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2018

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September 2018

Abstract

The paper reviews adjustment dynamics in the EMU on the basis of estimated DSGE models for four large EA Member States (DE, FR, IT, ES). We compare the response of the four countries to identical shocks and find a particularly strong response of employment and wages in ES, a high sensitivity of IT to investment-related shocks, and a comparatively strong impact of global shocks on the DE economy. We also perform counterfactual exercises that apply the estimated shocks and parameters for ES to DE, FR, and IT. The counterfactual simulations suggest that differences in shocks have been important for GDP growth differentials, and together with structural differences also contributed to differences in employment fluctuations across the four countries considered.

JEL classification: E32, F41, F44, F45
Keywords: Estimated DSGE; adjustment dynamics; business cycles; EMU; counterfactuals

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1. Introduction

Macroeconomic developments across Euro Area (EA) Member States (MS) have displayed a great deal of heterogeneity since the beginning of Economic and Monetary Union (EMU). The heterogeneity includes a boom in the EA "periphery" and rather meagre growth in the EA "core" in the early years of EMU, and a protracted recession in the EA "periphery" and much stronger recovery in the EA "core" after the global financial crisis.

Whether the heterogeneity across EA countries reflects differences in shocks, or differences in the transmission of shocks has been a longstanding question in the discussion of EMU. The general literature on the role of shocks and structure for business cycle differences across countries suggests that both elements are relevant. In an early paper on EMU, Bayoumi and Eichengreen (1992) find shocks in a VAR to be significantly more idiosyncratic across EU countries and the adjustment to shocks in EU to be slower compared to US regions. Abbritti and Mueller (2013) argue based on a calibrated DSGE model that structure matters for unemployment and inflation differentials in monetary union.

The role of shocks versus structure has also been discussed with respect to the relative performance of the EA and the US economies. Christiano et al. (2008) compare estimated DSGE models to show that shocks and structure matter for differences between EA and US aggregates. Sahuc and Smets (2008) perform counterfactuals with an estimated DSGE model and investigate whether differences in the interest rate policy between the ECB and the Fed are attributable to differences in the structure of the economies or differences in the size and nature of the shocks.

This paper contributes to the literature by looking at business cycle heterogeneity in EMU through the lenses of estimated DSGE models for individual EA MS. Following Albonico et al. (2017), we have estimated country blocks for the four largest economies in the EA, namely Germany, France, Italy, and Spain. The model is set up in a configuration with the respective MS (DE/FR/IT/ES), the rest of the EA (REA), and the rest of the world (RoW). The structure of the country blocks, i.e. model equations, time span, observable variables, prior parameter distributions, is ex-ante identical, so that differences between DE, FR, IT, and ES are expressed ex-post by differences in the estimated parameter values and the estimated shock processes.

The paper takes a comparative perspective to investigate the role of structural differences and idiosyncratic shocks. In a first step, we compare the adjustment dynamics of MS economies to identical shocks to assess the importance of structural differences. To the extent that outcomes
differ across countries in this scenario, the differences can be attributed to differences in the estimated economic structure. As an aside, we can compare the adjustment at country level to the adjustment of the EA aggregate to the same set of shocks to gauge whether the absence of nominal exchange rate adjustment and monetary policy independence has hampered macroeconomic adjustment at the level of MS significantly. In a second step, we perform counterfactuals (i) by imposing the estimated (smoothed) shocks from ES on the model blocks for DE, FR and IT and (ii) by imposing the estimated ES parameter with the country-specific estimated shocks such that the structure across MS is (largely) harmonised. Heterogeneity in outcomes in the latter case would be attributable to country-specific shocks rather than to differences in model structure.

Our estimated model is a standard New Keynesian model with price and wage rigidity. It emphasizes the role of price and wage adjustment in the transmission of shocks as a means to achieve relative price adjustment and contain fluctuations in economic activity in response to shocks. By contrast, financial frictions that have arguably played a substantial role for EA heterogeneity, notably the credit growth that has reinforced the boom and the deleveraging that has reinforced the recession in the EA "periphery", are not modelled as endogenous transmission channels, but rather reflected in the estimated (savings and investment) shocks.

We structure the remainder of the paper as follows: Section 2 presents stylized facts to motivate the paper. Sections 3 and 4 outline the structure of the model and the methodology for the model solution and estimation, respectively. Section 5 compares IRFs of individual EA countries and the EA aggregate for the shocks that have been main drivers of fluctuations in economic activity over the estimation horizon according to the estimated models, and it presents counterfactuals that illustrate the importance of differences in shocks versus structure (i.e. shock transmission) for macroeconomic heterogeneity over the sample period. Section 6 summarizes the findings and concludes.

2. Stylized facts
Macroeconomic developments across Euro Area (EA) Member States (MS) have displayed a large amount of heterogeneity since the beginning of Economic and Monetary Union (EMU). Figure 1 shows year-on-year real GDP growth for the EA aggregate and the four big MS (DE, FR, IT and ES) considered in this paper over the period 1991-2017. The Figure illustrates the difference in average growth rates over the sample, including since the start of EMU in 1999. The timing of
upturns and downturns in GDP growth appears to be fairly synchronised for our set of countries, whereas the strength of fluctuations in the growth rate differs.

**Figure 1: Real GDP growth 1991-2017 (year-on-year, %).**

The output gap as indicator of cyclical fluctuations around potential output shows a similar pattern as displayed in Figure 2. The timing of upswings and downswings in the business cycle is similar for DE, FR, IT, ES and the EA aggregate, but the amount of fluctuations varies. In particular, the range between maximum and minimum values of the output gap appears to be largest for Spain, and rather modest for France.

Table 1 presents summary statistics for the volatility of economic activity for 1999q1-2017q2 on a quarterly basis. The Table shows data moments for selected variables for DE, FR, IT, ES and the EA aggregate. In particular, it lists standard deviations and correlations with domestic real GDP growth. All variables are in quarter-on-quarter growth rates or first differences (real interest rate, TBY). Real GDP growth in DE, IT and ES has been more volatile than in the EA aggregate according to Table 1, contrary to FR. Contrary to the picture in Figures 1-2, GDP growth in ES does not stand out as particularly volatile based on quarterly data, which indicate a higher standard deviation for DE and IT. ES is characterised by strong movements in hours worked, real
wage growth, and consumption, however. Finally, the correlations between GDP growth, on the one hand, and inflation, REER, and TBY, on the other hand, change sign across countries, potentially indicating differences in the importance of demand versus supply shocks.

Figure 2: Annual output gap 1991-2017 (%).

Source: Ameco.

Table 1: Empirical business-cycle statistics, 1999q1-2017q2.

<table>
<thead>
<tr>
<th>Variables</th>
<th>EA</th>
<th>corr (x,GDP)</th>
<th>DE</th>
<th>corr (x,GDP)</th>
<th>FR</th>
<th>corr (x,GDP)</th>
<th>IT</th>
<th>corr (x,GDP)</th>
<th>ES</th>
<th>corr (x,GDP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real GDP</td>
<td>0.61</td>
<td>1</td>
<td>0.84</td>
<td>1</td>
<td>0.48</td>
<td>1</td>
<td>0.74</td>
<td>1</td>
<td>0.70</td>
<td>1</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.38</td>
<td>0.72</td>
<td>0.58</td>
<td>0.27</td>
<td>0.45</td>
<td>0.60</td>
<td>0.58</td>
<td>0.74</td>
<td>0.88</td>
<td>0.83</td>
</tr>
<tr>
<td>Investment</td>
<td>2.59</td>
<td>0.79</td>
<td>4.22</td>
<td>0.49</td>
<td>2.77</td>
<td>0.59</td>
<td>4.12</td>
<td>0.58</td>
<td>2.88</td>
<td>0.60</td>
</tr>
<tr>
<td>Hours worked</td>
<td>0.44</td>
<td>0.80</td>
<td>0.54</td>
<td>0.58</td>
<td>0.39</td>
<td>0.59</td>
<td>0.57</td>
<td>0.58</td>
<td>1.13</td>
<td>0.77</td>
</tr>
<tr>
<td>GDP deflator</td>
<td>0.26</td>
<td>0.02</td>
<td>0.35</td>
<td>-0.25</td>
<td>0.29</td>
<td>0.15</td>
<td>0.54</td>
<td>-0.10</td>
<td>0.47</td>
<td>0.57</td>
</tr>
<tr>
<td>CPI deflator</td>
<td>0.36</td>
<td>0.38</td>
<td>0.37</td>
<td>0.20</td>
<td>0.39</td>
<td>0.40</td>
<td>0.40</td>
<td>0.34</td>
<td>0.65</td>
<td>0.28</td>
</tr>
<tr>
<td>Real wage</td>
<td>0.29</td>
<td>-0.27</td>
<td>0.50</td>
<td>-0.10</td>
<td>0.43</td>
<td>-0.08</td>
<td>0.73</td>
<td>0.15</td>
<td>1.15</td>
<td>-0.15</td>
</tr>
<tr>
<td>Real interest rate</td>
<td>0.19</td>
<td>0.15</td>
<td>0.23</td>
<td>-0.07</td>
<td>0.16</td>
<td>-0.21</td>
<td>0.31</td>
<td>-0.21</td>
<td>0.19</td>
<td>-0.10</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>2.27</td>
<td>0.25</td>
<td>1.26</td>
<td>0.28</td>
<td>1.06</td>
<td>0.22</td>
<td>1.47</td>
<td>0.20</td>
<td>1.24</td>
<td>-0.11</td>
</tr>
<tr>
<td>... relative to std of real GDP</td>
<td>3.72</td>
<td>-</td>
<td>1.50</td>
<td>-</td>
<td>2.21</td>
<td>-</td>
<td>1.99</td>
<td>-</td>
<td>1.77</td>
<td>-</td>
</tr>
<tr>
<td>Trade balance to GDP</td>
<td>0.31</td>
<td>-0.09</td>
<td>0.67</td>
<td>0.34</td>
<td>0.39</td>
<td>-0.17</td>
<td>0.41</td>
<td>-0.19</td>
<td>0.63</td>
<td>-0.45</td>
</tr>
</tbody>
</table>

Note: First differences for real interest rate and trade balance; quarter-on-quarter growth rates for all other variables; an increase in the REER indicates real effective depreciation. Moments are computed on de-trended variables.
3. Model description
The analysis is based on the Global Multi-country (GM) model of the European Commission with an EA Member State (MS), the rest of the EA (REA), and the rest of the world (RoW) as building blocks (see Albonico et al. 2017). The EA MS block of the model is rather detailed, while the REA and RoW blocks are more stylized. The EA MS block assumes two (representative) households, firms and a government. EA MS households provide labour services to domestic firms. One of the two households (Ricardians) in each country has access to financial markets, owns her country’s firms and can smooth their consumption. The other household (liquidity-constrained) has no access to financial markets, does not own financial or physical capital, and consumes the disposable wage and transfer income in each period.

Final output in the EA MS is generated by perfectly competitive firms that combine domestic and imported intermediate inputs. Intermediates are produced by monopolistically competitive firms using local labour and capital as inputs. EA MS wages are set by monopolistic trade unions. Nominal differentiated goods prices and nominal wages are sticky. Governments purchase the local final good, make lump-sum transfers to local households, levy labour, consumption and capital taxes and issue debt. Given the monetary union setting, the European Central Bank (ECB) sets the nominal interest rate following a Taylor-type rule defined on EA aggregate inflation and the output gap. All exogenous random variables in the model follow independent autoregressive processes.

3.1. EA Member State households
The household sector consists of a continuum of households \( j \in [0; 1] \). There are two types of households, savers ("Ricardians", superscript \( s \)) who own firms and hold government and foreign bonds and liquidity-constrained households (subscript \( c \)) whose only income is labour income and who do not save. The share of savers in the population is \( \omega^s \).

Both households enjoy utility from consumption \( C^r_{jkt} \) and incur disutility from labour \( N^r_{jkt} \) \((r = s, c)\). On top of this, Ricardian’s utility depends also on the financial assets held. Date \( t \) expected life-time utility of household \( r \), is defined as:

\[
U^r_{jkt} = \sum_{s=t}^{\infty} \exp(\varepsilon^r_{kt}) \beta^{s-t} u^r_{jkt}(c)
\]
where $\beta$ is the (non-stochastic) discount factor (common for both types of households) and $\varepsilon_{kt}^c$ is the saving shock.

### 3.1.1. Ricardian households

The Ricardian households work, consume, own firms and receive nominal transfers $T_{jkt}$ from the government. Ricardians have full access to financial markets and are the only households who own financial assets, $\frac{A_{jkt}}{p_{kt}^{c,vat}}$, where $p_{kt}^{c,vat}$ is consumption price, including VAT.\(^1\) Financial wealth of household $j$ consists of bonds $\frac{B_{jkt}}{p_{kt}^{c,vat}}$ and shares $\frac{p_{kt}^S S_{jkt}}{p_{kt}^{c,vat}}$, where $p_{kt}^S$ is the nominal price of shares in $t$ and $S_{jkt}$ the number of shares held by the household:

$$\frac{A_{jkt}}{p_{kt}^{c,vat}} = \frac{B_{jkt}}{p_{kt}^{c,vat}} + \frac{p_{kt}^S S_{jkt}}{p_{kt}^{c,vat}}$$

It is assumed that households invest only in domestic shares. Bonds consist of government domestic, $\frac{B_{jkt}^g}{p_{kt}^{c,vat}}$, and foreign bonds\(^2\), $\frac{B_{jkt}^l}{p_{kt}^{c,vat}}$, and private risk-free bonds, $\frac{B_{jkt}^{rf}}{p_{kt}^{c,vat}}$ (in zero supply), with $e_{ikt}$ the bilateral exchange rate and $e_{kkt} \equiv 1$.

The period $t$ budget constraint of a saver household $j$ is:

$$(1 - \tau_k^N) \text{W}_{kt} \text{N}_{jkt} + (1 + i_{kt-1}^g) B_{jkt-1}^g + (1 + i_{kt-1}^{bw}) e_{ikt} B_{jkt-1}^{bw} + (1 + i_{kt-1}^{rf}) B_{jkt-1}^{rf}$$

$$+ (p_{kt}^S + p_{kt}^Y d_{kt}) S_{jkt-1} + T^s_{jkt} - \text{tax}^s_{jkt} = p_{kt}^{c,vat} c_{jkt}^s + A_{jkt}$$

where $\text{W}_{kt}$ is the nominal wage rate, $N_{jkt}^s$ is the employment in hours, and $\tau_k^N$ the labour tax rate. $p_{kt}^Y$, is the GDP price deflator. $i_{kt-1}^g$, $i_{kt-1}^{bw}$, $i_{kt-1}^{rf}$ are returns on domestic government bonds, foreign bonds of region $l$, and risk-free bonds, respectively. $T^s_{jkt}$ are government transfers to savers and $\text{tax}^s_{jkt}$ are lump-sum taxes paid by savers. Intermediate goods producers paying dividends $d_{kt}$ to savers.

We define the gross nominal return on domestic shares as:

$$1 + i_{kt}^s = \frac{p_{kt}^S + p_{kt}^Y d_{kt}}{p_{kt-1}^S}$$

\(^1\) Note that $p_{kt}^{c,vat}$ is related to $p_{kt}^c$, the private consumption deflator in terms of input factors, by the formula: $p_{kt}^{c,vat} = (1 + \tau_k^c) p_{kt}^c$ where $\tau_k^c$ is the tax on consumption.

\(^2\) We assume only one type of foreign bonds, $B_{RoWkt}^w$, issued by RoW and denominated in RoW currency.
The instantaneous utility functions of savers, $u^s(\cdot)$, is defined as:

$$u^s_{jk}(c^s_{jt}, n^s_{jt}, \frac{u^A_{jkt-1}}{p_{c,vat}^s}) = \frac{1}{1 - \theta_k}(c^s_{jt} - h_k c^s_{kt-1})^{1-\theta_k} - \frac{\omega^N_k \epsilon^u_{kt}}{1 + \theta^N_k} (c^s_{jt})^{1-\theta_k} (n^s_{jt})^{1+\theta_k}$$

where $c^s_{kt} = \int_0^1 c^s_{jkt} dj$, and $c_{kt} = \omega^s c^s_{kt} + (1 - \omega^s)c^c_{kt}$; $h_k \in (0; 1)$ measures the strength of external habits in consumption, $\omega^N_k$ the weight of the disutility of labour, and $\epsilon^u_{kt}$ captures a labour supply (or wage mark-up) shock.

The disutility of holding risky financial assets, $u^A_{jkt-1}$, is defined as:

$$u^A_{jkt-1} = (\alpha_k^b b^0 + \epsilon_k^b) B^g_{jkt-1} + \left((\alpha_k^b b^0 + \epsilon_k^b) e_{jkt-1} B^w_{jkt-1} + \frac{\alpha_k^b}{2} \left(\frac{e_{jkt-1} B^w_{jkt-1}}{p_k^y Y_{k-1}}\right)^2\right) + (\alpha_k^s S^0 + \epsilon_k^s) P_{st-1} S_{jkt-1}$$

Internationally traded bonds are subject to transaction costs in form of a function of the average net foreign asset position relative to GDP. The asset specific risk premium shock depends on an asset specific exogenous shock $\epsilon^x, x \in \{B, S, bw\}$ and an asset specific intercept $\alpha^x, x \in \{b^0, S^0, bw^0\}$. Similar to Krishnamurthy and Vissing-Jorgensen (2012) and Fisher (2015), the approach of modelling the disutility of holding risky assets captures the households’ preferences for the safe short term bonds, which generates endogenously a wedge between the return on risky assets and safe bonds.\(^3\) As in Benigno (2009) and Ratto et al. (2009), we assume that only the RoW bond is traded internationally. It follows that households in the Euro Area can invest in both national and foreign assets, while RoW households can only invest in domestic bonds.

The Ricardian household problem leads to the following first order conditions (FOC):

The FOC w.r.t. savers’ consumption produces:

$$\epsilon^c_{kt} (c^s_{kt} - h c^s_{kt-1})^{-\theta} = \lambda^s_{kt}$$

where $\lambda^s_{kt}$ is the Lagrange multiplier on the budget constraint.

\(^3\) This modification is along the lines of the money-in-utility approach by Sidrauski (1967), in which model agents derive utility from their holdings of money. In our model, it reflects the costs of holding risky assets relative to risk-free assets. A similar framework is used by Vitek (2017).
The FOC w.r.t. domestic risk-free bond:

$$\beta E_t \left[ \frac{1 + i_{r1}^{rf}}{\lambda_{kt} + \nu_{kt+1}} \right] = 1$$

The FOC w.r.t. domestic government bonds:

$$\beta E_t \left[ \frac{1 + i_{g}^{g} - \epsilon_{kt}^{B} - \alpha_{k1}^{b0}}{\lambda_{kt} + \nu_{kt+1}} \right] = 1$$

with $\pi_{kt}^{C, vat}$ the consumption deflator inflation rate and $\epsilon_{kt}^{B}$ the risk-premium on government bonds.

The FOC w.r.t. domestic stocks:

$$\beta E_t \left[ \frac{(1 + i_{s}^{s} - \epsilon_{kt}^{s} - \alpha_{k1}^{s0})}{\lambda_{kt} + \nu_{kt+1}} \right] = 1$$

where $\epsilon_{kt}^{s}$ the risk premium on stocks. The above optimality conditions are similar to a textbook Euler equation, but incorporate asset-specific risk premia that depend on an exogenous shock $\epsilon_{kt}^{A}$ as well as the size of the asset holdings as a share of GDP, see Vitek (2017) for a similar formulation. Taking into account the Euler equation for the risk-free bond and approximating, the equations simplify to the familiar expressions:

$$i_{kt}^{g} = i_{kt}^{rf} + rprem_{kt}^{g}$$

$$i_{kt}^{s} = i_{kt}^{rf} + rprem_{kt}^{s}$$

In the equations above, $rprem_{kt}^{g}$ is the risk premium on domestic government bonds, and $rprem_{kt}^{s}$ is a risk premium on domestic shares. It is introduced to capture, in a stylized manner, financial frictions.\(^4\)

Given the monetary union setting, we assume that an uncovered interest rate parity condition links the interest rate of EMU country to the EA interest rate (set by the central bank):

$$(1 + i_{kt}^{rf}) = (1 + i_{EA}) - \left( \alpha_{k1}^{bw1} \frac{e_{ROW, EA} B^{W}_{kt}}{p^{Y}_{kt} Y_{kt}} \epsilon_{kt}^{FQ} \right)$$

where $\alpha_{k1}^{bw1} e_{ROW, EA} B^{W}_{kt}$ captures a debt-dependent country risk premium on net foreign asset holdings as external closure to ensure long-run stability (see, e.g., Schmitt-Grohe and Uribe, 2003; Adolfson et al., 2008). Following Smets and Wouter (2007) we also introduce an additional risk premium shock, $\epsilon_{kt}^{FQ}$ (‘Flight to Safety’), which creates a wedge between the EA interest rate , $i_{EA}$, and

\(^4\) Observationally, this approach is equivalent to exogenous risk premia as well as risk premia derived in the spirit of Bernanke et al. (1996).
and the return on domestic risk-free assets, \( i_{kt}^{rf} \). Since a positive shock increases the required return on domestic assets and the cost of capital, it reduces current consumption and investment simultaneously and helps explaining the co-movement of consumption and investment.

### 3.1.2. Liquidity-constrained household

The liquidity-constrained household consumes her disposable after-tax wage and transfer income in each period of time ('hand-to-mouth'). The period \( t \) budget constraint of the liquidity-constrained household is:

\[
(1 + \tau_k^c)P_k^c C_j^{c,kt} = (1 - \tau_k^N)W_{kt} N_k^c + T_{kt}^c - tax^c_{jkt}.
\]

The instantaneous utility functions for liquidity-constrained households, \( u^c(\cdot) \), is defined as:

\[
u_j^{c,kt}(C_j^{c,kt}, N_j^{c,kt}) = \frac{1}{1 - \theta_k} \left( C_j^{c,kt} - h_k C_{kt-1}^{c} \right)^{1-\theta_k} - \left( C_k^{c,kt} \right)^{1-\theta_k} \frac{\omega_k^N \epsilon_{kt}^c}{1 + \theta_k^N} (N_j^{c,kt})^{1+\theta_k^N}
\]

with \( C_k^{c,kt} = \int_0^1 C_j^{c,kt} dj \).

### 3.1.3. Wage setting

Households are providing differentiated labour services, \( N_{jkt}^c \), in a monopolistically competitive market. We assume that there is a labour union that bundles labour hours provided by both types of domestic households into a homogeneous labour service and resells it to intermediate goods producing firms. We assume that Ricardian and liquidity-constrained households’ labour are distributed proportionally to their respective population shares, \( \omega_k^S \). Since both households face the same labour demand schedule, each household works the same number of hours as the average of the economy. It follows that the individual union's choice variable is a common nominal wage rate for both types of households.

The wage rule is obtained by equating a weighted average of the marginal utility of leisure, \( U_{kt}^N \), to a weighted average of the marginal utility of consumption, \( \lambda_{kt} \), times the real wage adjusted for a wage mark-up. Nominal rigidity in wage setting is introduced in the form of adjustment costs for changing wages. The wage adjustment costs are borne by the household. We also allow for real wage rigidity as in Blanchard and Galí (2007), where the slow adjustment of real wages occurs through distortions rather than workers’ preferences. The optimality condition is given by:
\[
\left( \frac{\mu_k^w}{\lambda_{kt}} \frac{U^N_{kt}}{P^Y_{kt}} \right)^{1-\gamma_k^{wr}} \frac{W_{kt-1}^{\gamma_k^{wr}}}{P^Y_{kt-1}} = \frac{W_{kt}}{P^Y_{kt}} \gamma_k^{wr} \left( 1 - \tau_N^k \right) \left( \frac{W_{kt}}{W_{kt-1}} - 1 - (1 - s_{fk}^w) (\pi_{kt-1}^Y - \bar{\pi}) - \pi^w \right)
\]

\[
= \gamma_k^{wr} E_t \left[ \frac{\lambda_{kt+1}}{\lambda_{kt}} \frac{N_{kt+1}}{N_{kt}} \frac{P^C_{kt+1} \psi_{kt+1}}{P^C_{kt} \psi_{kt}} \left( \frac{W_{kt+1}}{W_{kt}} - 1 - (1 - s_{fk}^w) (\pi_{kt}^Y - \bar{\pi}) \right) \right] - \pi^w \left( \frac{W_{kt+1}}{W^Y_{kt}} \frac{W_{kt}}{P^Y_{kt}} \right) + \varepsilon_{kt} \frac{W_{kt}}{P^Y_{kt}}
\]

where \( \mu_k^w \) is the wage mark-up, \( \gamma_k^{wr} \) is the degree of real wage rigidity, \( \gamma_k^w \) is the degree of nominal wage rigidity and \( s_{fk}^w \) is the degree of forward-lookingsness in the labour supply equation. \( U^N_{kt} \) is the marginal disutility of labour and defined as:

\[
U^N_{kt} = \omega_k^N (C_{kt})^{1-\theta_k} (N_{kt})^{-\theta_k^N}
\]

### 3.2. EA Member State production sector

#### 3.2.1. Total output demand

Total output, \( O_{kt} \), is produced by perfectly competitive firms by combining value added, \( Y_{kt} \), with energy input, \( Oil_{kt} \), using the following CES production function:

\[
O_{kt} = \left[ (1 - s_k^{oil}) \sigma_k^0 (Y_{kt})^{\sigma_k^{-1}} + (s_k^{oil}) \sigma_k^0 (Oil_{kt})^{\sigma_k^{-1}} \right]^{\frac{1}{\sigma_k}}
\]

where \( s_k^{oil} \) is the energy input share in total output and elasticity \( \sigma_k^0 \) is inversely related to the steady-state gross output price mark-up. It follows that the demand for \( Y_{kt} \) and \( Oil_{kt} \) by total output producers is, respectively:

\[
Y_{kt} = (1 - s_k^{oil}) \left( \frac{P^Y_{kt}}{P^Y_{oil_{kt}}} \right)^{-\sigma_k^0} O_{kt}
\]

\[
Oil_{kt} = s_k^{oil} \left( \frac{P^{oil}_{kt}}{P^Y_{kt}} \right)^{-\sigma_k^0} O_{kt}
\]

where \( P^Y_{kt} \) and \( P^{oil}_{kt} \) are price deflators associated with \( Y_{kt} \) and \( Oil_{kt} \), respectively. Since Oil is assumed to be imported from RoW, the oil price is given by:

\[
P^{oil}_{kt} = e_{Rowkt} P^{oil}_{Rowt} + \tau^{oil} P^Y_t\tau^0
\]

where \( \tau^{oil} P^Y_t \) is the excise duty. The price index of total output \( P^0_{kt} \) is:
\[ p_{kt}^0 = \left[ (1 - s_k^0 (p_{kt}^0))^{1-\sigma_k^0} + s_k^0 (p_{kt}^0)^{1-\sigma_k^0} \right]^{\frac{1}{1-\sigma_k^0}} \]

3.2.2. Intermediate goods producer

Each firm \( i \in [0;1] \) produces a variety of the domestic good which is an imperfect substitute for varieties produced by other firms. Given imperfect substitutability, firms are monopolistically competitive in the goods market and face a downward-sloping demand function for goods. Domestic final good producers then combine the different varieties into a homogenous good and sell them to domestic final goods producers and exporters.

Differentiated goods are produced using total capital, \( K_{ikt-1}^{tot} \), and labour, \( N_{ikt} \), which are combined in a Cobb-Douglas production function:

\[ Y_{ikt} = [A_k^Y (N_{ikt} - FN_{ikt})]^{\alpha} (cu_{ikt} K_{ikt-1}^{tot})^{1-\alpha} - A_k^Y FC_{ikt} \]

Where \( \alpha \) is the steady-state labour share, \( A_k^Y \) is labour-augmenting productivity common to all firms in the differentiated goods sector, and \( cu_{ikt} \) and \( FN_{ikt} \) are firm-specific level of capital utilization and labour hoarding, respectively.\(^5\) Total capital, \( K_{ikt}^{tot} \), is the sum of private installed capital, \( K_{ikt} \), and public capital, \( K_{ikt}^G \). \( FC_{ikt} \) captures fixed costs in production. Total Factor Productivity, \( TFP_{kt} \), can therefore be defined as:

\[ TFP_{kt} = (A_k^Y)^\alpha. \]

Since TFP is not a stationary process, we allow for two types of shocks that are related to a non-stationary process and its autoregressive component:

\[ \log(A_k^Y) - \log(A_{kt-1}^Y) = g_{kt}^{A^Y} + \varepsilon_{kt}^{L\bar{A}Y} \]

\[ g_{kt}^{A^Y} = \rho^{A^Y} g_{kt-1}^{A^Y} + (1 - \rho^{A^Y}) g_{kt}^{A^Y0} + \varepsilon_{kt}^{gA^Y} \]

where \( g^{A^Y} \) and \( g^{A^Y0} \) are the time-varying growth and the long-run growth of technology, respectively, and \( \varepsilon_{kt}^{L\bar{A}Y} \) is a permanent technology shock.

The monopolistically competitive producers maximize the real value of the firm, \( V_{kt} \), equal to a discounted stream of future dividends \( d_{kt} \), \( V_{kt} = d_{kt} + E_t[sdf_{kt+1} V_{kt+1}] \), with the stochastic discount factor

---

\(^5\) According to Burnside and Eichenbaum (1996), firms prefer not to layoff workers when the demand is temporarily low, because firing workers may be more costly than hoarding them. Additionally, the inclusion of labour hoarding allows to match the observed co-movement between output and working hours.
\[ sd_{kt} = (1 + i_{kt+1}^p)/(1 + \pi_{kt+1}^y) \approx (1 + r_{kt}^f + r_{prem_{kt}})/ (1 + \pi_{kt+1}^y) \]

which depends directly on the investment risk premium, \( r_{prem_{kt}} \). The dividends are defined as:

\[
d_{ikt} = (1 - \tau_k^Y) \left( \frac{P_{ikt}^Y}{P_{ikt-1}^Y} Y_{ikt} - W_{ikt} \right) + \tau_k^Y \delta_k \frac{P_{ikt}^K}{P_{ikt-1}^K} K_{ikt-1} - \frac{P_{ikt}^I}{P_{ikt-1}^Y} I_{ikt} - adj_{ikt}
\]

where \( I_{ikt} \) is physical investment, \( P_{ikt}^I \) is investment price, \( \tau_k^Y \) is the corporate tax, \( \delta_k \) is capital depreciation rate. Following Rotemberg (1982), firms face quadratic adjustment costs, \( adj_{ikt} \). Adjustment costs are associated with the output price, \( P_{ikt}^Y \), and labour input, \( N_{ikt} \), adjustment or moving capacity utilization, \( cu_{ikt} \), investment, \( I_{ikt} \), and labour hoarding, \( FN_{ikt} \), away from their optimal level:

\[ adj_{ikt} = adj(P_{ikt}^Y) + adj(N_{ikt}) + adj(cu_{ikt}) + adj(I_{ikt}) + adj(FN_{ikt}) \]

where

\[
adj_{ikt}^p = \frac{\gamma^p}{2} Y_{kt} \left( \frac{P_{ikt}^Y}{P_{ikt-1}^Y} - 1 \right)^2 \\
adj_{ikt}^N = \frac{\gamma^n}{2} Y_{kt} \left( \frac{N_{ikt} - FN_{ikt}}{N_{ikt-1} - FN_{ikt-1}} - 1 \right)^2 \\
adj_{ikt}^{cu} = \frac{P_{ikt}^I}{P_{ikt-1}^Y} K_{ikt-1} \left( \gamma^{u,1}(cu_{ikt} - 1) + \frac{\gamma^{u,2}}{2}(cu_{ikt} - 1)^2 \right) \\
adj_{ikt}^I = \frac{P_{ikt}^I}{P_{ikt-1}^Y} \left( \frac{\gamma^{I,1}}{2} K_{ikt-1} \left( \frac{I_{ikt}}{K_{ikt-1}} - \delta \right)^2 + \frac{\gamma^{I,2}}{2} \left( \frac{I_{ikt}}{K_{ikt-1}} - \frac{I_{ikt-1}}{K_{ikt-1}} \right)^2 \right) \\
adj_{ikt}^{FN} = Y_t \left[ \gamma_{k}^{FN,2} \left( \frac{FN_{ikt}}{Actr_{kt} Pop_{kt}} - FN \right) + \gamma_{k}^{FN,2} \left( \frac{FN_{ikt}}{Actr_{kt} Pop_{kt}} - FN \right)^2 \right]
\]

where \( \gamma \)-s capture the degree of adjustment costs, and \( Actr_{kt} Pop_{kt} \) is the active labour force. The maximization is subject to the production function, standard capital accumulation equation, \( K_{ikt} = (1 - \delta)K_{ikt-1} + I_{ikt} \), and the usual demand condition that inversely links demand for variety \( i \) goods and the price of the variety:

\[
Y_{ikt} = \left( \frac{P_{ikt}^Y}{P_{ikt-1}^Y} \right)^{-\sigma_Y} Y_{kt}
\]

The usual equality between the marginal product of labour and labour cost holds, with a wedge driven by the labour adjustment costs:

\[
\mu_{kt}^Y \frac{Y_{kt}}{N_{kt} - FN_{kt}} - adj_{ikt}^N = (1 - \tau_k^Y) \frac{W_{ikt}}{P_{ikt}^Y}
\]
with $\mu^y_{kt}$ being inversely related to the price mark-up. The capital optimality condition reflects the usual dynamic trade-off faced by the firm:

$$
\frac{1 + \pi^y_{kt+1}}{1 + i^s_{kt+1}} \frac{p^l_{kt+1}/p^y_{kt+1}}{p^l_{kt}/p^y_{kt}} \left( \mu^y_{kt+1} (1 - \alpha) p^Y_{kt+1} Y_{kt+1} + \tau^k \delta - \frac{adj^cu_{kt}}{K_{ikt}} + (1 - \delta) Q_{kt+1} \right) = Q_{kt}
$$

where $Q_{kt}$ has the usual Tobin's interpretation.

FOC w.r.t. investment implies that Tobin's Q varies due to the existence of investment adjustment costs:

$$
Q_{kt} = 1 + adj^i_{ikt}
$$

Firms adjust their capacity utilization and labour hoarding depending on the conditions on the market via the optimality condition, respectively:

$$
\frac{\mu^y_{kt}}{p^l_{kt}/p^y_{kt}} (1 - \alpha) \frac{Y_{kt}}{cu_{kt}} = adj^cu_{ikt}
$$

$$
\frac{\mu^y_{kt} \alpha}{N_{kt} - FN_{kt}} \frac{Y_{kt}}{adj^FN_{ikt}}
$$

Finally, the FOC w.r.t. differentiated output price pins down the price mark-up:

$$
\frac{\sigma^y}{\sigma^y - 1} \frac{\mu^y_{kt}}{p^l_{kt}/p^y_{kt}} (1 - \tau^k) + \frac{adj^py_{ikt}}{(\sigma^y - 1)} + \varepsilon^\mu_{kt}
$$

with $\varepsilon^\mu_{kt}$ being the mark-up shock. The latter equation, combined with the FOC w.r.t. labour implies the Phillips curve of the familiar form:

$$
\gamma^y_{kt} \sigma^y_{kt} = (1 - \tau^k_k) (\sigma^y_k - 1) + \gamma^p_k \sigma^y_k \frac{p^Y_{kt}}{p^Y_{kt-1}} [\pi^Y_{kt} - \bar{\pi}]
$$

$$
- \gamma^p_k \sigma^y_k \frac{1 + \pi^y_{kt+1}}{1 + i^s_{kt+1}} \frac{p^Y_{kt+1} Y_{kt+1}}{p^Y_{kt} Y_{kt}} (\pi^Y_{kt+1} - \bar{\pi}) + \sigma^y_k \varepsilon^\mu_{kt}
$$

where $\varepsilon^\mu_{kt}$ is the inverse of the price mark-up shock.

### 3.3. Trade

#### 3.3.1. Exchange rates

The nominal effective exchange rate, $e_{kt}$, measures the trade weighted average price of foreign currency in terms of domestic currency and is defined as:

$$
e_{kt} = \prod_l (e_{lkt})^{\omega_{lkt}}
$$
where $e_{lkt}$ are bilateral exchange rates between domestic country $k$ and foreign country $l$. Similarly, the real effective exchange rate, $rer_{kt}$, measures the trade weighted average price of foreign output in terms of domestic output:

$$rer_{kt} = \prod_{l} (rer_{lkt})^{w_{lkt}^T}$$

where $rer_{lkt}$ are bilateral real exchange rates between $k$ and $l$, $w_{lkt}^T$ is the trade weight of the foreign trade partner $l$ in the domestic trade and defined as:

$$w_{lkt}^T = \frac{1}{2} \left( \frac{P_{lkt}^X X_{lkt}}{P_{lkt}^X X_{kt}} + \frac{P_{lkt}^M \text{size}_{lk} M_{lkt}}{P_{kt}^M \text{tot} M_{kt}^\text{tot}} \right)$$

where $X_{lkt}$ and $M_{lkt}$ stand for domestic exports to and imports from country $l$, respectively, and $P_{lkt}^X$ and $P_{lkt}^M$ are the relevant price indices. $P_{kt}^M \text{tot} M_{kt}^\text{tot}$ includes oil imports from RoW and is defined as $P_{kt}^M \text{tot} M_{kt}^\text{tot} = P_{kt}^M M_{kt} + P_{kt}^0 \text{OIL}_{kt}$, $P_{kt}^X$ and $P_{kt}^M$ are the respective price aggregates and are defined in the next section.

### 3.3.2 Import sector

**Aggregate demand components**

The EA MS final aggregate demand component goods, $C_{kt}$ (private consumption good), $I_{kt}$, (private investment good), $G_{kt}$ (government consumption good), and $I_{kt}^G$ (government investment good), as well as $X_{kt}$ (export good) are produced by perfectly competitive firms by combining domestic output, $O_{kt}^Z$, with imported goods, $M_{kt}^Z$, where $Z = \{C, I, G, I^G, X\}$, using the following CES technology:

$$Z_{kt} = A_{kt}^{p^Z} \left[ \left( 1 - \varepsilon_{kt}^M s_k^{M,Z} \right)^{\sigma_k^{-1}} \left( \frac{O_{kt}^Z}{s_k^{M,Z}} \right)^{\sigma_k^{-1}} + \left( \varepsilon_{kt}^M s_k^{M,Z} \right)^{\sigma_k^{-1}} \left( \frac{M_{kt}^Z}{s_k^{M,Z}} \right)^{\sigma_k^{-1}} \right]$$

where $A_{kt}^{p^Z}$ is a shock to productivity in the sector producing goods, $\varepsilon_{kt}^M$ is a shock to the share, $s_k^{M,Z}$, of good-specific import demand components, and $\sigma_k^Z$ is the import elasticity of substitution between goods varieties. It follows that the demand for $O_{kt}^Z$ and imported goods $M_{kt}^Z$ are given by:

$$O_{kt}^Z = \left( A_{kt}^{p^Z} \right)^{\sigma_k^{-1}} \left( 1 - \varepsilon_{kt}^M s_k^{M,Z} \right)^{\sigma_k^{-1}} \left( \frac{P_{kt}^O}{P_{kt}^Z} \right)^{-\sigma_k^Z} Z_{kt}$$

$$M_{kt}^Z = \left( A_{kt}^{p^Z} \right)^{\sigma_k^{-1}} \varepsilon_{kt}^M s_k^{M,Z} \left( \frac{P_{kt}^M}{P_{kt}^Z} \right)^{-\sigma_k^Z} Z_{kt}$$
where $P_{kt}^O$ and $P_{kt}^M$ are the price deflators associated with $O_{kt}^Z$ and $M_{kt}^Z$, respectively, and the total final good deflator $P_{kt}^Z$ is such that:

$$P_{kt}^Z = \left( A_{kt}^{pZ} \right)^{-1} \left[ (1 - \epsilon_{kt}^{M} S_k^{M,Z}) (P_{kt}^O)^{1-\sigma_k^Z} + \epsilon_{kt}^{M} S_k^{M,Z} (P_{kt}^M)^{1-\sigma_k^Z} \right]^{1-\sigma_k^Z}$$

**Economy-specific final imports demand**

Final imported goods, $M_{kt}$, are produced by perfectly competitive firms combining economy-specific final imports goods, $M_{lkt}$, using CES production function:

$$M_{kt} = \left[ \sum_{l} (s_{lkt}^{M} \frac{\sigma_{FM}^{M}}{\sigma_{FM}^{M}-1} \left( \frac{M_{lkt} \frac{size_{k}}{size_{l}}}{M_{kt} \frac{size_{k}}{size_{l}}} \right) \right]^{\frac{1}{\sigma_{FM}^{M}-1}}$$

where $\sigma_{FM}^{M}$ is the price elasticity of demand for country $l$'s goods and $\sum l s_{lkt}^{M} = 1$ are import shares. The demand for goods from country $l$ is then:

$$M_{lkt} = s_{lkt}^{M} \left( \frac{P_{lkt}^{M}}{P_{kt}^{M}} \right)^{-\alpha_{FM}^{M}} M_{kt} \frac{size_{k}}{size_{l}}$$

while the imports price:

$$P_{kt}^{M} = \left[ \sum_{l} s_{lkt}^{M} (P_{lkt}^{M})^{1-\alpha_{FM}^{M}} \right]^{\frac{1}{1-\alpha_{FM}^{M}}}$$

with $P_{lkt}^{M}$ being the country-specific import goods prices. Since all products from country $l$ are initially purchased at export price $P_{lt}^{X}$, the economy-specific import goods price can be expressed as:

$$P_{lkt}^{M} = \epsilon_{lkt} P_{lt}^{X}$$

**3.3.3. Export sector**

The exporting firms are competitive and export a good that is a combination of domestic output and import content. The corresponding export price is given by:

$$P_{kt}^{X} = \exp(\epsilon_{kt}^{X}) \left[ (1 - s_{k}^{M,Z} S_k^{M,Z}) (P_{kt}^O)^{1-\sigma_k^Z} + s_{k}^{M,Z} (P_{kt}^M)^{1-\sigma_k^Z} \right]^{1-\sigma_k^Z}$$

where $\epsilon_{kt}^{X}$ captures an export-specific price shock.
3.4. EA Member State policy

3.4.1. EA monetary policy

Monetary policy is modelled by a Taylor rule where the ECB sets the policy rate \( i_{\text{EA}t} \) in response to the annualized EA-wide inflation gap, \( \pi_{\text{EA}t}^{\text{c,vat,QA}} \), and the annualized EA output gap. The policy rate adjusts sluggishly to deviations of inflation from their respective target level and to the output gap; it is also subject to random shocks:

\[
\begin{align*}
    i_{\text{EA}t} - \bar{i} &= \rho_i(i_{\text{EA}t-1} - \bar{i}) \\
    &+ (1 - \rho_i) \left[ \eta^{\pi}_{\text{EA}} 0.25 \left( \pi_{\text{EA}t}^{\text{c,vat,QA}} - \bar{\pi}_{\text{EA}}^{\text{c,vat,QA}} \right) \\
    &+ \eta^{ly}_{\text{EA}} \left( \log \left( 0.25 \sum_{r=1}^{4} Y_{\text{EA}t-r} \right) - \log \left( 0.25 \sum_{r=1}^{4} \gamma_{\text{EA}t-r}^{\text{pot}} \right) \right) \right] + \epsilon_{\text{EA}t}
\end{align*}
\]

where \( \bar{i} = r + \bar{\pi}^{Y_{\text{obs}}} \) is the steady-state nominal interest rate, equal to the sum of the steady-state real interest rate and GDP inflation. The policy parameters \( (\rho_i, \eta^{\pi}, \eta^{ly}) \) capture interest rate inertia and the response to the annualized inflation gap and output gap, respectively.

3.4.2. Member State fiscal policy

The government collects taxes on labour, \( \tau^N_k \), capital, \( \tau^K_k \), and consumption, \( \tau^C_k \), as well as lump-sum taxes, \( t\text{ax}_k \), and issues one-period bonds, \( B^G_{kt} \), to finance government consumption, \( G_{kt} \), investment, \( I^G_{kt} \), transfers, \( T_{kt} \), and the servicing of the outstanding debt. The government budget constraint is:

\[
B^G_{kt} = (1 + i^g_{kt-1})B^G_{kt-1} - R^G_{kt} + P^G_{kt}G_{kt} + P^{I^G}_{kt}I^G_{kt} + T_{kt}
\]

where nominal government revenues, \( R^G_{kt} \), are defined as:

\[
R^G_{kt} = \tau^K_k (p^N_{kt} Y_{kt} - W_{kt} N_{kt} - p^I_{kt} K_{kt-1}) + \tau^N_k W_{kt} N_{kt} + \tau^C_k p^C_{kt} C_{kt} + \tau^y_{kt} p^{Y_0}_{kt} Oi\delta_{kt} + \text{tax}_{kt}.
\]

Excise duty on oil imports from RoW, \( \tau^Y_{kt} p^{Y_0}_{kt} \), are determined exogenously. Government consumption, investment and transfers follow autoregressive processes. Government expenditure and receipts can deviate temporarily from their long-run levels in systematic response to budgetary or business-cycle conditions and in response to idiosyncratic shocks.
Government uses lump-sum taxes as budget closure and increases (decreases) taxes when the level of government debt and government deficit is above (below) the debt, $\bar{B}_k^G$, and deficit target, $DEF_k^T$, respectively:

$$\frac{t^{ax}_{kt}}{P_{kt}^Y Y_{kt}} = \rho^{tax} \left( \frac{t^{ax}_{kt-1}}{P_{kt-1}^Y Y_{kt-1}} - \bar{t}^{ax} \right) + \eta_{kt}^{DEF} \left( \frac{\Delta B_{kt-1}^G}{P_{kt-1}^Y Y_{kt-1}} - DEF_k^T \right) + \eta_k^B \left( \frac{\Delta \bar{B}_{kt-1}^e}{P_{kt-1}^Y Y_{kt-1}} - \bar{B}_k^G \right) + \epsilon_{kt}^{tax}$$

The model uses a measure of discretionary fiscal effort (DFE) as defined by the European Commission (2013):

$$DFE_{kt} = \frac{R_{kt}^G}{Y_{kt}} - \frac{\Delta E^G_{kt} - (\Delta Y_{kt}^{pot} - 1) E^G_{kt-1}}{Y_{kt}}$$

where $E^G_{kt}$ is the adjusted nominal expenditure aggregate, $Y_{kt}^{pot}$ is the medium-term nominal potential output.\(^6\) In order to be consistent with the definition of DFE, which is defined with respect to all primary adjusted government expenditures, we define the aggregate nominal expenditure as:

$$E^G_{kt} = P_{kt}^G G_{kt} + P_{kt}^IG I_{G_{kt}} + P_{kt}^Y T_{kt}$$

We use the following DFE rules for government consumption, investment and transfers:

$$\frac{\Delta P_{kt}^G G_{kt}}{P_{kt}^Y Y_{kt}} = \left( \frac{\Delta Y_{kt}^{pot}}{P_{kt}^Y Y_{kt}} - 1 \right) \frac{P_{kt-1}^G G_{kt-1}}{P_{kt-1}^Y Y_{kt-1}} - \alpha_G \left\{ \frac{P_{kt-1}^G G_{kt-1}}{P_{kt-1}^Y Y_{kt-1}} - \bar{G} \right\} + \epsilon_{kt}^G$$

$$\frac{\Delta P_{kt}^IG I_{G_{kt}}}{P_{kt}^Y Y_{kt}} = \left( \frac{\Delta Y_{kt}^{pot}}{P_{kt}^Y Y_{kt}} - 1 \right) \frac{P_{kt-1}^IG I_{G_{kt-1}}}{P_{kt-1}^Y Y_{kt-1}} - \alpha_{IG} \left\{ \frac{P_{kt-1}^IG I_{G_{kt-1}}}{P_{kt-1}^Y Y_{kt-1}} - \bar{I}^G \right\} + \epsilon_{kt}^{IG}$$

$$\frac{\Delta P_{kt}^Y T_{kt}}{P_{kt}^Y Y_{kt}} = \left( \frac{\Delta Y_{kt}^{pot}}{P_{kt}^Y Y_{kt}} - 1 \right) \frac{P_{kt-1}^Y T_{kt-1}}{P_{kt-1}^Y Y_{kt-1}} - \alpha_T \left\{ \frac{P_{kt-1}^Y T_{kt-1}}{P_{kt-1}^Y Y_{kt-1}} - \bar{T} \right\} + \epsilon_{kt}^T$$

where $\epsilon_{kt}^G, \epsilon_{kt}^{IG}, \epsilon_{kt}^T$ are shocks to government consumption, investment and transfers, respectively. The parameters $\alpha_G, \alpha_{IG}, \alpha_T > 0$ ensure long-run stability of the model.

### 3.5. Closing the economy

Market clearing requires that:

$$Y_{kt} P_{kt}^Y + \tau_{kt}^Oil Oil_{kt} P_{kt}^{Y_O} = P_{kt}^C C_{kt} + P_{kt}^I l_{kt} + P_{kt}^IG I_{G_{kt}} + TB_{kt}$$

where the trade balance, $TB_{kt}$, is defined as the difference between exports and imports with domestic importers buying the imported good at the price $P_{kt}^X$:

\(^6\) The adjusted nominal expenditure removes interest payments and non-discretionary unemployment expenditures from total nominal expenditure. We define potential output as the output level that would prevail if labour input equaled steady-state per capita hours worked, capital stock was utilized at full capacity and TFP equaled its trend component.
\[ TB_{kt} = p^X_{kt}X_{kt} - \sum_{l}^{size_t} e_{lkt}p^X_{lt}M_{lkt} - e_{Rowkt}p^{Oil}_{Rowt}OIL_{Rowt} \]

Export is a sum of imports from the domestic economy by other countries:

\[ X_{kt} = \sum_{l} M_{lkt} \]

where \( M_{lkt} \) stands for imports of economy \( l \) from the domestic economy \( k \). Total imports are defined as:

\[ P_{kt}^{M_{tot}}M_{kt}^{tot} = p^M_{kt}M_{kt} + p^{oil}_{kt}OIL_{kt} \]

where non-oil imports are \( P^M_{kt}M_{kt} = P^M_{kt}(M^C_{kt} + M^I_{kt} + M^G_{kt} + M^{IG}_{kt}) \).

Net foreign assets, \( B^w_{kt} \), evolve according to:

\[ e_{Rowk,t}B^w_{kt} = (1 + i^w_{t-1})e_{Rowk,t}B^w_{kt-1} + TB_{kt} + ITR_kp^Y_{kt}Y_{kt} \]

Since we allow for a non-zero trade balance in the steady-state, we include an international transfer, \( ITR_k \), calibrated to satisfy zero NFA in equilibrium. Finally, net foreign assets of each country \( l \) sum to zero:

\[ \sum_{l} NFA_{lt}size_t = 0. \]

### 3.6. The REA and RoW blocks

The model of the REA and RoW blocks (subscript \( k=\text{REA, RoW} \)) is simplified in structure. Specifically, it consists of a budget constraint for the representative household, demand functions for domestic and imported goods (derived from CES consumption good aggregators), a production technology for manufacturing output that uses labour as the only factor of production, a New Keynesian Phillips curve, and a Taylor rule. The REA and RoW blocks abstract from capital accumulation. There are shocks to labour productivity, price mark-ups for manufacturing output, the subjective discount rate, the relative preference for domestic vs. imported goods, as well as monetary policy shocks in the REA and RoW.

Since RoW is an oil exporter, the budget constraint for the representative household is:

\[ p^Y_{Rowt}Y_{Rowt} + p^{oil}_{Rowt}Oil_{lt} = p^C_{Rowt}C_{Row,t} + TB_{Rowt} \]

where \( p^Y_{Rowt} \) and \( Y_{Rowt} \) are price and volume of RoW final goods output, \( p^{oil}_{Rowt} \) and \( Oil_{Rowt} \) are price and volume of oil exports to country \( l=\text{(EMU country)} \), and \( TB_{Rowt} \) is the trade balance.

The budget constraint for the representative household in REA, as an oil importer, is:
where \( \tau_{oil} p^Y_t o i l_{\text{REA}} \) captures the excise duty.

The intertemporal equation for aggregate demand follows from the FOC for consumption:

\[
\beta_t \frac{\lambda_{kt+1}}{\lambda_{kt}} \frac{1 + i_{kt}}{1 + \pi^C_{kt+1}} = 1
\]

with \( \beta_t = e^{\varepsilon^C_{kt} \beta}, (C_{kt} - h_{C_{kt-1}})^{-\theta}_k = \lambda_{kt} \) and \( \varepsilon^C_{kt} \) as the REA and RoW demand shock.

Final aggregate demand \( C_{kt} \) (in the absence of investment and government spending in the REA and RoW block) is a combination of domestic output, \( Y_{kt} \), and imported goods, \( M_{kt} \), using the following CES function:

\[
C_{kt} = A^C_{kt} \left[ \left( 1 - \varepsilon^M_{kt} \right) \left( \frac{1}{\sigma^C_{kt}} + \left( \varepsilon^M_{kt} \right)^{\sigma^C_{kt} \sigma^C_{kt}} \right) \right]^{\sigma^C_{kt} \sigma^C_{kt}-1}
\]

where \( \varepsilon^M_{kt} \) is a shock to input components and \( s_M^C \) the import share. From profit maximization we obtain the demand for domestic and foreign goods:

\[
Y^C_{kt} = \left( A^C_{kt} \right)^{\sigma^C_{kt}-1} \left( 1 - \varepsilon^M_{kt} \right) \left( \frac{P^Y_{kt}}{P^C_{kt}} \right)^{-\sigma^C_{kt}} \lambda_{kt}
\]

\[
M^C_{kt} = \left( A^C_{kt} \right)^{\sigma^C_{kt}-1} \varepsilon^M_{kt} \left( \frac{P^M_{kt}}{P^C_{kt}} \right)^{-\sigma^C_{kt}} \lambda_{kt}
\]

where the consumer price deflator, \( P^C_{kt} \), is given by:

\[
P^C_{kt} = \left( A^C_{kt} \right)^{-1} \left[ \left( 1 - \varepsilon^M_{kt} \right) \left( P^Y_{kt} \right)^{1-\sigma^C_{kt}} \right]^{\frac{1}{1-\sigma^C_{kt}}}
\]

The intermediate good producers use labour to manufacture domestic goods given a linear production function:

\[
Y_{kt} = A^Y_{kt} N_{kt}
\]

where \( A^Y_{kt} \) captures a trend in productivity.

Price setting follows a New Keynesian Phillips curve:

\[
\pi^Y_{kt} - \bar{\pi}^Y_k = \beta \frac{\lambda_{kt+1}}{\lambda_{kt}} \left[ s f p_k (\pi^Y_{kt+1} - \bar{\pi}^Y_k) + (1 - s f p_k)(\pi^Y_{kt-1} - \bar{\pi}^Y_k) \right] + \varphi^Y_k \ln(Y_{kt} - \bar{Y}_k) + \varepsilon^Y_{kt}
\]

where \( \lambda_{kt} = (C_{kt} - h_k C_{kt-1})^{-\theta_k} \) is the marginal utility of consumption, \( s f p_k \) is the share of forward-looking price-setters, and \( \varepsilon^Y_{kt} \) is a cost push shock.

Monetary policy in RoW follows a Taylor-type rule:
\[ i_{RoWt} - i = \rho_{RoW}^i (i_{RoWt-1} - i) \]

\[ + (1 - \rho_{RoW}^i) \left[ \eta_{RoW}^{\pi} 0.25 (\pi_{RoW}^{c,vat,QA} - \pi_{RoW}^{c,vat,QA}) \right] \]

\[ + \eta_{RoW}^{iy} \left( \log \left( 0.25 \sum_{r=1}^{4} Y_{RoWt-r} \right) - \log \left( 0.25 \sum_{r=1}^{4} Y_{RoWt-r}^{pot} \right) \right) + \varepsilon_{RoWt} \]

Oil is considered to be an unstorable exogenous endowment of RoW and is supplied inelastically:

\[ Oil_{RoWt} = \sum_{l} \frac{size_l}{size_{RoW}} Oil_{l, RoWt} \]

where net oil exporting firms’ revenues in RoW are driven only by its price, \( P_{RoWt}^{Oil} \), which is assumed to be determined in RoW currency:

\[ P_{RoWt}^{Oil} = \frac{P_{t}^{y0}}{A^{Poil}} \]

Total nominal exports of final goods for REA and RoW are defined as:

\[ P_{kt}^{X} X_{kt} = \sum_{l} P_{lkt}^{X} M_{lkt} \]

with the bilateral export price being defined as the domestic price subject to a bilateral price shock:

\[ p_{lkt}^{X} = \exp(\varepsilon_{lkt}^{X}) p_{kt}^{Y} \]

We combine the FOCs with respect to international bonds of REA and RoW to obtain the uncovered interest parity (UIP) condition:

\[ E_t \left[ \frac{e_{RoW,EAt+1}}{e_{RoW,EAt}} \right] (1 + i_{RoWt}) = (1 + i_{EAt}) + \varepsilon_{EAt}^{bw} + \alpha_{EA}^{bw0} + \alpha_{EA}^{bw1} \frac{e_{RoW,EAt} B_{EAt}}{p_{EAt}^{Y} Y_{EAt}} \]

where \( \varepsilon_{EAt}^{bw} \) captures a bond premium shock between EA and RoW (exchange rate shock), and \( \alpha_{EA}^{bw1} \) is a debt-dependent country risk premium on net foreign asset holdings to ensure long-run stability (see, e.g., Schmitt-Grohe and Uribe, 2003; Adolfson et al., 2008).
4. **Model solution and econometric approach**

We compute an approximate model solution by linearizing the model around its deterministic steady-state. We calibrate a subset of parameters to match long-run data properties, and we estimate the remaining parameters using Bayesian techniques.\(^7\) We combine prior information about structural parameters and data likelihood to construct the posterior kernel and maximize it. The estimated model assumes 39 exogenous shocks, as it appears that many shocks are needed to capture the key dynamic properties of macroeconomic and financial data (e.g. Kollmann et al. 2015). The large number of shocks is also dictated by the fact that we use a large number of observables (38) for estimation.\(^8\) Note that the number of shocks has to be at least as large as the number of observables to avoid stochastic singularity of the model. The observables employed in the estimation are listed in the Data Appendix A.2.

The estimation uses quarterly data for the period 1999q1-2017q2. The model has been estimated using the slice sampler algorithm proposed by Neal (2003).\(^9\) The calibrated parameters, steady-state ratios and trade shares match average historical ratios of the four EA Member States considered in the paper and can be found in the Appendix A.1. Along the deterministic steady-state all real variables (deflated by the GDP deflator) are assumed to grow at a rate of 1.3% per year (the average growth rate of EA output over the sample period). Prices grow at an EA inflation rate of 2% per year, adjusted by country-specific average productivities for the demand components (private and public consumption and investment). Population is detrended by the EA average rate of population growth (0.4% per year). The steady-state share of Ricardian households is set according to the survey evidence in Dolls et al. (2012).

5. **Estimation results**

5.1. **Posterior parameter estimates**

The posterior estimates of key model parameters for the four EA MS, and the EA aggregate for comparison, are reported in Table 2. The EA posterior estimates are obtained from a two-region (EA-RoW) model version of the GM model. The model properties discussed in what follows are evaluated at the posterior mode of the model parameters.

\(^7\) We use the DYNARE software (Adjemian et al. 2011) to solve the linearized model and to perform the estimation.

\(^8\) According to Kollmann et al. (2015), we assume an additional exogenous shock for Germany by introducing an observable proxy (benefit replacement rate) for the labour market reform (‘Hartz reform’).

\(^9\) See also Planas et al. (2015) for a detailed description on the theory and practice of slice sampling.
### Table 2: Posterior estimates of key estimated parameters.

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<th>Posterior Distribution</th>
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<th>Mode (Std.)</th>
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Note: Cols. (1) lists model parameters. Cols. (2-3) indicate the prior distribution functions (B: Beta distribution; G: Gamma distribution). Identical priors are set for country-specific parameters. Cols. (4)-(8) show the mode and the standard deviation (Std.) of the posterior distributions of the Euro Area, Germany, France, Italy and Spain, respectively.

The estimated habit persistence in consumption is particularly high in IT (0.80), indicating a sluggish adjustment of consumption to income shocks, similar to the EA aggregate. The risk aversion coefficient is in the same range for the EA aggregate and all four countries, varying from 1.38 in FR to 1.55 in ES. In DE, we observe slightly more elastic labour supply. The total import price elasticity (elasticity between imports and domestic output) is higher in FR (1.38), the EA aggregate (1.38) and DE (1.30), and lower in IT (1.17), whereas the bilateral import price elasticity (the elasticity between imports from different sources) is significantly higher in FR (2.18). The
model estimates also suggest substantial differences in price and wage rigidities. The model estimate for IT suggests much lower price rigidity (10.89) compared to FR (35.65) and the EA average (44.70). Nominal wage adjustment costs seem to be lower in IT (2.30) and ES (2.43), while real wage inertia (wage norm) is high for all countries. The most striking difference in the posterior estimates possibly concerns employment adjustment costs, where FR (107.97) and ES (9.31) strongly contrast with the middle position of DE, IT and the EA average. Estimated low employment adjustment frictions in ES are in line with the finding of a strong employment response over the business cycle, and notably in the Great recession, which Bentolila et al. (2012) relate to the importance of temporary contracts and associated low firing costs in ES. Adjustment costs on the capital stock are particularly low in IT (13.91) and appear comparable across DE, FR, ES and the EA aggregate. However, investment adjustment costs indicate substantially higher rigidity in ES (35.89) and the EA aggregate (47.86). The fiscal feedback rule on lump-sum taxes exhibits relatively high persistence for FR (0.96), ES (0.92), and the EA average (0.91), implying a more drawn-out response to debt and deficit levels in this cases.

5.2 Model fit

In order to evaluate the capability of the model to fit the data, Table 3 compares sample and model-implied moments for a subset of key statistics. We focus on volatilities of real GDP, consumption, investment, employment, the trade balance-to-GDP ratio and the GDP deflator as well as the cross-correlation of GDP with its main components. The estimated models tend to overestimate the volatility of real variables. However, the relative magnitudes seem to be preserved, e.g. \( \text{std(GC)}/\text{std(GY)} \). Of particular note is the high volatility of investment, which is in line with the data patterns. Most of the correlations between GDP growth and its components are well captured. More precisely, all country models replicate well the correlation of consumption, investment and employment with output. Moreover, our estimated models are able to replicate both positive (Germany) and negative (France and Spain) correlations between the trade balance and GDP growth. However, the GDP deflator is mostly negatively correlated with output, but matches the data pattern only for Germany and Italy fairly well. Overall, the theoretical moments give additional credit to the plausibility of the estimated structural models to replicate key features of EA Member State business cycles. Figures A.4a-A.4d in the Appendix A.4 also show the Bayesian 1-step-ahead prediction for key observables for DE, FR, IT and ES.
Table 3: Theoretical moments.

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<th>Variable</th>
<th>Model Std (%)</th>
<th>Model Corr (x,GDP)</th>
<th>Data Std (%)</th>
<th>Data Corr (x,GDP)</th>
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Note: We use first differences for the trade balance-to-GDP ratio and quarter-on-quarter growth rates for all other variables. The model-predicted moments are generated by a version of the linearized model in which the covariance matrix of all exogenous variables is set at the covariance matrix of the smoothed estimates of the innovations.

5.3 Impulse responses (structural differences across countries)

Looking at impulse response functions (IRFs) is helpful to better understand the role of structural differences for the transmission of shocks in the model. In case of structural similarity one would expect to see a similar transmission of shocks across the four countries. This sub-section presents IRFs for four exemplary drivers (consumption and investment demand, foreign demand, and labour supply) of economic dynamics. The focus on demand shocks derives from the fact that they have
been the main driver of business-cycle volatility and divergence in the EA in our models, as discussed in Kollmann et al. (2015), Kollmann et al. (2016), and in’t Veld et al. (2014). We normalise the shock size across countries to 1% and set the AR(1) parameter to 0.8 for this exercise.

Figure 3 shows the response to a positive private saving shock (savings increase), which is modelled as a persistent fall in the subjective rate of time preference of MS households. The shock triggers a persistent reduction in aggregate consumption. With sluggish price and wage adjustment, the domestic GDP and employment decline. The shock triggers a fall in the policy and a decline in the real interest rate in the medium term, leading to an increase in investment. The trade balance improves on impact due to a combination of lower import (domestic demand contraction) and stronger export demand (real exchange rate depreciation).

Figure 3: Positive shock to the saving rate (decline in consumption demand).

![Graphs showing the response to a positive saving shock]

Note: The IRFs display percentage deviations (GDP, consumption, investment, hours, real wage, and real exchange rate) and percentage-point deviations from steady-state (real interest rate, GDP inflation, and trade balance-to-GDP ratio). A positive change in the real exchange rate corresponds to real effective depreciation.
Figure 3 also shows that the negative shock to domestic consumption has particularly negative consequences on activity and employment in ES, whereas the decline in employment in other MS is dampened (but also made more persistent) by stronger employment adjustment frictions (labour hoarding). Consequently, real wages in ES decline, whereas real wages increase elsewhere due to less decline in labour demand and higher estimated wage stickiness. Additionally, lower capital stock adjustment costs in IT lead to a stronger increase in investment compared to the other MS.

Figure 4 presents the dynamic adjustment to an increase in the investment risk premium (financing costs), which leads to a decline in domestic investment demand. The decline of investment lowers aggregate demand, GDP and employment; domestic price inflation also declines. The decline in demand and prices triggers a reduction in the risk free interest rate, which strengthens the demand for consumption.

**Figure 4: Positive shock to the investment risk premium (decline in investment demand).**
The decline in investment demand and activity is particularly pronounced in IT, for which the estimation indicates comparatively low costs of adjusting the stock of capital and the amount of investment. The strong response of activity in IT leads to comparatively strong REER depreciation and TBY improvement, which follows from less import demand and increasing price competitiveness. ES shows a similar decline in employment for less investment decline due to its estimated lower labour adjustment costs, which translates into declining real wages in ES.

**Figure 5: Positive shock to the RoW saving rate (decline in foreign demand).**

Figure 5 presents dynamic responses to a foreign demand shock, namely a positive shock to savings (negative shock to private demand) in the RoW. Analogously to the MS saving shocks, the positive RoW savings shock is modelled by a rise in the subjective discount rate in the RoW and its illustrative size chosen to generate 1% of GDP RoW consumption decline on impact. The shock lowers RoW demand and activity in combination with real effective appreciation in the EA and EA MS. The reduction in policy and real interest rates in response to lower output and inflation in the EA strengthens consumption and investment demand in EA MS. The fall in the domestic
savings rate (consumption increase) and the increase in the investment rate lead to a decline in the trade balance in the EA. The more pronounced real exchange rate appreciation in the EA is driven by a stronger nominal exchange rate appreciation of the euro.

Among the EA MS, DE experiences the strongest negative response of activity, inflation, the REER and TBY due to its stronger trade openness to the RoW. At the same time, falling inflation comes with a pronounced rise in DE real wages as nominal wages adjust less rapidly than prices. Low employment and investment adjustment costs in IT lead to a less pronounced decline in domestic activity.

**Figure 6: Positive shock to labour supply.**

Figure 6 finally shows dynamic responses to an increase in domestic labour supply in the EA and the EA MS respectively. The labour supply shock reduces the real wage and increases employment and GDP. The improved profitability and the reduction in the policy rate strengthen investment demand. The REER depreciates and the TBY improves, where the competitiveness gain outweighs the impact of higher domestic demand.
ES and IT show the strongest positive employment, GDP and domestic demand effects, which are due to the particularly low estimated wage and labour adjustment costs in ES, and low costs of adjusting the stock of capital and the amount of investment in IT. The negative price responses imply a stronger initial increase in the real interest rate, which somewhat dampens investment growth. Comparatively strong downward price adjustment in ES leads to pronounced REER depreciation compared to other EA MS.

In sum, the preceding IRFs indicate broadly similar adjustment behaviour to shocks across EA MS. There are some marked differences, however. In particular, demand and supply shocks generate a comparatively strong response of employment and real wages in ES given low degrees of estimated wage and labour adjustment costs. Shocks to investment conditions show a particularly pronounced response of activity and inflation in IT due to relatively low estimated values of capital stock and investment adjustment costs in IT. Finally, DE appears to be particularly sensitive to shocks in the RoW given its higher openness to extra-EA trade.

5.4 Counterfactuals
In this subsection, we show results from counterfactual simulations to assess the importance of differences in goods and factor market adjustment for the resilience of the different EA MS to shocks. In the first counterfactual, we take the estimated (smoothed) shock processes for ES (including also the foreign shocks in the ES-REA-RoW model) and simulate the models for the other three EA MS considered with the same shocks. In the second counterfactual, we simulate the models with the estimated country-specific shocks, but imposing the estimated parameters for ES (ES structure) on the DE, FR and IT model blocks.10

Figure 7 shows the counterfactuals for real GDP growth (year-on-year) in DE, FR and IT. The plots depict the actual Spanish (black dotted line) and actual MS (blue solid line) GDP growth together with the two counterfactuals of imposing either the estimated ES shocks (red solid line) or the estimated ES structure (dashed red line). Figure 7 suggests that differences in the estimated shocks have a comparably strong impact on MS real GDP growth compared to ES real GDP growth. More precisely, the counterfactual GDP growth in DE, FR and IT would have been closer

---

10 Christiano et al. (2008) and Sahuc and Smets (2008) present similar counterfactual for a comparison of EA and US dynamics. Christiano et al. (2008) assess the importance of differences in shocks and structure for the outcomes in the EA and US after 2001, while Sahuc and Smets (2008) analyse the driving forces behind the different interest rate behaviour in the EA and US.
to the actual GDP growth in ES had the former countries experienced the same shocks. Structural differences, to the contrary, seem to explain much less of the gap between real activity in the respective MS and ES.

**Figure 7: Real GDP growth counterfactuals (y-o-y).**

Note: Since we assume in our estimations the same global trend across countries, we take into account the estimated country-specific initial conditions based on the pre-sample period 1995q1 – 1998q4.

In DE, e.g., the less pronounced fall in GDP growth in 2009 in the counterfactual that imposes ES shocks (red line) is due to the fact that trade and foreign demand shocks have made a considerably negative contribution to DE real GDP growth during the financial crisis and the global recession,
whereas the lower GDP growth in ES was mainly driven by domestic demand, particularly by an increase in investment risk premia and positive saving shocks (see Figure A.3a and A.3d in the Appendix A.3). On the other hand, the fast recovery in DE after the crisis would have been mitigated and the post-crisis slump much more pronounced compared to actual GDP growth (blue line). A similar picture for the post-crisis slump can be seen in FR, whereas real GDP growth in the last three years would have been higher with the imposed ES shocks due to strong domestic demand recovery and labour market reforms in ES (see Figures A.3b and A.3d in the Appendix).

Table 4 compares the standard deviations of real GDP growth, employment (hours worked) and inflation (GDP deflator) as measures of macroeconomic volatility in the counterfactuals (CF) with (i) ES shocks and (ii) ES structure, respectively, to the standard deviations of the respective variables in the estimated models for DE, FR, and IT (country-specific shocks and structure).

### Table 4: Counterfactual moments with estimated ES shocks and structure.

<table>
<thead>
<tr>
<th>Variables</th>
<th>DE std (%)</th>
<th>DE std (%)</th>
<th>FR std (%)</th>
<th>FR std (%)</th>
<th>IT std (%)</th>
<th>IT std (%)</th>
<th>ES std (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>0.84</td>
<td>0.48</td>
<td>0.74</td>
<td>0.56</td>
<td></td>
<td></td>
<td>0.70</td>
</tr>
<tr>
<td>CF (ES shocks)</td>
<td>0.55</td>
<td>0.45</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>CF (ES structure)</td>
<td>0.91</td>
<td>0.51</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hours worked</td>
<td>0.54</td>
<td>0.39</td>
<td>0.57</td>
<td>1.12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CF (ES shocks)</td>
<td>0.67</td>
<td>0.79</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CF (ES structure)</td>
<td>0.93</td>
<td>0.66</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP inflation</td>
<td>0.35</td>
<td>0.29</td>
<td>0.54</td>
<td>0.47</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CF (ES shocks)</td>
<td>0.45</td>
<td>0.55</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>CF (ES structure)</td>
<td>0.40</td>
<td>0.33</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Note: The Table reports the comparison of standard deviations (std in %) between the baseline with estimated country-specific shocks and the counterfactual versions (CF) imposing estimated shocks for the ES model on DE, FR and IT ("ES shocks") and imposing estimated structural parameters for ES on DE, FR and IT ("ES structure") respectively. Quarter-on-quarter growth rates for real GDP and hours worked.

Table 4 shows that imposing the shocks estimated for ES reduces the GDP growth volatility in DE and IT compared to the data. By contrast, imposing ES parameter values increases the volatility of GDP growth in DE and FR compared to the data due to the particularly low labour market frictions (low estimated employment adjustment costs) in ES, which leads to higher short-run volatility of employment and consumption demand. In the counterfactuals for employment (hours worked), structural differences with respect to ES matter for all three MS countries, which is illustrated by the fact that imposing parameter estimates for ES moves the three countries closer to the degree of
employment fluctuations observed in ES. According to the simulations, GDP price inflation would have been more volatile in DE and FR with both ES shocks and ES structure compared to actual data, while GDP inflation in IT shows the opposite result and would have been less volatile with the imposed ES structure. The latter result reflects the relatively low degree of estimated price adjustment costs in IT (Table 2).

In sum, Table 4 suggests that there is no clear pattern between the three EA MS whether differences in shocks or differences in the transmission of shocks (structural differences in goods and labour market) matter more for explaining macroeconomic volatility compared to ES. Counterfactuals for real GDP growth in DE and IT suggest that differences from ES are driven mainly by differences in shocks. Concerning the labour market, however, structural differences appear to be the main driver of the pronounced increase in employment (hours worked) volatility in DE and IT in the counterfactuals.

It is important to mention that imposing ES shocks and ES parameter estimates does not make DE, FR and IT replicate the model results for ES fully. Imposing the same values for the parameters governing the model dynamics does not lead to strictly identical country blocks. A number of (policy-invariant) differences across countries remain that affect the response to shocks. These differences notably include country size, which implies a different weight in EA aggregates and EA monetary policy according to the EA Taylor rule, and trade openness, which implies differences in the transmission of foreign shocks and the strength of the trade channel.

**Figure 8: Policy-invariant differences across countries.**

Figure 8 shows the gap between our counterfactuals for DE and the data for ES. In particular, the counterfactual real GDP growth for DE imposing both estimated ES shocks and ES structural
parameter estimates does not fully match real GDP growth in ES. The combination of ES shocks and structure closely tracks ES data, however, in particular when compared to data for DE.

6. Conclusions
The paper has reviewed adjustment dynamics in EMU through the lenses of estimated multi-country DSGE models for four large (individual) Euro Area (EA) Member States (MS), the rest of the EA (REA), and the rest of the world (RoW). In particular, we have analysed to what extent goods and factor market frictions (price, nominal wage, and real rigidities) differ across countries (DE, FR, IT, and ES) and, hence, generate different or similar dynamic responses to identical shocks. Our results suggest broadly similar adjustment behaviour to shocks across EA MS. There are some marked differences, however. In particular, demand and supply shocks generate a comparatively strong response of employment and real wages in ES given low estimated values of wage and labour adjustment costs. Shocks to investment conditions show a particularly pronounced response of activity and inflation in IT due to relatively low estimated values of capital stock and investment adjustment costs in IT. Finally, DE appears to be particularly sensitive to shocks in the RoW given its higher openness to extra-EA trade.

In counterfactual simulations we have imposed, first, estimated shocks and, second, estimated parameters for the Spanish (ES) economy on the models for Germany (DE), France (FR), and Italy (IT) to assess whether ES shocks (including financial ones) or ES structural characteristics would have led to similar fluctuation in economic activity in these countries. The results from the counterfactuals do not provide a uniform picture: Imposing ES shocks reduces the volatility of real GDP growth in DE, FR and IT to below the observed value for ES. Imposing ES parameters raises GDP growth volatility in DE and FR, which reflects stronger employment fluctuations and the associated stronger fluctuation in domestic demand in ES. Concerning employment, ES shocks and parameter values lead to higher volatility of hours worked in DE, FR and IT, which a stronger contribution of structural differences. GDP inflation volatility in DE and FR increases with ES shocks and structure, whereas the volatility in IT drops relative to the data in line with relatively low estimated price stickiness in IT. The results are in line with the literature that stresses both structural differences in the transmission of shocks and idiosyncratic shocks as driver of business-cycle heterogeneity.
The analysis in the paper is subject to limitations calling for further work. The limitations include the simple structure of the financial sector in the model, which omits amplifying mechanisms such as credit expansion and deleveraging dynamics. Financial frictions enter our model (mainly) through shocks rather than as part of the shock transmission. Future work should complement the finding of differences in the parameter estimates with a more thorough discussion of structural differences in goods and labour markets across the four countries. The (omitted) particular nature of shocks may affect the parameter estimates. An example is the sectoral composition of expansions and recessions that may shape the employment multiplier. Finally, the current analysis does not account for asymmetries in the model dynamics, which could derive from a temporarily binding zero lower bound on nominal interest rates, or asymmetric (downward) price or wage rigidity.
References
Abbritti, M., A. Mueller (2013): Asymmetric labor market institutions in the EMU and the volatility of inflation and unemployment differentials, Journal of Money, Credit and Banking 45(6), 1165-1186.
### Appendix A:
#### A.1 Calibrated parameters and steady-state ratios of EA Member States

<table>
<thead>
<tr>
<th>Description</th>
<th>DE</th>
<th>FR</th>
<th>IT</th>
<th>ES</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>EA Monetary Policy</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nominal interest rate in SS</td>
<td>0.005</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CPI inflation in SS</td>
<td>0.004</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest rate persistence</td>
<td>0.838</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Response to inflation</td>
<td></td>
<td>1.548</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Response to GDP</td>
<td>0.086</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td><strong>Preferences</strong></td>
<td></td>
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<td></td>
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<tr>
<td>Consumption share of Ricardian households</td>
<td>0.61</td>
<td>0.66</td>
<td>0.67</td>
<td>0.69</td>
</tr>
<tr>
<td>Intertemporal discount factor</td>
<td>0.9975</td>
<td>0.9975</td>
<td>0.9975</td>
<td>0.9975</td>
</tr>
<tr>
<td>Degree of openness</td>
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<td>0.28</td>
<td>0.26</td>
<td>0.29</td>
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<tr>
<td>Preference for imports from RoW</td>
<td>0.53</td>
<td>0.42</td>
<td>0.50</td>
<td>0.46</td>
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<tr>
<td>Preference for imports from REA</td>
<td>0.47</td>
<td>0.58</td>
<td>0.50</td>
<td>0.54</td>
</tr>
<tr>
<td>Preference for gov’t bonds</td>
<td>-0.0003</td>
<td>-0.0011</td>
<td>0.0009</td>
<td>0.0006</td>
</tr>
<tr>
<td>Preference for stocks</td>
<td>-0.0002</td>
<td>-0.0009</td>
<td>-0.0007</td>
<td>-0.0026</td>
</tr>
<tr>
<td>Preference for foreign bonds</td>
<td>-0.0048</td>
<td>-0.0048</td>
<td>-0.0048</td>
<td>-0.0048</td>
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<tr>
<td>Weight of disutility of labour</td>
<td>2.98</td>
<td>4.62</td>
<td>7.79</td>
<td>7.62</td>
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<tr>
<td><strong>Production</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cobb-Douglas labour share</td>
<td>0.65</td>
<td>0.65</td>
<td>0.65</td>
<td>0.65</td>
</tr>
<tr>
<td>Depreciation of private capital stock</td>
<td>0.0142</td>
<td>0.0150</td>
<td>0.0136</td>
<td>0.0123</td>
</tr>
<tr>
<td>Depreciation of public capital stock</td>
<td>0.0142</td>
<td>0.0150</td>
<td>0.0136</td>
<td>0.0123</td>
</tr>
<tr>
<td>Share of oil in total output</td>
<td>0.015</td>
<td>0.015</td>
<td>0.015</td>
<td>0.015</td>
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<tr>
<td>Linear capacity utilization adj. costs</td>
<td>0.0147</td>
<td>0.0166</td>
<td>0.0143</td>
<td>0.0140</td>
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<tr>
<td><strong>Fiscal policy</strong></td>
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<tr>
<td>Consumption tax</td>
<td>0.20</td>
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<td>0.20</td>
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<tr>
<td>Corporate profit tax</td>
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<td>0.30</td>
<td>0.30</td>
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<td>Labour tax</td>
<td>0.40</td>
<td>0.54</td>
<td>0.45</td>
<td>0.35</td>
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<td>Deficit target</td>
<td>0.019</td>
<td>0.021</td>
<td>0.032</td>
<td>0.017</td>
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<tr>
<td>Debt target</td>
<td>2.59</td>
<td>2.86</td>
<td>4.48</td>
<td>2.31</td>
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<td><strong>Steady-state ratios</strong></td>
<td></td>
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<td></td>
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<tr>
<td>Private consumption share in SS</td>
<td>0.57</td>
<td>0.55</td>
<td>0.60</td>
<td>0.60</td>
</tr>
<tr>
<td>Private investment share in SS</td>
<td>0.17</td>
<td>0.19</td>
<td>0.17</td>
<td>0.20</td>
</tr>
<tr>
<td>Gov’t consumption share in SS</td>
<td>0.19</td>
<td>0.23</td>
<td>0.19</td>
<td>0.18</td>
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<tr>
<td>Gov’t investment share in SS</td>
<td>0.02</td>
<td>0.04</td>
<td>0.03</td>
<td>0.04</td>
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<tr>
<td>Transfers share in SS</td>
<td>0.17</td>
<td>0.18</td>
<td>0.18</td>
<td>0.13</td>
</tr>
<tr>
<td><strong>Others</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Size of the country (% of world)</td>
<td>4.50</td>
<td>3.45</td>
<td>2.81</td>
<td>1.80</td>
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<tr>
<td>Trend of total factor productivity</td>
<td>0.0029</td>
<td>0.0018</td>
<td>0.0027</td>
<td>0.0013</td>
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<tr>
<td>Trend of private consumption specific productivity</td>
<td>0.0009</td>
<td>0.0008</td>
<td>-0.0006</td>
<td>-0.0014</td>
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<tr>
<td>Trend of gov’t consumption specific productivity</td>
<td>0.0008</td>
<td>0.0002</td>
<td>-0.0007</td>
<td>-0.0004</td>
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<tr>
<td>Trend of private investment specific productivity</td>
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<td>0.0010</td>
<td>-0.0006</td>
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<td>Trend of gov’t investment specific productivity</td>
<td>-0.0011</td>
<td>0.0010</td>
<td>-0.0006</td>
<td>0.0020</td>
</tr>
</tbody>
</table>
A.2 Data
A.2.1 Data sources
Data for the EA Member States (quarterly national accounts, fiscal aggregates, quarterly interest and exchange rates) are taken from Eurostat. Corresponding data for the US (which is part of RoW) come from the Bureau of Economic Analysis (BEA) and the Federal Reserve. Bilateral trade flows are based on trade shares from the GTAP trade matrices for trade in goods and services. ROW series are constructed on the basis of the IMF International Financial Statistics (IFS) and World Economic Outlook (WEO) databases.

A.2.2 Constructing of data series for ROW variables
Series for GDP and prices in the ROW starting in 1999 are constructed on the basis of data for the following 58 countries: Albania, Algeria, Argentina, Armenia, Australia, Azerbaijan, Belarus, Brazil, Bulgaria, Canada, Chile, China, Colombia, Croatia, Czech Republic, Denmark, Egypt, Georgia, Hong Kong, Hungary, Iceland, India, Indonesia, Iran, Israel, Japan, Jordan, Korea, Lebanon, Libya, FYR Macedonia, Malaysia, Mexico, Moldova, Montenegro, Morocco, New Zealand, Nigeria, Norway, Philippines, Poland, Romania, Russia, Saudi Arabia, Serbia, Singapore, South Africa, Sweden, Switzerland, Syria, Taiwan, Thailand, Tunisia, Turkey, Ukraine, United Arab Emirates, United Kingdom, and Venezuela. The ROW data are annual data from the IMF International Financial Statistics (IFS) and World Economic Outlook (WEO) databases.

A.2.3 List of observables
The estimation uses the following time series for the EA Member States: Total factor productivity, real GDP, GDP deflator, population, total employment, employment rate, relative prices with respect to GDP deflator (VAT-consumption, government consumption, private investment, government investment, export and import), nominal EA policy rate, and nominal shares of GDP (consumption, government consumption, investment, government investment, government interest payment, transfers, public debt, wage bill and exports). The list of observables also includes the oil price and the nominal effective EUR exchange rate. For REA, we use data on total factor productivity, population, GDP, GDP deflator, trade balance-to-GDP ratio, export share to GDP as well as export and import prices relative to GDP deflator. Note that we observe the series for EA19 and calculate within the model the consistent REA series. For RoW, we use data on total factor productivity, population, GDP, GDP deflator and the nominal policy rate.
A.2.4 Construction of Rest-of-World (ROW) aggregates

The series for ROW real GDP (GDPR) is constructed as follows. First, we normalise the series for GDP in national currency (NAC) at constant prices for each country (i) at the common base year t=0:

$$\frac{GDPR_i^t}{GDPR_i^t} = \prod_{k=1}^{i} \left( \frac{GDPR_k^t}{GDPR_k^t} \right)$$

Then we calculate the time-varying share of each country in the block based on nominal GDP (GDPN) in USD. Finally, we compute ROW GDPR as the GDPN-weighted average of the 58 countries, which gives the ROW GDPR index with base year t=0:

$$GDPR_{ROW}^t = \sum_{i=1}^{58} \frac{GDPN_{USD,i}^t}{GDPN_{USD,ROW}^t} GDPR_i^t$$

The aggregation applies time-varying weights in order to account for changes in the relative economic weight of individual ROW countries over the sample period. ROW GDPR is normalised to 1 in year 2005.

The series for the ROW GDP deflator (PGDP) is constructed analogously to the ROW GDPR series. First, we normalise the series for the PGDP for each country (i) to base year t=0:

$$\frac{PGDP_i^t}{PGDP_i^t} = \prod_{k=1}^{i} \left( \frac{PGDP_k^t}{PGDP_k^t} \right)$$

Then we calculate the time-varying share of each country in the block based on GDP in USD and compute the ROW PGDP as the GDP-weighted average of the 58 country series, which gives the ROW GDPR index with base year t=0:

$$PGDP_{ROW}^t = \sum_{i=1}^{58} \frac{GDPN_{USD,i}^t}{GDPN_{USD,ROW}^t} PGDP_i^t$$

ROW GDPR is normalised to 1 in year 2005. An index of ROW nominal GDP (GDPN) with base year 2005 can be calculated by multiplying ROW GDPR with ROW PGDP.

The ROW block in the model has a flexible nominal exchange rate. The ROW nominal exchange rate to the USD (e) is calculated as GDP-weighted average of bilateral exchange rates against the USD for the 58 countries. As for GDPR and PGDP above, we normalise bilateral USD exchange rates in each country to the base year t=0:
The ROW nominal exchange rate to the USD with base year \( t=0 \) is then calculated as GDP-weighted average of the 58 country series:

\[
\frac{e_i^{i,s}}{e_0^{i,s}} = \prod_{k=1}^{t'} \left( \frac{e_k^{i,s}}{e_{k-1}^{i,s}} \right)
\]

The ROW exchange rate to the USD is normalised to 1 in 2005. The exchange rate series includes exchange rate movements between members of the ROW group instead of attributing them to the ROW price index.

The short-term interest rate for the ROW is the GDP-weighted average of interest rate series for countries (i) in the ROW. The sample is reduced to 47 countries due to limited data availability and the GDP weights are adjusted accordingly.

The ROW trade balance (TB) balances international trade flows:

\[
TB_i^{ROW} = -(TB_i^{EA} + TB_i^{US})
\]

ROW exports equal the sum of EA and US imports from the ROW. The bilateral imports from the ROW are obtained by subtracting imports from the US (EA) from total EA (US) imports based on trade matrices for international good and service trade. Analogously, imports of the ROW equal EA plus US exports to the ROW.
A.3 Historical shock decompositions for DE, FR, IT and ES

To quantify the role of different shocks as drivers of endogenous variables in the period 2000-2017 we plot the estimated contribution of these shocks to historical time series. Figures A.3a-A.3d show historical shock decompositions of year-on-year growth rates of real GDP for the four estimated EA MS, namely DE, FR, IT, and ES. In each sub-plot, the continuous black line shows the historical time series, from which sample averages have been subtracted. The vertical black bars show the contribution of different groups of exogenous shocks (see below) to the historical data, while stacked light bars show the contribution of the remaining shocks. Bars above the horizontal axis represent positive shock contributions, while bars below the horizontal axis show negative contributions.

Given the large number of shocks, we plot the contributions of the following groups of exogenous variables: (1) ‘TFP’ represents the contribution of permanent shocks to productivity; (2) ‘Goods and Labour market’ captures supply side shocks (price and wage mark-up shocks, in DE also the shock to the unemployment benefit ratio to account for the Hartz reform); (3) ‘Aggregate Demand’ includes demand-side shocks (household savings shock, fiscal shocks, investment risk premium shocks); (4) ‘Monetary EA’ represents the contribution of monetary policy shocks; (5) ‘Bond premium EA vs RoW’ represents shocks to the EA uncovered interest parity condition (exchange rate shocks); (6) ‘Trade shocks’ capture the worldwide relative preference for domestic-produced goods and foreign goods as well as price mark-up shocks for exports and imports; (7) ‘Shocks REA’ capture the remaining shocks originating in the rest of the Euro Area (demand and supply shocks); (8) ‘Shocks RoW’ represent the shocks originating in RoW (demand and supply shocks); (9) ‘Oil’ captures shocks to the Oil price.
Figure A.3a: Historical decomposition of real GDP growth in Germany.

Figure A.3b: Historical decomposition of real GDP growth in France.
Figure A.3c: Historical decomposition of real GDP growth in Italy.

Figure A.3d: Historical decomposition of real GDP growth in Spain.
A.4 Model fit of EA Member States
In order to assess the fit of the model we can compare the estimates of endogenous variables with its observable counterparts. Figures A.4 show the Bayesian 1-step-ahead prediction for key observable variables for DE, FR, IT and ES. The red line indicates the observed series from 1999q1-2017q2 (quarter-on-quarter growth rates). The black line depicts the model-consistent estimate of the 1-step-ahead forecast of the endogenous variable calculated by the Kalman filter. The grey confidence bounds represent posterior parameter uncertainty. A specific credit on the plausibility of our estimated model to replicate key features of EA business cycles is to focus on capacity utilization. While capacity utilization is not directly measurable in national account statistics, we use a ‘model-free’ or reduced form proxy that has been constructed to compare the model-based and model-free estimate of capacity utilization. The lower right panel in Figures A.4 plot the times series of capacity utilization implied by the constructed proxy (black dotted line) and the model-implied one computed via Kalman filter. Even without directly observing capacity utilization, the two measures coincide and give additional credit on the plausibility of the estimated model to replicate key features of EA business cycles.
Figure A.4a: Bayesian 1-step-ahead prediction of Germany.
Figure A.4b: Bayesian 1-step-ahead prediction of France.
Figure A.4c: Bayesian 1-step-ahead prediction of Italy.
Figure A.4d: Bayesian 1-step-ahead prediction of Spain.
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