( \frac{y^pot_{k,t-1}}{y^pot_{k,t-1}} - 1 \right) E^G_{k,t-1}}{Y_{k,t}},
\]

where \( R^G_{k,t} \) stands for government revenues in nominal terms, \( E^G_{k,t} \) is the adjusted nominal expenditure aggregate, \( y^pot_{k,t} \) is the medium-term nominal potential output, and \( Y_{k,t} \) is nominal GDP.\(^{12}\) In order to be consistent with the definition of DFE, which is defined with respect to all primary adjusted government expenditures, we define the aggregate nominal expenditure as:

\[
E^G_{k,t} = P^G_{k,t} G_{k,t} + P^IG_{k,t} I^G_{k,t} + P^Y_{k,t} T_{k,t}.
\]

We use the following feedback rules for government consumption, \( G_{k,t} \), investment, \( I^G_{k,t} \), and

---

\(^{12}\)The adjusted nominal expenditure removes interest payments and non-discretionary unemployment expenditures from total nominal expenditure.
transfers, \( T_{k,t} \):

\[
\begin{align*}
G_{k,t} P^G_{k,t} - G_{k,t-1} P^G_{k,t-1} &= \left( \frac{Y^\text{pot}_{k,t}}{Y^\text{pot}_{k,t-1}} \exp(\pi^y_{k,t}) - 1 \right) G_{k,t-1} P^G_{k,t-1} Y_{k,t} \,, \\
I^G_{k,t} P^G_{k,t} - I^G_{k,t-1} P^G_{k,t-1} &= \left( \frac{Y^\text{pot}_{k,t}}{Y^\text{pot}_{k,t-1}} \exp(\pi^y_{k,t}) - 1 \right) I^G_{k,t-1} P^G_{k,t-1} Y_{k,t} \,, \\
T_{k,t} P^Y_{k,t} - T_{k,t-1} P^Y_{k,t-1} &= \left( \frac{Y^\text{pot}_{k,t}}{Y^\text{pot}_{k,t-1}} \exp(\pi^y_{k,t}) - 1 \right) T_{k,t-1} P^Y_{k,t-1} Y_{k,t} \,.
\end{align*}
\]

where \( \varepsilon^G_{k,t}, \varepsilon^IG_{k,t}, \varepsilon^T_{k,t} \) are shocks to government consumption, investment and transfers, respectively. The parameters \( \alpha^G_{k,t}, \alpha^IG_{k,t}, \alpha^T_{k,t} > 0 \) are estimated policy feedback parameters to ensure long-run stability of the model.

2.5. Monetary policy

Monetary policy is modelled with a Taylor rule where the ECB sets the policy rate, \( i_{E,A,t} \), in response to the annualized EA-wide inflation, \( \pi^\text{c,vat,QA}_{E,A,t} \), and the annualized EA output gap (Taylor, 1993).\(^{13}\) The policy rate adjusts sluggishly to deviations of inflation from their respective target level and to the output gap and is subject to random shocks:

\[
i_{E,A,t} - \bar{i} = \rho^i \left( i_{E,A,t-1} - \bar{i} \right) + \left( 1 - \rho^i \right) \left[ \eta^\pi > 0.25 \left( \pi^C_{E,A,t} - \bar{\pi}^C_{E,A} \right) \right] + \eta^y \left( \log \left( \frac{1}{4} \sum_{r=1}^{4} Y_{E,A,t-r} \right) - \log \left( \frac{1}{4} \sum_{r=1}^{4} Y^\text{pot}_{E,A,t-r} \right) \right) + \varepsilon^i_{E,A,t}, \tag{15}
\]

where \( \bar{i} = \bar{\pi}^Y_{\text{ obs}} \) is the steady state nominal interest rate, equal to the sum of the steady state real interest rate and GDP inflation. Quarterly annualized inflation is defined as:

\[
\pi^C_{E,A,t} = \log \left( \sum_{r=0}^{3} P^C_{E,A,t-r} \right) - \log \left( \sum_{r=4}^{7} P^C_{E,A,t-r} \right).
\]

The policy parameters \( (\rho^i, \eta^\pi, \eta^y) \) capture interest rate inertia and the response to annualized inflation and output gap, respectively. It is assumed that the rate of the risk-free bonds is equal to the EA policy rate, \( i^f_{k,t} \equiv i_{E,A,t} \).

\(^{13}\)We define potential output, \( Y^\text{pot}_{E,A,t} \), at date \( t \) as the output level that would prevail if labor input equaled steady state per capita hours worked, capital stock were utilized at full capacity and TFP equaled its trend component.
2.6. Closing the economy

Market clearing requires that:

\[ Y_{k,t}P^Y_k + r^{Oil}Oil_{k,t}P^{Oil}_t = P^C_kC_{k,t} + P^I_kI_{k,t} + P^{IG}_kIG_{k,t} + P^G_kG_{k,t} + TB_{k,t}, \]

where the trade balance, \( TB_{k,t} \), is defined as the difference between exports and imports:

\[ TB_{k,t} = P^X_kX_{k,t} - \sum_l \frac{size_l}{size_k} P^M_{l,k,t}M_{l,k,t} - P^{Oil}_{RoW,k,t}OIL_{RoW,k,t}e_{RoW,k,t}. \]

EMU-country exports are a sum of imports from the domestic economy by other countries:

\[ X_{k,t} = \sum_l M_{l,k,t}, \]

where \( M_{l,k,t} \) stands for imports of economy \( l \) from EMU country \( k \). Net foreign assets, \( B^W_{k,t} \), evolve according to:

\[ e_{RoW,k,t}B^W_{k,t} = (1 + i^w_{t-1})e_{RoW,k,t}B^W_{k,t-1} + TB_{k,t} + ITR_kP_kY_k, \]

where \( ITR_k \) represents international transfers, which are calibrated to allow for a non-zero steady state of the trade balance.

Finally, net foreign assets of all the countries sum to zero:

\[ \sum_l NFA_{l,t}size_l = 0. \]

2.7. The REA and RoW block

The model of the REA and RoW blocks is simplified in structure and with fewer shocks. Specifically, REA and RoW consist of a budget constraint for the representative household (Ricardian), demand functions for domestic and imported goods (derived from CES consumption good aggregators), a production technology that uses only labor as input factor, a New Keynesian Phillips curve, and a Taylor rule. The REA and RoW blocks abstract from capital accumulation. There are shocks to labor productivity, price mark-ups, the subjective discount rate, the relative preference for domestic and imported goods as well as monetary policy shocks. Unless otherwise specified, subindex \( k \) corresponds to REA and RoW.

Since RoW is an oil exporter, the resource constraint for the representative household
\[(k = RoW) \text{ is:} \]
\[
P^Y_{RoW,t} Y_{RoW,t} + P^{Oil}_{RoW,t} OIL_{RoW,t} = P^C_{RoW,t} C_{RoW,t} + P^X_{RoW,t} X_{RoW,t} - \sum_l \frac{size_l}{size_{RoW}} e_{l, RoW,t} P^X_{l,t} M_{l, RoW,t},
\]

where \(X_{RoW,t}\) are non-oil exports by RoW. The resource constraint for the representative household in REA (\(k = REA\)), as an oil importer, is:

\[
P^Y_{REA,t} Y_{REA,t} + \tau^{Oil}_{t} P^{Y0}_{t} OIL_{REA,t} = P^C_{REA,t} C_{REA,t} + P^X_{REA,t} X_{REA,t} - \sum_l \frac{size_l}{size_{REA}} e_{l, REA,t} P^X_{l,t} M_{l, REA,t},
\]

where \(\tau^{Oil}_{t} P^{Y0}_{t} OIL_{REA,t}\) captures the excise duty.

Final aggregate demand \(C_{k,t}\) (in the absence of investment and government spending in REA and RoW) is a combination of domestic output, \(Y_{k,t}\) and imported goods, \(M_{k,t}\), using the following CES function:

\[
C_{k,t} = A_{k,t}^P \left[ (1 - u_{k,t}^{MC} s_k^M) \frac{1}{\sigma_k} (Y^C_{k,t})^{\frac{\sigma_k}{\sigma_k - 1}} + (u_{k,t}^{MC} s_k^M)^{\frac{1}{\sigma_k}} (M^C_{k,t})^{\frac{\sigma_k}{\sigma_k - 1}} \right]^{\frac{1}{1 - \sigma_k}},
\]

where \(u_{k,t}^{MC}\) is a shock to input components and \(s_k^M\) the import share. From profit maximization we obtain the demand for domestic and foreign goods:

\[
Y^C_{k,t} = (A_{k,t}^P)^{\frac{1}{\sigma_k} - 1} (1 - u_{k,t}^{MC} s_k^M) \left( \frac{P^Y_{k,t}}{P^C_{k,t}} \right)^{\frac{1}{\sigma_k}} C_{k,t},
\]

\[
M^C_{k,t} = (A_{k,t}^P)^{\frac{1}{\sigma_k} - 1} u_{k,t}^{MC} s_k^M \left( \frac{P^M_{k,t}}{P^C_{k,t}} \right)^{\frac{1}{\sigma_k}} C_{k,t},
\]

where the consumer price deflator \(P^C_{k,t}\) satisfies:

\[
P^C_{k,t} = \frac{1}{A_{k,t}^P} \left[ (1 - u_{k,t}^{MC} s_k^M) (P^Y_{k,t})^{1 - \frac{1}{\sigma_k}} + u_{k,t}^{MC} s_k^M (P^M_{k,t})^{1 - \frac{1}{\sigma_k}} \right]^{\frac{1}{1 - \sigma_k}}.
\]

The intermediate good producers use labor to manufacture domestic goods (non-oil output) according to a linear production function:

\[
Y_{k,t} = A_{k,t}^Y N_{k,t},
\]
where $A_{k,t}^Y$ captures a trend in the productivity and $N_{k,t} = Actr_{k,t} Pop_{k,t}$ is the active population in the economy. Price setting for non-oil output follows a New Keynesian Phillips curve:

$$
\pi_{k,t}^Y - \bar{\pi}_k^Y = \beta_{k,t} \lambda_{k,t+1} \frac{\lambda_{k,t}}{\lambda_{k,t}} [sfp_k (\pi_{k,t+1}^Y - \bar{\pi}_k^Y) + (1 - sfp_k) \pi^*_k] + \phi^y_k \log \frac{Y_{k,t}}{Y_k} + \varepsilon^Y_{k,t},
$$

where $\lambda_{k,t} = (C_{k,t} - hC_{k,t-1})^{-\theta}$ is the marginal utility of consumption, $\varepsilon^Y_{k,t}$ is a cost push shock, $sfp_k$ is the share of forward looking price setters, and $\pi^*_k$ measures the weight of backward looking price setters according to:

$$
\pi^*_k = \rho^*_k (\pi_{k,ss} - \bar{\pi}) + (1 - \rho^*_k) (\pi_{k,t-1} - \bar{\pi}).
$$

Monetary policy in RoW ($k = RoW$) follows a Taylor-type rule as in equation (15):

$$
i_{RoW,t} - \bar{i} = \rho^i_{RoW} (i_{RoW,t-1} - \bar{i}) + (1 - \rho^i_{RoW}) \left[ \eta^i_{RoW} 0.25 \left( \pi_{C,QA}^{RoW,t} - \pi_{RoW,t} \right) + \eta^i_{RoW} \left( \log \left( 0.25 \sum_{r=1}^4 Y_{RoW,t-r} \right) - \log \left( 0.25 \sum_{r=1}^4 Y_{pot,RoW,t-r} \right) \right) \right] + \varepsilon^i_{RoW,t}.
$$

Oil is considered to be an unstorable exogenous endowment of RoW ($k = RoW$) and it is supplied inelastically:

$$
OIL_{RoW,t} = \sum_l \frac{\text{size}_l}{\text{size}_{RoW}} OIL_{l,RoW,t},
$$

where net oil exporting firms’ revenues in RoW are driven only by its price, $P_{RoW,t}^{Oil}$, which is assumed to be denominated in RoW currency:

$$
P_{RoW,t}^{Oil} = \frac{P_{t}^{Y0}}{AP_{Out}^t}.
$$

Total nominal exports for REA and RoW are defined as:

$$
P_{k,t}^X X_{k,t} = \sum_l P_{l,k,t}^X M_{l,k,t},
$$

with the bilateral export price being defined as the domestic price subject to a bilateral price shock:

$$
P_{l,k,t}^X = \exp(\varepsilon_{l,k,t}) P_{k,t}^Y.
$$

We combine the FOCs with respect to international bonds of REA and RoW to obtain the
uncovered interest parity (UIP) condition:

\[
E_t \left[ \frac{\varepsilon_{RoW,EA,t+1}^{bw}}{\varepsilon_{RoW,EA,t}} \right] (1 + i_{RoW,t}) = (1 + i_{EA,t}) + \varepsilon_{EA,t}^{bw} + \alpha_{EA} \frac{\varepsilon_{RoW,EA,t}^{bw} B_{EA,t}^{W}}{P_{EA,t}^{Y} Y_{EA,t}},
\]

where \( \varepsilon_{EA,t}^{bw} \) captures a bond premium shock between EA and RoW (exchange rate shock), and \( \alpha_{EA}^{bw} \) is the debt-dependent country risk premium on net foreign asset holdings to ensure long-run stability (see, e.g., Schmitt-Grohe and Uribe, 2003; Adolfson et al., 2008).

3. Model solution and econometric approach

The model is solved by linearizing it around its deterministic steady state. A subset of parameters is calibrated at quarterly frequency to match long-run properties, the remaining parameters are estimated using Bayesian methods.\(^{14}\) As in Bayesian practice, the likelihood function (evaluated by implementing the Kalman Filter) and the prior distribution of the parameters are combined to calculate the posterior distribution. The posterior Kernel is then simulated numerically using the slice sampler algorithm as proposed by Planas et al. (2015).\(^{15}\)

The estimation uses quarterly and annual data for the period 1999q1 to 2016q4.\(^{16}\) Data for EMU countries and the Euro Area are taken from Eurostat (in particular, from the European System of National Accounts ESA95), while the Rest of the World series are constructed using IMF International Financial Statistics (IFS) and World Economic Outlook (WEO) databases. Appendix A provides a detailed description of data sources, definitions and transformations. The estimated model uses 38 observed series and assumes 38 exogenous shocks. On the one hand, the large number of shocks is dictated by the fact that we use a large number of observables for estimation. On the other hand, many shocks are needed to capture key dynamic properties of macroeconomic and financial data (see Kollmann et al., 2015).

The steady-state of the model is calibrated quarterly to match long-run data properties. Along the deterministic steady state all real variables are assumed to grow along the balanced growth path at a rate of 1.3% per year (the average growth rate of EA output over the sample

\(^{14}\)We use the Dynare software 4.5 to solve the linearized model and to perform the estimation (see Adjemian et al., 2011).

\(^{15}\)The slice sampler algorithm was introduced by Neal (2003). Planas et al. (2015) reconsider the slices along the major axis of the ellipse to better fit the distribution than any of Euclidean slices. The slice sampler has shown to be more efficient and offer better mixing properties than the Metropolis-Hastings sampler (Căles et al., 2017).

\(^{16}\)The model is estimated at quarterly frequency, interpolating annual data for the series that are not available at higher frequency.
Table 1: Selected calibrated structural parameters

<table>
<thead>
<tr>
<th></th>
<th>DE</th>
<th>FR</th>
<th>IT</th>
<th>ES</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>EA Monetary Policy</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nominal interest rate in SS</td>
<td>(i_{EA})</td>
<td>0.005</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CPI inflation in SS</td>
<td>(\pi_{EA})</td>
<td>0.004</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest rate persistence</td>
<td>(\rho_{EA})</td>
<td>0.838</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Response to inflation</td>
<td>(\eta^{\pi}_{EA})</td>
<td>1.548</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Response to GDP</td>
<td>(\eta^{y}_{EA})</td>
<td>0.086</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| **Preferences**              |     |     |     |     |
| Intertemporal discount factor| \(\beta\) | 0.997 | 0.997 | 0.997 | 0.997 |
| Degree of openness           | \(s_{M}^{M}\) | 0.35 | 0.28 | 0.26 | 0.29 |
| Preference for imports from RoW | \(s_{M}^{RoW}\) | 0.53 | 0.42 | 0.50 | 0.46 |
| Preference for imports from REA | \(s_{M}^{REA}\) | 0.47 | 0.58 | 0.50 | 0.54 |
| Preference for govern’t bonds | \(\alpha^{\delta}_{M}\) | -0.0003 | -0.0011 | 0.0009 | 0.0006 |
| Preference for stocks        | \(\alpha^{S}_{M}\) | -0.0002 | -0.0009 | -0.0007 | -0.0026 |
| Preference for foreign bonds | \(\alpha^{bw}_{M}\) | -0.0048 | -0.0048 | -0.0048 | -0.0048 |
| Weight of disutility of labor| \(\omega^{N}\) | 2.98 | 4.62 | 7.79 | 7.62 |

| **Production**               |     |     |     |     |
| Cobb-Douglas labor share     | \(\alpha\) | 0.65 |       | 0.65 | 0.65 |
| Depreciation of private capital stock | \(\delta\) | 0.0142 | 0.0150 | 0.0136 | 0.0123 |
| Depreciation of public capital stock | \(\delta^{G}\) | 0.0142 | 0.0150 | 0.0136 | 0.0123 |
| Share of oil in total output | \(s_{Oil}\) | 0.0150 | 0.0150 | 0.0150 | 0.0150 |
| Linear capacity utilization adj. costs | \(\gamma^{u,1}_{N}\) | 0.0147 | 0.0166 | 0.0143 | 0.0140 |

| **Fiscal policy**            |     |     |     |     |
| Consumption tax              | \(\tau^{C}\) | 0.20 | 0.20 | 0.20 | 0.20 |
| Corporate profit tax         | \(\tau^{K}\) | 0.30 | 0.30 | 0.30 | 0.30 |
| Labor tax                    | \(\tau^{N}\) | 0.40 | 0.54 | 0.45 | 0.35 |
| Deficit target               | def | 0.019 | 0.021 | 0.032 | 0.017 |
| Debt target                  | \(BG\) | 2.591 | 2.860 | 4.477 | 2.309 |

| **Steady state ratios**      |     |     |     |     |
| Private consumption share in SS | \(C/Y\) | 0.57 | 0.55 | 0.60 | 0.60 |
| Private investment share in SS | \(I/Y\) | 0.17 | 0.19 | 0.17 | 0.20 |
| Gov’t consumption share in SS | \(C^{G}/I\) | 0.19 | 0.23 | 0.19 | 0.18 |
| Gov’t investment share in SS  | \(I^{G}/Y\) | 0.02 | 0.04 | 0.03 | 0.04 |
| Transfers share in SS        | \(T/Y\) | 0.17 | 0.18 | 0.18 | 0.13 |

| **Others**                   |     |     |     |     |
| Size of the country (% of world) | \(size\) | 4.50 | 3.45 | 2.81 | 1.80 |
| Trend of total factor productivity | \(g^{AV}_{0}\) | 0.0029 | 0.0018 | 0.0027 | 0.0013 |
| Trend of private consumption specific productivity | \(g^{APC}_{0}\) | 0.0009 | 0.0008 | -0.0006 | -0.0014 |
| Trend of gov’t consumption specific productivity | \(g^{APG}_{0}\) | 0.0008 | 0.0002 | -0.0007 | -0.0004 |
| Trend of private investment specific productivity | \(g^{API}_{0}\) | -0.0011 | 0.0010 | -0.0006 | 0.0020 |
| Trend of gov’t investment specific productivity | \(g^{APIG}_{0}\) | -0.0011 | 0.0010 | -0.0006 | 0.0020 |

The nominal variables grow at an EA average inflation rate of 1.57% per year. Population is detrended by the EA average rate of population growth (0.3% per year), the
demand components by the country-specific average productivity. The steady-state ratios of main economic aggregates to GDP are calibrated to match historical ratios for each country over the sample period.

Table 1 provides an overview of the calibrated parameters. The discount factor at quarterly frequency is set to 0.997 for all countries in order to match an annual interest rate of 1%. Given the monetary union setting, we calibrate the aggregate Taylor rule parameters according to their estimated values based on a three-region setting of the Global Multi-country (GM) model (EA-US-RoW) (see Kollmann et al., 2016). The interest rate persistence is 0.838, coefficients for the response to inflation and output gap are 1.548 and 0.086, respectively. Trade related parameters such as the degree of openness or preferences for imports are set to match the average share of total imports of each country. The debt targets are set to match the average values of the debt-to-GDP ratios over the sample. Preferences for bonds and stocks are consistent with their related risk premia.

4. Estimation results

In this section, we discuss the main issues related to fitting observed data, and the associated modelling and estimation approaches that are applied. Next, we present the posterior estimates of key model parameters, the ability of the model to fit the data and impulse response functions. We also evaluate the drivers of business cycles fluctuations in each country by analyzing the historical decomposition of real GDP growth and trade balance-to-GDP ratio.

As usual in empirical work, are critical areas that may require modelling extensions, in the attempt to improve the model fit to data. The general idea is that the extensions should nest the core model specification, in such a way that they are used only when the fit improves significantly, by satisfying some information criterion. In Bayesian terms, the whole estimation process can be seen as including an identification step that selects the extensions only when the loss in parsimony is justified by an appropriate gain in fit.

As discussed in Section 2, there are three ingredients in the model that we identified for possible extensions.

1. There is an overall tendency to over-predict inflation, or a too fast bouncing back of inflation forecasts after supply (mark-up in particular) and demand shocks. To address this, we have identified an extended formulation of the hybrid Phillips curve, as discussed in equation (11). This feature is selected for Italy and Spain.

2. In the core model, the dynamic adjustment in the cyclical component component in the production function is driven exclusively by capacity utilization. This cyclical term
is also linked to the price equation, as part of the system of equations defining the firms
decision rules. This can be another source of lack of fit in the model: to fix this we
introduced the endogenous labor hoarding (6). This, jointly with the extended Phillips
curve, can help in improving the fit of the equations int he supply side. This feature
is selected for France, Italy and Spain.

3. Fitting trade equations to data usually provides very large import demand shocks: this
is often related to trends in the import share and to the trade-off between short and
long term elasticities.

- Import demand shocks can feature a clear trend component. One possible way
to reduce the size of this shock is to interpret part of them as deriving from a
gain/loss of competitiveness associated to diverging trends in productivity. This
has been implemented in equations (12) and (14) and was selected for Italy and
Spain.

- There is usually a trade-off between short and long term elasticities, and the data
may have more to say about the former than the latter. We tried some ways
to address explicitly this in the model, but so far without success, so the model
features only one elasticity. To allow for more flexible fit to the data, however, we
allow, in the priors for France Italy and Spain, this parameter to range below the
theoretical lower bound of 1 for long term elasticity. This may also help in fitting
better the trade equations. Implications on irfs thereof will be discussed later.

4.1. Posterior estimates

The posterior estimates (with HPD intervals) of key model parameters are reported
in Table 2. The estimated habit persistence is relatively high in Germany and Italy, which
implies a slow adjustment of consumption to changes in income. Risk aversion coefficients are
relatively similar for all countries and range between 1.4 - 1.5. The labor supply elasticity is
relatively high in Germany (2.8) and rather small in Italy (1.97). The import price elasticity
coefficient varies between the countries. While in Germany it has been estimated above
unity (1.3), in France, Spain and Italy the coefficient is ranging from 0.3 to 0.7. Price
adjustment costs are rather low in Italy (11.04) compared to the other countries. Real wage
rigidity is high for all countries. Employment adjustment costs vary significantly among the
countries. The labor market rigidity is linked to the two adjustment cost parameters in labor
demand and labor hoarding. The former appears to be relatively rigid in France (128.76)
compared to Spain (10.19) and Italy (24.62), while in Germany its low value (0.47) needs
to be considered in combination to the totally rigid labor hoarding component (∞). The
latter features similar levels in France (1.13) and Spain (1.03), with a somewhat larger level
in Italy (1.51). Capacity utilization and investment adjustment costs are relatively similar across the four countries. The fiscal feedback rule on lump-sum taxes exhibits relatively high persistence for France (0.94) and Spain (0.91), implying a more drawn-out response to debt and deficit levels in this cases. The estimated responses of taxes to deficit and debt targets are in the same order of magnitude across countries.
### Table 2: Prior and posterior distribution of key estimated model parameters

<table>
<thead>
<tr>
<th>Prior distribution</th>
<th>Posterior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Distr</td>
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<tr>
<td><strong>Preferences</strong></td>
<td></td>
</tr>
<tr>
<td>Consumption habit persistence</td>
<td>$H$</td>
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<td></td>
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<tr>
<td>Savers share</td>
<td>$\omega^S$</td>
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<td></td>
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<tr>
<td>Risk aversion</td>
<td>$\theta$</td>
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<td></td>
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<tr>
<td>Inverse Frisch elasticity of labor supply</td>
<td>$\phi^N$</td>
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<td></td>
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</tr>
<tr>
<td>Import price elasticity</td>
<td>$\sigma^z$</td>
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<td></td>
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<tr>
<td><strong>Nominal and real frictions</strong></td>
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<tr>
<td>Price adjustment cost</td>
<td>$\gamma^P$</td>
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<tr>
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<tr>
<td>Nominal wage adjustment cost</td>
<td>$\gamma^w$</td>
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<tr>
<td>Real wage rigidity</td>
<td>$\gamma^{wr}$</td>
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<tr>
<td>Employment adjustment cost</td>
<td>$\gamma^N$</td>
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<tr>
<td>Labor hoarding quadratic adj cost</td>
<td>$\gamma^{FN}$</td>
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<td></td>
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<tr>
<td>Capacity Utilization quadratic adj cost</td>
<td>$\gamma^{CU}$</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Investment adjustment cost</td>
<td>$\gamma^I$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Fiscal policy</strong></td>
<td></td>
</tr>
<tr>
<td>Lump sum taxes persistence</td>
<td>$\rho^T$</td>
</tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>Lump sum taxes response to deficit</td>
<td>$\eta^{DEF}$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Lump sum taxes response to debt</td>
<td>$\eta^B$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>adj to deviation of Gov spending from trend</td>
<td>$\alpha_G$</td>
</tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>adj to deviation of Gov investment from trend</td>
<td>$\alpha_{IG}$</td>
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<tr>
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</tr>
<tr>
<td>adj to deviation of Gov transfers from trend</td>
<td>$\alpha_T$</td>
</tr>
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</tbody>
</table>
4.2. Model fit

In order to evaluate the model’s capability to fit the data, Table 3 compares sample and model-implied moments for a subset of key statistics. In particular, we focus on volatilities and persistence of real GDP, consumption, investment, employment, trade balance-to-GDP ratio and GDP deflator as well as the cross-correlation of GDP with its main components. We use first differences for the trade balance-to-GDP ratio and quarter-on-quarter growth rates for all other variables. It shows that all estimated models tend to overestimate the volatilities of real variables. However, the relative magnitudes seem to be preserved, e.g. \( \text{std}(\text{GC})/\text{std}(\text{GY}) \). Of particular note is the high volatility of investment, which is in line with the data patterns. Most of the correlations between GDP growth and its components are fairly well captured. More precisely, all country models replicate the correlation of consumption, investment and employment with output. In our model the trade balance is positively correlated with output, but matches the data pattern only for Germany. Moreover, all estimated models generate a negative correlation between GDP inflation and GDP growth, which is only the case in the data for Germany and Italy. First-order autocorrelations are particularly well seized in France and Spain, whereas Germany and Italy display a more differentiated picture.

The last two columns in Table 3 report the \( R^2 \), in particular the 1-year and 2-year ahead forecast. We define the \( R^2 \) as follows:

\[
R^2 = 1 - \frac{e_j'y_j}{y_j'y_j}
\]

where \( y_j = [y_{1,j}, \ldots, y_{T,j}]' \) is the country-specific j-th time series in deviation from the model implied steady state and \( e_j = [e_{1,j}, \ldots, e_{T,j}]' \) is the associate the k-step ahead forecast error obtained from the Kalman filter recursions. The \( R^2 \) defined in this way has an upper bound located at 1 and is unbounded from below. This means that in the perfect case that the model generates no forecast error, the \( R^2 \) is one and it declines monotonically as the forecast error increases. Since the volatility of the forecast error could be larger than the volatility of the observed time series, the \( R^2 \) can be negative. In that case, a constant forecast centered on the sample mean would do a better job since its \( R^2 \) coincides with zero. The graphical representation of the k-step ahead forecast, i.e. the 1-year and 2-year ahead forecast at each point in time, can be found in Figures B.16 - B.19 in AppendixB.1.

The 1-year ahead \( R^2 \) is mostly positive for all analyzed countries, indicating that the model forecast errors are not very large. Even the 2-year ahead forecast provides a relatively good fit, especially for IT and ES. A different picture arises for Germany, for which the estimated model delivers a poor (in-sample) forecast accuracy, particularly for consumption.
and the GDP deflator.\footnote{Similar issues in fitting consumption behavior in Germany are reported in Kollmann et al. (2015).}

Table 3: Theoretical moments and model fit

<table>
<thead>
<tr>
<th>Variable</th>
<th>Std(%)</th>
<th>AR(1)</th>
<th>Corr (x, GY)</th>
<th>r2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>Germany</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP growth (GY)</td>
<td>0.86</td>
<td>1.80</td>
<td>0.42</td>
<td>0.20</td>
</tr>
<tr>
<td>Consumption growth (GC)</td>
<td>0.60</td>
<td>1.70</td>
<td>-0.13</td>
<td>0.73</td>
</tr>
<tr>
<td>std(GC)/std(GY)</td>
<td>0.70</td>
<td>0.94</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment growth</td>
<td>4.31</td>
<td>5.64</td>
<td>-0.02</td>
<td>0.12</td>
</tr>
<tr>
<td>GDP deflator</td>
<td>0.32</td>
<td>0.87</td>
<td>0.27</td>
<td>0.73</td>
</tr>
<tr>
<td>Hours growth</td>
<td>0.63</td>
<td>1.40</td>
<td>-0.01</td>
<td>0.31</td>
</tr>
<tr>
<td>ΔTrade balance to GDP</td>
<td>0.67</td>
<td>1.32</td>
<td>-0.08</td>
<td>0.09</td>
</tr>
<tr>
<td>France</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP growth</td>
<td>0.49</td>
<td>1.08</td>
<td>0.57</td>
<td>0.09</td>
</tr>
<tr>
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<td>0.46</td>
<td>1.00</td>
<td>0.13</td>
<td>0.70</td>
</tr>
<tr>
<td>std(GC)/std(GY)</td>
<td>0.94</td>
<td>0.93</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment growth</td>
<td>2.82</td>
<td>3.39</td>
<td>0.15</td>
<td>0.27</td>
</tr>
<tr>
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<td>0.56</td>
<td>0.60</td>
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<tr>
<td>Hours growth</td>
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<td>0.68</td>
<td>0.59</td>
<td>0.31</td>
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<tr>
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<td>0.39</td>
<td>0.89</td>
<td>-0.14</td>
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<tr>
<td>Italy</td>
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<tr>
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<td>1.36</td>
<td>0.71</td>
<td>0.05</td>
</tr>
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<td>0.94</td>
<td>0.65</td>
<td>0.73</td>
</tr>
<tr>
<td>std(GC)/std(GY)</td>
<td>0.76</td>
<td>0.69</td>
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<tr>
<td>Investment growth</td>
<td>4.45</td>
<td>5.62</td>
<td>-0.02</td>
<td>0.14</td>
</tr>
<tr>
<td>GDP deflator</td>
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<td>0.70</td>
<td>-0.25</td>
<td>0.34</td>
</tr>
<tr>
<td>Hours growth</td>
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<td>0.76</td>
<td>0.34</td>
<td>0.36</td>
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<td>ΔTrade balance to GDP</td>
<td>0.44</td>
<td>0.97</td>
<td>0.15</td>
<td>-0.02</td>
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<td>Spain</td>
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<tr>
<td>GDP growth</td>
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<td>1.36</td>
<td>0.91</td>
<td>0.12</td>
</tr>
<tr>
<td>Consumption growth</td>
<td>0.90</td>
<td>1.33</td>
<td>0.64</td>
<td>0.70</td>
</tr>
<tr>
<td>std(GC)/std(GY)</td>
<td>1.27</td>
<td>0.98</td>
<td></td>
<td></td>
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<tr>
<td>Investment growth</td>
<td>2.79</td>
<td>3.51</td>
<td>0.40</td>
<td>0.40</td>
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<tr>
<td>GDP deflator</td>
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<td>0.60</td>
<td>0.83</td>
<td>0.79</td>
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<tr>
<td>Hours growth</td>
<td>1.15</td>
<td>1.21</td>
<td>0.27</td>
<td>0.35</td>
</tr>
<tr>
<td>ΔTrade balance to GDP</td>
<td>0.62</td>
<td>1.08</td>
<td>0.21</td>
<td>-0.01</td>
</tr>
</tbody>
</table>

An additional way to assess the fit of the model is to compare the estimates of endogenous variables with its observable counterparts. For example, capacity utilization is an endogenous variable defined in the GM3-EMU model and it is treated as an endogenous state in the state space representation. As a consequence, the Kalman filter allows us to retrieve a model-consistent estimate of capacity utilization over the business cycles. While capacity
utilization is not directly measurable in national account statistics, we use a ‘model-free’ or reduced form proxy that has been constructed to compare the model-based and model-free estimate of capacity utilization.\textsuperscript{18} Figure 1 plots the times series of capacity utilization implied by the reduced form model and the GM implied one computed via Kalman filter. As the differences are minimal and the two measures coincide, it gives additional credit on the plausibility of the estimated structural models to replicate key features of EA member state business cycles. It seems useful to underline that this match is improved for France, Italy and Spain by the use of the labor hoarding extension.

4.3. Dynamic transmission of shocks

Figures 2 - 7 show the dynamic responses of main variables to domestic supply (TFP), domestic demand (private saving and government spending), EA monetary policy, foreign demand shocks and exchange rate shock. All figures report the response of a temporary shock of 1\% except for TFP, where it is a temporary shock to the growth rate (i.e. permanent to the level). In all cases we report expansionary shocks. Each panel shows, for the four countries, the dynamic response of the following endogenous variables: GDP, private consumption, private investment, total hours worked, real wages, real interest rate, GDP inflation, real

\textsuperscript{18}For details on the construction of the capacity utilization series, see Havik et al. (2014).
effective exchange rate, and the trade balance-to-GDP ratio. Real variables are presented as percentage deviation from their steady state. GDP inflation and the trade balance-to-GDP ratio are expressed in percentage point deviations. Real interest rates are shown in absolute annualized deviations from their steady state.

**Figure 2: Permanent positive TFP shock**

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**Permanent positive TFP shock.** A one-time increase in the level of TFP lowers the marginal costs of production (Figure 2). As a result firms lower prices, GDP inflation goes down and real wages and income increase. The delayed increase in consumption is due to habit persistence in preferences. Expected higher returns increase the investment in Italy. Higher investment adjustment cost in Germany, France and Spain prevents the increase of investment on impact. Employment temporarily decreases in Germany and Spain, but reacts positively in Italy and France, where the labor demand adjustment costs are much more elevated. The exchange rate depreciates and the trade balance improves temporarily due to substitution of imports by domestic demand. However, on impact, the relatively slow adjustment in prices and increased demand induce a negative trade balance. As Italy’s demand responds much stronger, the trade balance deficit is also larger.

**Negative private saving shock (positive shock to consumption demand).** A negative shock to the saving rate, which is modeled as a persistent increase in the subjective rate of time preference of households, boosts domestic consumption with a concomitant increase in domestic output and prices (Figure 3). The shock triggers a rise in the policy rate and an increase
in the real interest rate in the medium term, leading to a decline in investment. The trade balance deteriorates on impact due to a combination of higher import (domestic demand expansion) and lower export demand (real exchange rate appreciation). Figure 3 also shows

Figure 4: Positive government purchases shock
that the positive shock to domestic consumption has particularly positive consequences on activity and employment in Spain. Relatively low labour market frictions (labour hoarding and wage stickiness) increase real wages in Spain, whereas real wages decrease elsewhere.

*Government expenditure shock.* An increase in government expenditure raises domestic output and crowds out consumption and investment given the increase in the real interest rate. Upward pressure of prices leads to a real exchange rate appreciation and a deterioration of the trade balance. The fiscal multiplier is less than one on impact and similar in size across the four countries. However, non negligible differences in the adjustment process can be seen particularly for Spain and France. While Spain shows a more hump-shaped response in the adjustment of employment, France illustrates a highly persistent positive response of employment and output mostly due to the very small estimated feedback parameter, $\alpha_G$, which triggers a very persistent government spending shock.

*EA monetary policy shock.* An expansionary monetary policy (lowering the annualized interest rate by 1pp) implies an increase in aggregate demand components (Figure 5). Investment raises substantially due to a decline in real interest rates. Higher domestic demand induces firms to increase labor demand which results in higher employment. The real exchange rate depreciates due to a strong initial depreciation of the Euro. Despite gaining competitiveness the trade balance-to-GDP ratio deteriorates due to the sharp increase of GDP and the low estimated price elasticity of trade ($<1$). As mentioned before, relaxing
the lower bound to this elasticity is allowed to address the fact that data rather reflect short than long term behavior. Indeed, the data show a very strong pro-cyclical correlation between imports and domestic demand components and a decline on impact of the trade balance after a demand shock tends to reflect such an evidence. This may be at the cost of an underestimation of long run effects of permanent shocks. Note, however, that the pure exchange rate channel (see Figure 7) features a standard behavior.

**Negative shock to the RoW savings rate (positive shock to foreign demand).** Figure 6 presents dynamic responses to a positive foreign demand shock, namely a negative shock to RoW savings. Analogously to domestic saving shocks, the negative RoW savings shock is modeled by a decline in the subjective discount rate. The shock increases RoW demand and activity in combination with real effective depreciation in the four countries, leading to trade balance improvements. The rise in policy and real interest rates in response to higher output and inflation in the EA dampens consumption and investment demand. Germany experiences the strongest positive response of activity, inflation and the trade balance due to its stronger trade openness to the RoW.

**Positive shock to preferences for international bond (Euro depreciation).** Figure 7 presents dynamic responses to a Euro depreciation shock. The rise in competitiveness increases trade balance via a rise in exports and a reduction imports demand. Consequently, domestic GDP and labor supply increase. The real interest rate is almost neutral on impact but rises due to monetary response to the rise of inflation, which is primarily caused by a deterioration of the
terms of trade. Consumption falls on impact because of the reduction of its imported component, while investment decreases on impact due to outflow of capital (preference shock towards foreign assets). Subsequently, both consumption and investment return to equilibrium following a path determined by the intertemporal substitution implied by the real interest rate. The overall effect on trade balance is positive for all countries, even for low estimated values of trade elasticity.

4.4. Historical Shock Decompositions

This section highlights the estimated contribution of different shocks to historical time series in the period 2000-2016. Figures 8-15 plot the historical decomposition of the four countries (DE, FR, IT, ES) for two macroeconomic variables, namely the growth rate of real GDP and the trade balance-to-GDP ratio. The black line depicts the observed data and the vertical bars show the contribution of different groups of shocks to movements in the data. Vertical bars above the horizontal line (steady state) represent positive shock contribution, while bars below the line represent negative contributions. The sum of all shock contributions equals the historical data.

We plot the contributions of the following (groups of) exogenous variables originating in the respective domestic country: (1) permanent shocks to TFP; (2) fiscal policy shocks; (3) EA monetary policy shocks; (4) price mark-up shocks; (5) interest parity shocks between the domestic country and EA versus RoW (‘bond premium shock’); (6) shocks to
the subjective discount factors of domestic households (‘private saving shock’); (7) shocks to the domestic investment risk premium; (8) domestic wage mark-up shocks; (9) domestic labor demand shocks; (10) permanent productivity shocks to private and public consumption (‘other shocks’); (11) shocks to the worldwide relative preference for domestic-produced goods and foreign goods as well as price mark-up shocks for exports and imports (‘trade shocks’); (12) the remaining shocks originating in the rest of EA; (13) the shocks originating in RoW; (14) shocks to the Oil price; (15) shocks to the risk free rate in the domestic country that represent the risk premium (‘flight to safety shock’); (16) for Germany we also plot the shock to the unemployment benefit ratio to account for the Hartz reform.

**Shock decomposition of real GDP growth.** The most pronounced shock that drives GDP fluctuations in all four countries is the investment risk premium. In Spain, it is often accompanied by private saving shocks, which, among others, have contributed to the housing boom before the global financial crisis. In Germany and France, trade shocks and price mark-up shocks have played an additional important role. In Germany, the ‘Hartz Reform’ was another positive contributor.

The growth slowdown during the financial crisis is largely associated with an increase in investment risk premia. In Spain, it was accompanied by a negative contribution of
consumption (positive saving shock or ‘deleveraging’), in France, Germany and Italy by price mark-up and trade shocks. In contrast, expansionary monetary and fiscal policy had a noticeable stabilizing effect on the respective domestic GDP growth during the 2008-09 financial crisis. The subsequent fiscal austerity due to the sovereign debt and banking crisis has the strongest negative contribution in Spain.

In 2010, the crisis was followed by a relatively rapid partial recovery due to a fall in investment risk premia and foreign demand shocks. The main drivers during this period were relatively homogeneous across the four countries. The post crisis slump in France, Italy and Spain was mainly driven by an increase in investment risk premia and negative trade shocks. The main drivers of above-trend GDP growth in Germany during the last years have been the fall in oil prices, positive trade shocks, positive foreign demand as well as the depreciation of the euro (explained in the model by an increase in the risk premium on euro-denominated bonds). The recovery in Spain and Italy in recent years has also been mainly driven by positive domestic consumption (negative shocks to private saving), i.e. by a normalisation of domestic demand compared to the crisis years.

Our estimates suggest that EA monetary policy shocks had a relatively weak stabilizing effect on GDP growth, particularly in times of binding constraints. This is somehow obvious in a linearized version of the model, where the zero lower bound on the nominal interest
Figure 10: Historical decomposition of real GDP growth in Italy

Figure 11: Historical decomposition of real GDP growth in Spain
rate is not imposed as a constraint on monetary policy. It has to be stressed, however, that "monetary policy" in the decomposition only refers to the Taylor rule shock and excludes non-conventional measures that are rather be part of receding investment risk premia, declining savings rates, and exchange rate depreciation shocks in the logic of the model.

It is interesting to notice that the GM model attributes the subdued levels of the Italian output growth over the full sample to a sequence of persistent negative TFP shocks which act as a persistent drag to the economy. TFP shocks are reduced form representation for whatever is left out from combining capital and labor inputs and their intensity in utilization. Therefore, one can think of total factor productivity as bundling together intangible assets (i.e. unobservable or difficult to measure quantities) such as technological innovation and/or input mis-allocations. In light of this, the decomposition of the Italian output growth offers a narrative which is coherent with other studies that, by exploiting the cross-sectional variation, explain the Italian low productivity in terms of limited ICT investment and penetration, e.g. Pellegrino and Zingales (2017) and Hassan and Ottaviano (2013).

**Shock decomposition of the trade balance-to-GDP ratio.** The steady state of the trade balance-to-GDP ratios are set to the mean of the observed country-specific time series. Therefore, the trade balance steady state in Germany is around 5 percent of GDP, in France around -1 percent, in Italy close to zero and in Spain around -2 percent of GDP.
Germany’s trade balance surplus has been accumulated since the beginning of the 2000s. Only during the global 2008-09 crisis, Germany’s trade balance declined because of the simultaneous contraction of RoW real activity and global trade. Beside the traditionally high saving rate in Germany, the increase in global trade and RoW demand, the depreciation of the euro (explained in the model by an increase in the risk premium on euro-denominated bonds) and the decline in oil prices lead to an even more pronounced increase of Germany’s trade balance surplus.

While France shows a gradual and persistent trade balance deterioration since the beginning of the sample, Italy and Spain experienced a rapid trade balance reversal since 2011. France suffered substantially more than the other estimated EMU countries from the decline of REA demand after the financial crisis. An increase in the risk premium on investment (explained in the model by lower import demand) as well as the euro depreciation and increased demand from RoW contributed to a stabilization of the trade balance in France.

Italy and Spain show similar pattern in terms of main drivers during the last years. Both countries suffered from negative trade shocks and low foreign demand. The Spanish pre-crisis boom, characterized by strong domestic consumption and investment, has driven the trade balance deficit in the early years. The trade balance improvements were mainly driven by an increase in savings and investment risk premia, the depreciation of the euro as well as
Particularly in the post-crisis period, the ‘flight to safety’ shock’s contribution to the trade balance-to-GDP ratio was considerably positive across the countries, especially in Italy and Spain. It captures an additional risk premium to the domestic interest rate and implies capital outflows of the domestic country, which improves the trade balance.

Summarizing the key patterns of the historical decomposition across the countries, the model suggests that:

(1) The GDP growth slowdown during the 2008-2009 financial crisis was largely due to an increase in investment risk premia and negative shocks to foreign demand and trade. The positive contribution of stabilizing fiscal policy during the financial crisis is visible across countries.

(2) The partial recovery in the aftermath of the crisis was due to a fall in investment risk premia and a recovery of world trade and demand, particularly for Germany.

(3) During the last years, the main drivers of GDP growth were a normalisation of consumption after a period of post-crisis deleveraging, the fall in oil prices, positive trade development and the euro depreciation.\footnote{We already noted discussing impulse responses that the standard exchange rate channel is present in the} Expansionary fiscal shocks contributed con-
siderably to GDP growth in Spain.

(4) The trade balance development in Germany differs substantially from those of the other countries. The improvement of the trade balance-to-GDP ratios after the financial crisis are mainly driven by traditionally higher private savings (domestic demand), an increase in investment risk premia as well as the depreciation of the euro and a recovery of world demand and trade. Weak foreign demand from REA has weighted negatively in the trade balance of the countries.

model for all countries, even where the estimated import elasticities are small.
5. Conclusion

This paper presents the European Commission’s Global Multi-country model (GM model), a fully estimated structural macroeconomic model, developed jointly by the Joint Research Centre and DG ECFIN. It is used for spillover analysis, forecasting and medium term projections.

The GM model is designed to be flexible for alternative country configurations. This paper presents the GM model constructed for member countries of the monetary union (GM3-EMU), which has been estimated for the four largest European economies (Germany, France, Italy, and Spain). We keep the core structure of the country blocks, i.e. model equations, time span, observable variables, prior parameter distributions and shocks, ex-ante identical and we allow specific modelling features, that nest the core specifications, to be selected when they satisfy information criteria. So, differences between the countries are expressed ex-post by differences in the estimated parameter values and the estimated shock processes, as well as in the selected model features. This allows us to analyse and compare business cycle properties and heterogeneity in the transmission mechanism of fundamental shocks and policy interventions.

The paper comprises the detailed theoretical model specification and the estimation results. We analyse the fit of the model for the four selected countries (Germany, France, Italy, Spain), highlight the transmission mechanisms of shocks and explain their contribution in determining the observed behavior over time of real GDP growth and the trade balance-to-GDP ratio.

Further steps in the development of the GM model have been made to aggregate the estimated EMU countries in the so-called GM7 model (Germany, France, Italy, Spain, Rest of the Euro Area, US, and Rest of the World), which is particularly interesting for analyzing and assessing cross-country spillovers.

The paper discusses also issues concerning to the model fit, which mainly concern inflation, consumption and trade. Therefore, lines of development of the GM model project, aimed at addressing such issues, include: better characterizing trade shocks and short and long term import elasticities; allowing for good specific import shares; introduce the possibility of pricing to market in export pricing; better characterize labor supply and possibly (downward) wage rigidities); better address non-Ricardian behavior; the role of different distortionary taxation schemes in fiscal stabilization rules; better endogenize monetary policy features like forward guidance.
References


Appendix A. Data source and transformations

The estimation uses quarterly and annual data for the period 1999q1 to 2016q4. Data for EMU countries and the Euro Area aggregate (EA19) are taken from Eurostat (in particular, from the European System of National Account ESA95). Bilateral trade flows are based on trade shares from the GTAP trade matrices for trade in goods and services. The Rest of the World (RoW) data are annual data and are constructed using IMF International Financial Statistics (IFS) and World Economic Outlook (WEO) databases.

Series for GDP and prices in the RoW start in 1999 and are constructed on the basis of data for the following 58 countries: Albania, Algeria, Argentina, Armenia, Australia, Azerbaijan, Belarus, Brazil, Bulgaria, Canada, Chile, China, Colombia, Croatia, Czech Republic, Denmark, Egypt, Georgia, Hong Kong, Hungary, Iceland, India, Indonesia, Iran, Israel, Japan, Jordan, Korea, Lebanon, Libya, FYR Macedonia, Malaysia, Mexico, Moldova, Montenegro, Morocco, New Zealand, Nigeria, Norway, Philippines, Poland, Romania, Russia, Saudi Arabia, Serbia, Singapore, South Africa, Sweden, Switzerland, Syria, Taiwan, Thailand, Tunisia, Turkey, Ukraine, United Arab Emirates, United Kingdom, and Venezuela.

When not available, quarterly-frequency data are obtained by interpolating annual data. We seasonally adjust the following time series using the TRAMO-SEATS package developed by Gómez and Maravall (1996): nominal public investments (for EA19, Germany, France, Italy, and Spain), nominal social benefits other than transfers in kind (for EA19, Germany, France, Italy, and Spain), government interest expenditure (for EA19, Germany, France, Italy, and Spain), compensation of employees (for Germany, France, Italy, and Spain), general government net lending (for Italy and Spain), employees (for EA19, Germany, France, Italy, and Spain).

Table A.4 lists the observed time series. GDP deflators and relative prices of aggregates are computed as the ratios of current price value to chained indexed volume. The trend component of total factor productivity is computed using the DMM package developed by Fiorentini et al. (2012). The obtained series at quarterly frequency is then used to estimate the potential output. In Germany, in order to account for the ‘Hartz’ reforms, we observe the historical unemployment benefit ratio (constructed as the ratio of unemployment benefits to the wage rate).

We make a few transformations to the raw investment series. In particular, we compute the deflator of public investments based on annual data and then obtain its quarterly frequency counterpart through interpolation. This series together with nominal public investments is then used to compute real quarterly public investments. In order to assure consistency between nominal GDP and the sum of the nominal components of aggregate demand, we impute change in inventories to the series of investments.
### Table A.4: List of observables

**EMU countries**

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<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Description</th>
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<tr>
<td>LACTR</td>
<td>$\log(\text{actr}_k)$</td>
<td>Log of active rate population</td>
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<td>LBGY</td>
<td>$\log \left( \frac{P^y}{P_e} \right)$</td>
<td>Log of Nominal gov. bonds share</td>
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<tr>
<td>LCGY</td>
<td>$\log \left( \frac{P^y}{P_c} \right)$</td>
<td>Log of Nominal gov. consumption share</td>
</tr>
<tr>
<td>LCY</td>
<td>$\log \left( \frac{P^y}{P} \right)$</td>
<td>Log of Nominal consumption share</td>
</tr>
<tr>
<td>LICY</td>
<td>$\log \left( \frac{Y}{Y} \right)$</td>
<td>Log of Nominal gov. interest payments share</td>
</tr>
<tr>
<td>LIGY</td>
<td>$\log \left( \frac{I}{I} \right)$</td>
<td>Log of Nominal gov. investment share</td>
</tr>
<tr>
<td>LITOTY</td>
<td>$\log \left( \frac{I}{I} \right)$</td>
<td>Log of Nominal investment share</td>
</tr>
<tr>
<td>LN</td>
<td>$\log(N)$</td>
<td>Log of hours</td>
</tr>
<tr>
<td>LPCVATPYOBS</td>
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<td>Log of consumption price final to observed GDP price</td>
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<td>LPGPYOBS</td>
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<td>Log of gov. observed price to observed GDP price</td>
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<td>$\log \left( \frac{P^y}{P^G} \right)$</td>
<td>Log of govt. investment price to observed GDP price</td>
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<td>LPITOTPYOBS</td>
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<td>Log of import price to observed GDP price</td>
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<td>$\log \left( \frac{P}{P} \right)$</td>
<td>Log of population</td>
</tr>
<tr>
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<td>$\log \left( \frac{P^X}{P} \right)$</td>
<td>Log of export price to GDP price</td>
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<tr>
<td>LPYOBS</td>
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<td>Log of observed GDP price</td>
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<td>LTFPTREND</td>
<td>$\log(tfp)$</td>
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<td>$\log \left( \frac{Y}{Y} \right)$</td>
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<td>Nominal wage share</td>
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<td>Log of Nominal export share</td>
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<td>LYOBS</td>
<td>$\log(Y)$</td>
<td>Log of observed GDP</td>
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<tr>
<td>TBY</td>
<td>$\log \left( \frac{TB}{Y} \right)$</td>
<td>Nominal trade balance share</td>
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**Rest of the Euro Area**

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<td>INOMEA</td>
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<td>EA Nominal Interest rate</td>
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<td>$\log(\epsilon_{RoW,EA})$</td>
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<td>Log of import price to observed GDP price</td>
</tr>
<tr>
<td>LPOP,EA</td>
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<td>Log of population</td>
</tr>
<tr>
<td>LPXOPYOBS,EA</td>
<td>$\log \left( \frac{P^X}{P} \right)$</td>
<td>Log of export price to GDP price</td>
</tr>
<tr>
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<td>Log of observed GDP price</td>
</tr>
<tr>
<td>LYOBS,EA</td>
<td>$\log(Y)$</td>
<td>Log of observed GDP</td>
</tr>
<tr>
<td>LYTREND,EA</td>
<td>$\log(Y)$</td>
<td>Log of GDP trend</td>
</tr>
<tr>
<td>TB,EA</td>
<td>$\log \left( \frac{TB}{Y} \right)$</td>
<td>Nominal trade balance share</td>
</tr>
<tr>
<td>XY,EA</td>
<td>$\log \left( \frac{X}{Y} \right)$</td>
<td>Nominal export share of EA</td>
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**Rest of the World**

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<th>Description</th>
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</thead>
<tbody>
<tr>
<td>INOMRoW</td>
<td>$i_{RoW}$</td>
<td>RoW Nominal Interest rate</td>
</tr>
<tr>
<td>LPOILRoW</td>
<td>$\log(P^{oil})$</td>
<td>Log of oil price</td>
</tr>
<tr>
<td>LPOP,RoW</td>
<td>$\log(P)$</td>
<td>Log of population</td>
</tr>
<tr>
<td>LPYOBS,RoW</td>
<td>$\log \left( \frac{P^Y}{P^I} \right)$</td>
<td>Log of observed GDP price</td>
</tr>
<tr>
<td>LYOBS,RoW</td>
<td>$\log(Y)$</td>
<td>Log of observed GDP</td>
</tr>
<tr>
<td>LYTREND, RoW</td>
<td>$\log(Y)$</td>
<td>Log of GDP trend</td>
</tr>
</tbody>
</table>
Table A.5: Prior and posterior distributions of key model parameters and innovations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Distr Mean St.dev DE FR IT ES</td>
<td></td>
</tr>
<tr>
<td><strong>Autocorrelations of forcing variables</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subjective discount factor $\rho^{UC}$</td>
<td>Beta 0.5 0.87 0.65 0.77 0.87</td>
<td></td>
</tr>
<tr>
<td>Investment risk premium $\rho^S$</td>
<td>Beta 0.85 0.96 0.93 0.96</td>
<td></td>
</tr>
<tr>
<td>Domestic price mark-up $\rho^{MUY}$</td>
<td>Beta 0.05 (0.93, 0.98) (0.92, 0.97) (0.91, 0.98) (0.94, 0.98)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labor demand $\rho^{ND}$</td>
<td>Beta 0.85 0.93 0.95 0.97</td>
<td></td>
</tr>
<tr>
<td>Flight to safety $\rho^{FQ}$</td>
<td>Beta 0.05 (0.86, 0.96) (0.90, 0.97) (0.92, 0.98) (0.94, 0.99)</td>
<td></td>
</tr>
<tr>
<td>Trade share $\rho^M$</td>
<td>Beta 0.45 0.95 0.94 0.77 0.83</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>* When we adopt the extended Phillips curve, we set persistence of the shock to 0, for the sake of parsimony.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| **Standard deviations (%) of innovations to forcing variables** | |
| ---- | |
| Monetary policy $\varepsilon^i$ | Gamma 0.1 0.009 0.009 0.008 0.009 |
| Government consumption $\varepsilon^G$ | Gamma 0.01 0.0009 0.001 0.001 0.001 |
| Gov transfers $\varepsilon^T$ | Gamma 0.004 0.0008 0.0011 0.0008 |
| Temporary TFP $\varepsilon^{LAY}$ | Gamma 0.0004 0.0009 0.0012 0.0012 |
| Subjective discount factor $\varepsilon^{UC}$ | Gamma 0.004 0.0015 0.0015 0.0015 |
| Investment risk premium $\varepsilon^S$ | Gamma 0.0004 0.0003 0.0002 0.0002 |
| Price mark-up $\varepsilon^{MUY}$ | Gamma 0.008 0.0016 0.0016 0.0016 |
| labor supply $\varepsilon^U$ | Gamma 0.01 0.01 0.01 0.01 |
| Fligth to safety $\varepsilon^S$ | Gamma 0.0004 0.0008 0.0008 0.0008 |
| Trade share $\varepsilon^M$ | Gamma 0.01 0.025 0.0199 0.0251 |

* When we adopt the extended Phillips curve, we set persistence of the shock to 0, for the sake of parsimony.
Appendix B. Additional estimation results

Appendix B.1. Annual Fit

Figures B.16 - B.19 show the unconditional 1- and 2-year ahead forecast of selected observed variables for the four EMU countries. The solid blue line depicts the observed annual time series, the red solid line shows the unconditional model-implied 1- and 2-year ahead prediction at each point (year) in time. The dashed slim green and blue lines connect the 1- and 2-year predictions, respectively.

This graphical representation of the 1- and 2-year ahead forecast error, discussed in section 4.2, suggest that our estimated models deliver a relatively good (in-sample) forecast accuracy. For example, looking closer to the huge drop in real GDP growth during the global financial crisis for the four estimated countries, the models 2-year ahead predictions in 2008 forecast a further decrease in GDP growth in 2009 before it forecasts a recovery in 2010. We are also able to fit fairly well nominal and real export and import growth across countries. However, we face some difficulties in delivering a well-performing (in-sample) forecast accuracy, e.g., for consumption growth and GDP inflation in Germany and France.

Appendix B.2. Cross correlation

Figures B.20 - B.23 depict the lead-lag structure of real GDP growth with its main components (consumption, investment, employment, and the trade balance) and GDP inflation for the four EMU countries. We use first differences for the trade balance-to-GDP ratio and quarter-on-quarter growth rates for all other variables. It compares the model-generated cross correlations (black) (auto-correlation for GDP growth) with the ones of the observed data (blue). The horizontal dashed red lines represent the 95% confidence bounds.

In the figures, lag refers to the timing of the second argument of the couple, where GDP is always the first. Looking for example at the subplot of consumption growth in B.20, it provides information on the cross-correlation of consumption growth, ranging from \( t - 5 \) to \( t + 5 \), on contemporaneous GDP growth at time \( t \): when lag is positive, consumption leads GDP by lag periods; when lag is negative, consumption lags GDP by lag periods.

Therefore, the cross-correlation of consumption growth in \( t + 2 \) on GDP growth in \( t \) can also be interpreted as the cross-correlation of GDP growth in \( t - 2 \) on consumption growth in \( t \).

The figures suggest that most of the correlations between GDP growth and its components are fairly well captured. More precisely, all country models replicate the contemporaneous correlation of consumption, investment and employment with output. In our model the trade balance is positively correlated with output, but matches the data pattern only for Germany. Moreover, all estimated models generate a negative contemporaneous correlation.
between GDP inflation and GDP growth, which matches the data only in Germany and Italy. Persistency patterns are particularly well seized in Spain.
Figure B.16: Annual fit in Germany

Note: The solid blue line depicts the observed annual series, the red solid line shows the unconditional 1- and 2-year ahead prediction.
Figure B.17: Annual fit in France
Figure B.18: Annual fit in Italy
Figure B.19: Annual fit in Spain
Figure B.20: Lead-Lag structure of output growth with its main component and GDP inflation for Germany

Figure B.21: Lead-Lag structure of output growth with its main component and GDP inflation for France
Figure B.22: Lead-Lag structure of output growth with its main component and GDP inflation for Italy

Figure B.23: Lead-Lag structure of output growth with its main component and GDP inflation for Spain
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