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A Minimum Distance Estimator for Dynamic Conditional Correlations

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Abstract

The crucial problem in estimating dynamic conditional correlation models is the need to guarantee a positive-definite covariance matrix. In order to avoid any violation of this property, many estimators impose strong restrictions on the model. In addition, many models do not parameterize the correlations directly but the covariance. This paper avoids this problem in proposing a minimum distance estimator (MDE) to estimate dynamic conditional correlations or multivariate GARCH models. The model allows full flexibility in the estimation. Violations of the positive-definiteness of the covariance matrices are part of the specification tests. A simulation study shows the performance of the estimator and the empirical section compares estimates of the MDE with the DCC estimator of Engle (2002). This paper does not only demonstrate an alternative estimator but also outlines how this model can be used to analyze the influence of the restrictions on the estimates in multivariate GARCH models.

JEL classification: **C32, C52**

KEYWORDS: Multivariate GARCH, BEKK, Covariance Models, Time-varying Correlations

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Contents

1 Introduction

One of the most recent papers on Multivariate GARCH models and time-varying correlation, that is, dynamic correlation estimation is a survey by Bauwens, Laurent and Rombouts (2005). This paper shows the developments in multivariate GARCH models initiated by Robert Engle in 1982 with a univariate ARCH model (Engle, 1982). Bollerslev (1986) generalized the ARCH framework to GARCH which was followed by the first multivariate GARCH model in 1988 (Bollerslev, Engle and Wooldridge, 1988). This first multivariate model was improved by Baba, Engle, Kraft and Kroner (1991), the BEKK model, and Engle and Kroner (1995). A model introducing asymmetries in covariances was suggested by Kroner and Ng (1998).

Research has always focussed around the problem to guarantee positive-definite covariance matrices for a larger number of assets and flexible structures. One of the most flexible models is the DCC estimator of Engle (2002)[1] which has a forerunner introduced by Tse and Tsui (2002). The DCC estimator can easily handle a large number of time-series, e.g. asset returns. It is also flexible with regard to the specification of individual univariate GARCH processes and the dynamic covariance equation. This is due to the two-step estimation which also renders the optimization procedure rather fast.

Interestingly, all parametric models use Maximum-Likelihood methods for the estimation process. Consequently, restrictions have to be applied to guarantee positive-definite covariance matrices since this is an essential part in the likelihood function. Furthermore, almost all parametric models parameterize the covariance matrix but not the correlation matrix.[2]

This paper contributes to the literature in two respects: first, it offers an alternative estimation technique based on a minimum distance estimator (MDE) and second, the model parameterizes the conditional dynamic correlations directly and without any restrictions.

[1]The theoretical properties are elaborated in Engle and Sheppard (2001).

[2]We are not aware of any model but the constant correlation model that employs a direct parameterization.

Violations to the positive-definiteness are tested a posteriori. This study could also be seen as a pure specification test of the restrictions employed in multivariate GARCH models such as the DCC model of Engle (2002).

The paper is structured as follows: first, we present the Minimum Distance Estimator. Second, we perform a simulation study and third, we show in an empirical analysis the flexibility of the model and the differences to the DCC model of Engle (2002). Section 5 concludes.

2 Minimum Distance Estimator

A simple multivariate GARCH(1,1) model can be written as follows

$$\boldsymbol{H}_t = \boldsymbol{\Omega} + \boldsymbol{A}\boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}_{t-1}' + \boldsymbol{B}\boldsymbol{H}_{t-1} \tag{1}$$

where all elements are $N \times N$ matrices. More specifically, $\boldsymbol{H}_t$ and $\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t'$ is the covariance matrix and the matrix of residuals, respectively. The remaining matrices are parameter matrices to be estimated. These parameter matrices have to fulfill two main requirements: first, they should flexibly model the dynamic process and second, they should guarantee a positive-definite covariance matrix. These two conditions usually constitute a trade-off. Full flexibility risks indefinite covariance matrices and restricted flexibility risks an inadequate modelling of the conditional covariance. This trade-off is partially eliminated by the model explained below.

The model that is estimated here is based on the following basic specification presented for two times series, i.e. $N = 2$:

$$\boldsymbol{H}_t = \boldsymbol{\Omega} + \begin{pmatrix} b_{11} & b_{21} \\ b_{21} & b_{22} \end{pmatrix} \otimes \begin{pmatrix} \epsilon_{1,t-1}^2 & \epsilon_{t-1}\epsilon_{2,t-1} \\ \epsilon_{1,t-1}\epsilon_{2,t-1} & \epsilon_{2,t-1}^2 \end{pmatrix} + \begin{pmatrix} c_{11} & c_{21} \\ c_{21} & c_{22} \end{pmatrix} \otimes \boldsymbol{H}_{t-1} \tag{2}$$

where $\boldsymbol{H}_t = \begin{pmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{pmatrix}$. $\boldsymbol{\Omega}$ is also a $N \times N$ parameter matrix whose elements are given by a_{ij} for $i, j = 1, 2$. $\otimes$ denotes the Kronecker product. The only restriction that will be posed on this model is that the sum of the parameters in B and C governing one element of $\boldsymbol{H}_t$ (conditional volatility or covariance) is less than one in order to guarantee stationarity. The above model represents a dynamic covariance specification. The dynamic correlation process is not modelled directly in such a specification. A dynamic correlation model is written as follows

$$\boldsymbol{R}_t = \boldsymbol{\Omega} + \begin{pmatrix} 1 & b_{21} \\ b_{21} & 1 \end{pmatrix} \otimes \begin{pmatrix} 0 & z_{1,t-1}z_{2,t-1} \\ z_{1,t-1}z_{2,t-1} & 0 \end{pmatrix} + \begin{pmatrix} 1 & c_{21} \\ c_{21} & 1 \end{pmatrix} \otimes \boldsymbol{R}_{t-1} \quad (3)$$

where $\boldsymbol{R}_t = \begin{pmatrix} 1 & \rho_{12,t} \\ \rho_{21,t} & 1 \end{pmatrix}$ is the conditional (time-varying) correlation matrix, ρ its elements and $\boldsymbol{\Omega}$ is the unconditional correlation matrix. In this model, the restriction of the parameters also extends to the $\boldsymbol{\Omega}$ matrix. Thus, the sum of the elements governing one process of all three parameter matrices must be smaller than one.

The parameter matrices have zero elements on the main diagonal since the elements on the main diagonal are not estimated. All main-diagonal elements are one. Their values are one. Note also that the symmetry of the covariance and correlation matrices is represented by the symmetric parameter matrices.

We aim to estimate the time-varying $N \times N$ correlation matrix $\boldsymbol{R}_t$ for N time-series with a minimum distance estimator (MDE) as follows. Assume $\boldsymbol{H}_t$ is the corresponding covariance matrix which depends on a parameter vector $\boldsymbol{\theta}$. The essence of the estimator is based on the idea that the covariance matrix is given by

$$E_{t-1}(\boldsymbol{\epsilon}_t \boldsymbol{\epsilon}_t') = \boldsymbol{H}_t \quad (4)$$

and that the correlation matrix can be written as

$$E_{t-1}(\boldsymbol{z}_t\boldsymbol{z}_t') = \boldsymbol{R}_t \tag{5}$$

where ϵ_t and $\boldsymbol{z}_t$ are $N \times 1$ vectors consisting of the raw time series and the standardized residuals at time t, respectively.

The above relations lead to the minimization of the difference $\boldsymbol{D}_t$

$$\boldsymbol{D}_t(\boldsymbol{\theta}) = \boldsymbol{H}_t(\boldsymbol{\theta}) - \boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t' \tag{6}$$

for a covariance estimator or

$$\boldsymbol{D}_t(\boldsymbol{\theta}) = \boldsymbol{R}_t(\boldsymbol{\theta}) - \boldsymbol{z}_t\boldsymbol{z}_t' \tag{7}$$

for a correlation estimator.

$\boldsymbol{H}_t$, $\boldsymbol{R}_t$ and $\boldsymbol{D}_t$ are all $N \times N$ matrices.

The moment conditions are obtained by transforming this $N \times N$ matrix into a vector of size $N^2 \times 1$. Since negative and positive differences between the matrices should not compensate each other, we use the sum of the absolute values of the elements of this vector as follows

$$m_t(\theta) = |vec(\boldsymbol{D}_t)|'\iota \tag{8}$$

where vec transforms the matrix into a vector and ι is a corresponding vector of ones.

The moment conditions are

$$\bar{\boldsymbol{m}}(\theta) = \frac{1}{N^2}\boldsymbol{m}(\theta) \tag{9}$$

where $\bar{\boldsymbol{m}}$ is a $T \times 1$ vector.

Finally, we minimize the sum of squares of $\bar{\boldsymbol{m}}(\theta)$ as shown below

$$q = (1/T)\bar{\boldsymbol{m}}(\theta)'\bar{\boldsymbol{m}}(\theta) \tag{10}$$

Under normal conditions this estimator is consistent but inefficient. Different weighting matrices $\boldsymbol{W}$ in

$$q = \bar{\boldsymbol{m}}(\theta)'\boldsymbol{W}\bar{\boldsymbol{m}}(\theta) \tag{11}$$

to yield efficient estimates can be introduced. The estimator is implemented in MATLAB and the code can be obtained from the author.

3 Simulation Study

In this section we analyze the mean absolute error and the correlation of different simulated correlation processes with the estimated correlation processes obtained with different specifications of the Minimum Distance Estimator. The simulated correlation processes and conditional volatilities are similar to the simulation study in Engle (2002).

We simulate constant correlations, fast and slow correlation variations via sinus functions, a trend and a regime switching constant correlation model. The conditional volatilities are assumed to follow GARCH(1,1) processes.

Table 1 contains the simulation results of 200 iterations for each model and each process. For every model, that is, a scalar covariance MDE, a full covariance MDE and a full correlation MDE (equal to a scalar correlation MDE in the bivariate case), the mean absolute error (MAE) and the correlation coefficient of the simulated process with the estimated correlation process is tabulated. In table 2 the fraction of remaining autocorrelation in the squared standardized residuals is reported. Both tables contain two panels for $T = 1000$ and $T = 2000$.

Finally, table 3 contains a special characteristic of the mean distance estimator. Since it is estimated without the imposition of any restrictions, violations against the positive-

Table 1: **Simulation Results - Mean Absolute Error and Correlation**

Panel A: $T = 1000$						
corr. process	scalar covariance MDE		full covariance MDE		full correlation MDE	
	MAE	corr	MAE	corr	MAE	corr
constant	0.0682	.	0.0566	.	0.0303	.
fast sine	0.2019	0.7706	0.1840	0.7896	0.1867	0.7827
slow sine	0.2818	0.7935	0.2620	0.7850	0.2716	0.7620
trend	0.1321	0.5767	0.1118	0.6856	0.1065	0.7155
ramp	0.1595	0.7044	0.1244	0.7947	0.1137	0.8341
Panel B: $T = 2000$						
corr. process	scalar covariance MDE		full covariance MDE		full correlation MDE	
	MAE	corr	MAE	corr	MAE	corr
constant	0.0653	.	0.0493	.	0.0248	.
fast sine	0.1926	0.7894	0.1515	0.8582	0.1529	0.8567
slow sine	0.2430	0.8621	0.2120	0.8611	0.2242	0.8413
trend	0.1289	0.5989	0.0918	0.7895	0.0808	0.8486
ramp	0.1504	0.7413	0.1118	0.8335	0.0877	0.8973

definiteness of the resulting covariance matrices for some t in the interval $[1, T]$ can occur. Furthermore, correlations larger than one in absolute terms can also result for some t. The table contains the sum of these violations for every t in every iteration. For example, a value of 100 for positive-definiteness means that 100 violations occurred for covariance matrices at some time t of $T = 1000$ or $T = 2000$ in 200 iterations. In other words, a value of 100 means for $T = 1000$ that a fraction of $100/(1000 \cdot 200) = 0.0005$ experienced positive indefinite covariance matrices.

Table 1 shows that the mean absolute error is smallest for constant correlations and generally below 0.2. For a larger number of observations these values decrease clearly. The correlations of the simulated series with the estimated series are around 0.8 for most of the estimated models and simulated processes which is consistent with the results of the mean absolute error. Furthermore, it is evident that a larger number of observations (doubling the number from 1000 to 2000) considerably increases the degrees of correlation.

Table 2 shows a similar picture for the fraction of remaining autocorrelations.

Table 2: **Simulation Results - Fraction of remaining autocorrelation in squared standardized residuals**

Panel A: $T = 1000$			
corr. process	scalar covariance MDE	full covariance MDE	full correlation MDE
constant	0.15	0.11	0.10
fast sine	0.11	0.09	0.24
slow sine	0.10	0.28	0.28
trend	0.12	0.05	0.09
ramp	0.11	0.26	0.27
Panel B: $T = 2000$			
corr. process	scalar covariance MDE	full covariance MDE	full correlation MDE
constant	0.10	0.09	0.11
fast sine	0.10	0.10	0.17
slow sine	0.12	0.41	0.38
trend	0.12	0.14	0.09
ramp	0.15	0.43	0.29

Table 3: **Simulation Results - Absolute number of violations of positive-definiteness (PD) and absolute value of correlation (CORR) smaller than one**

Panel A: $T = 1000$						
corr. process	scalar covariance MDE		full covariance MDE		full correlation MDE	
	PD	CORR	PD	CORR	PD	CORR
constant	0	0	0	15	0	8
fast sine	0	0	0	6	476	149
slow sine	0	0	0	243	354	2097
trend	0	0	0	8	73	3
ramp	0	0	0	458	31	190
Panel B: $T = 2000$						
corr. process	scalar covariance MDE		full covariance MDE		full correlation MDE	
	PD	corr	PD	corr	PD	corr
constant	0	0	0	0	0	0
fast sine	0	0	0	8	755	80
slow sine	0	0	0	241	531	3801
trend	0	0	0	71	165	7
ramp	0	0	0	1037	28	234

The fraction of violations per run is given by the above number divided by $(T \cdot 200)$ (the number of iterations).

Finally, table 3 shows the intuitive result that more flexibility increases the risk of violations. The scalar covariance model does not exhibit one single violation. The full covariance model only exhibits violations of the correlation boundaries, that is the estimated correlation coefficient is not always between -1 and $+1$. The last simulated process (ramp) performs worst and has 458 violations for $T = 1000$ observations and 1037 violations for $T = 2000$ observations. The fraction of observations is small given that it is the sum of 200 simulation runs. In the first case and in the second case, the fraction is smaller than 0.5%. The violations never exceed 1%. The highest number of violations is for the slow sine case for $T = 2000$ where 3801 violations are accounted. This is equal to a fraction of 0.95%.

We have additionally simulated a correlation process following $R_t = 0.01 + 0.05 z_{t-1} z'_{t-1} + 0.80 R_{t-1}$. The results of these simulations are striking. The correlations are well above 0.95 and close to 1 for processes with $T = 2000$. The fraction of remaining autocorrelation in squared residuals and violations are both negligible.

Since the model imposes no restrictions, violations can occur. In a second step, these violations could be further examined and reduced or eliminated by modifying or augmenting the basic model. For example, a term accounting for asymmetries of positive and negative shocks could be introduced or dummy variables capturing extreme movements. When conditional correlations are estimated directly violations could also be caused by an inadequate conditional volatility estimation in the first stage. For example, if conditional volatilities do not include an asymmetric term, this could also affect conditional correlation estimates and potentially cause violations of the positive-definiteness of the correlation matrices or lead to levels of correlations outside the boundaries.

It is important to mention that a scalar and a full version of the conditional correlations are equal in the bivariate case. The models are only different if the number of series N is larger than two.

Finally, in order to assess the specification of the model, that is, the adequateness of the moment conditions, the distribution of the function value q in equation 10 could be computed and a test statistic derived. The distribution of the correlation estimator was exemplary computed for the scalar version of the covariance MDE. The median value was 1 with values of 1.25 and 1.30 for the 95% and 99% quantile, respectively. Obviously, these results are very preliminary and have to be extended.

4 Empirical Analysis

This section aims to demonstrate the flexibility of the Mean Distance Estimator. We use nine (arbitrary) stocks of the German stock index DAX to show an application of the estimator to an empirical data set. The stocks are Allianz, Bayerische Hypo Vereinsbank, Bayer, BMW, Deutsche Bank, Munich Re, SAP, Siemens and Volkswagen. The data span a period of 10 years, commencing June 1, 1995 until May 31, 2005. The number of observations for each stock is $T = 2622$.

In order to demonstrate the flexibility of the estimator, we estimate time-varying covariance matrices for three stocks, four stocks and five stocks simultaneously. The analysis is limited to five stocks for presentation purposes only. It will become clear that the estimated parameter matrices are too large for more than five stocks. Therefore, a scalar version of the dynamic covariance and correlation model is estimated for all stocks and compared to the results obtained with the DCC model of Engle (2002). However, the correlation estimates in contrast to the parameter estimates are reported for the scalar and the full specification of the MDE.

The parameter estimates for the first three and four stocks of the sample are shown in table 4. The results for the first five stocks are presented in table 5 and a comparison of the MDE and DCC estimator for a scalar version of the two models is shown in table 6. The bivariate correlation plots corresponding to the tables are given by figures 1, 2 and 3, respectively. The standard errors and the resulting t-statistics are obtained with a block bootstrap for the scalar dynamic covariance and correlation model. The bootstrap is based on 100 runs with a randomly selected start date between 1 and 500 and a randomly selected sample length of 1000 and 2622 minus the start date. The bootstrapped standard errors are robust to a variation of the number of runs.

Table 4 shows the common finding that variances and covariances are persistent. The table further clarifies that there is considerable variation within the estimated parameter matrices. For example, in the upper panel of the table where the estimates for Allianz, Bayerische Hypo Bank and Bayer are presented, the elements of the B matrix vary between 0.8579 and 0.9228. In the lower panel, this variation is even larger and between 0.8121 for the covariance between Bayer and BMW and 0.9555 for the variance of BMW.

Table 5 confirms the discussed findings from above. Variances and covariances are highly persistent with a considerable degree of heterogeneity within the estimated parameter matrices. A comparison of the estimates for three, four and five asset returns also shows that the estimates are stable. For example, the variances and covariance parameter estimates for the first three stocks are almost equal among all tables (see upper left square of B matrices for example).

The comparison of the MDE estimator and the DCC estimator in table 6 shows that there are considerable differences regarding the parameter estimates. The graph (figure 3)with the plots of the estimated dynamic correlations illustrates that the differences are small and that the MDE estimator does not systematically deviate from the DCC estimates. It is also clear, however, that the MDE estimates are more volatile which can be explained with the more flexible specification. Differences can also be due to the direct specification of the correlation process in the MDE correlation model.

Finally, figure 4 presents the correlation estimates of the full correlation minimum distance estimator for the first five bivariate correlations as used above. The figure shows remarkable differences among the correlations through time and among the two different MDE specifications, i.e. the scalar and the full MDE yield different results due to the higher flexibility of the latter. The correlation estimates also show violations of the correlation boundaries in several cases. For example, the first bivariate correlation series exhibit

Table 4: **Empirical Results - Three and Four stocks simultaneously**

Allianz, Bayerische Hypo Bank, Bayer			
$\boldsymbol{\Omega}$			
	0.0225	0.0157	0.0208
	0.0157	0.0308	0.0323
	0.0208	0.0323	0.0454
$\boldsymbol{A}$			
	0.0782	0.0614	0.0566
	0.0614	0.0928	0.0737
	0.0566	0.0737	0.0967
$\boldsymbol{B}$			
	0.8992	0.9228	0.9226
	0.9228	0.8764	0.8938
	0.9226	0.8938	0.8579

Allianz, Bayerische Hypo Bank, Bayer, BMW				
$\boldsymbol{\Omega}$				
	0.0211	0.0149	0.0204	0.0182
	0.0149	0.0308	0.0326	0.0097
	0.0204	0.0326	0.0438	0.0890
	0.0182	0.0097	0.0890	0.0187
$\boldsymbol{A}$				
	0.0770	0.0606	0.0577	0.0609
	0.0606	0.0928	0.0743	0.0454
	0.0577	0.0743	0.0947	0.0988
	0.0609	0.0454	0.0988	0.0258
$\boldsymbol{B}$				
	0.9018	0.9245	0.9219	0.9209
	0.9245	0.8763	0.8931	0.9449
	0.9219	0.8931	0.8615	0.8121
	0.9209	0.9449	0.8121	0.9555

Table 5: **Empirical Results - Five stocks simultaneously**

Allianz, Bayerische Hypo Bank, Bayer, BMW, Deutsche Bank					
$\boldsymbol{\Omega}$					
	0.0229	0.0157	0.0208	0.0179	0.0238
	0.0157	0.0304	0.0326	0.0104	0.0277
	0.0208	0.0326	0.0462	0.0844	0.0465
	0.0179	0.0104	0.0844	0.0191	0.0237
	0.0238	0.0277	0.0465	0.0237	0.0555
$\boldsymbol{A}$					
	0.0788	0.0617	0.0566	0.0607	0.0719
	0.0617	0.0925	0.0735	0.0445	0.0867
	0.0566	0.0735	0.0974	0.0962	0.1036
	0.0607	0.0445	0.0962	0.0259	0.0704
	0.0719	0.0867	0.1036	0.0704	0.1267
$\boldsymbol{B}$					
	0.8983	0.9226	0.9225	0.9214	0.9042
	0.9226	0.8771	0.8938	0.9450	0.8855
	0.9225	0.8938	0.8564	0.8194	0.8499
	0.9214	0.9450	0.8194	0.9550	0.9058
	0.9042	0.8855	0.8499	0.9058	0.8178

Table 6: **Empirical Results - Comparison scalar covariance MDE, scalar correlation MDE and scalar DCC estimator - All nine stocks**

Estimate	MDE covariance estimator		MDE correlation estimator		DCC estimator	
$\boldsymbol{\Omega}$	0.0222	(3.1858)	0.0012	(0.5663)	.	
$\boldsymbol{A}$	0.0515	(11.8201)	0.0129	(2.1385)	0.0138	(1.9710)
$\boldsymbol{B}$	0.9263	(49.7344)	0.9816	(12.6115)	0.9755	(5.5003)

t-statistics (bootstrapped for the MDE with 100 runs) in parenthesis

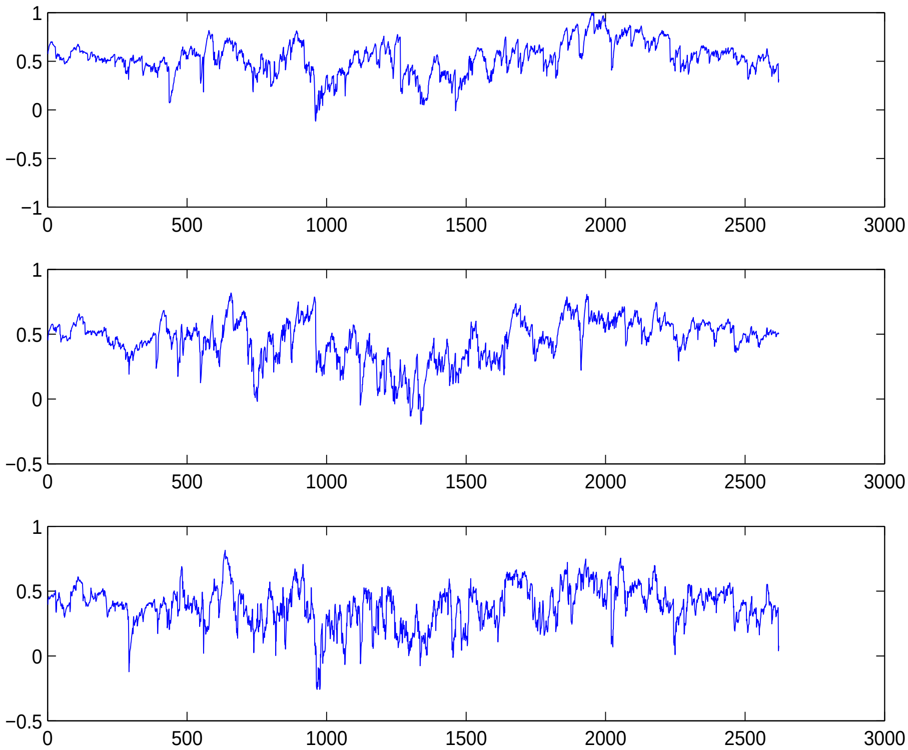

Figure 1: Bivariate Correlations - Allianz, Bayerische Hypo Bank and Bayer

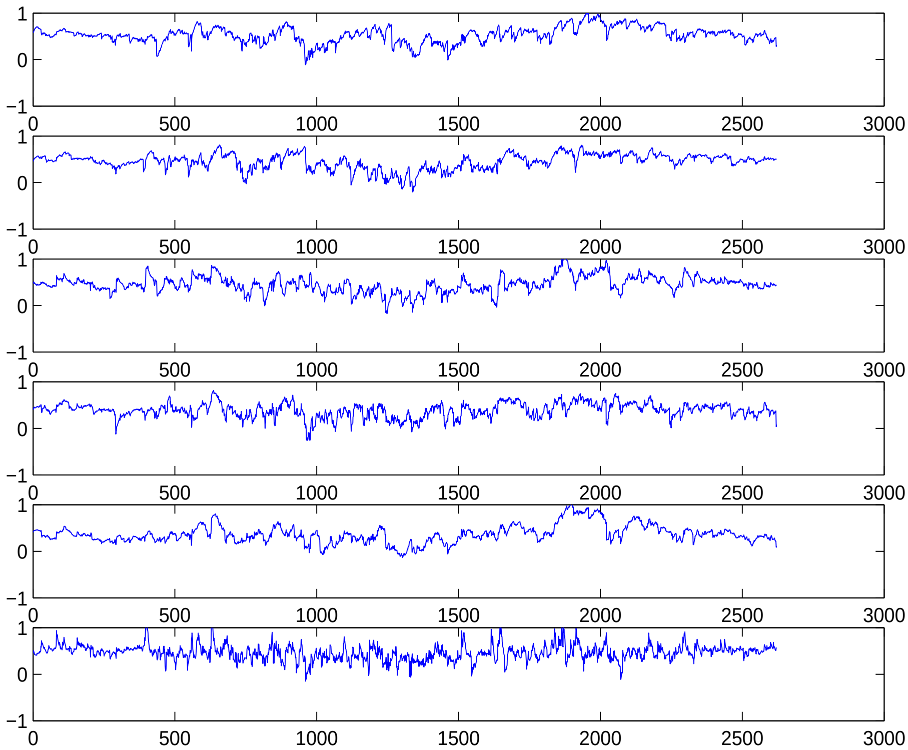

Figure 2: Bivariate Correlations - Allianz, Bayerische Hypo Bank, Bayer and BMW

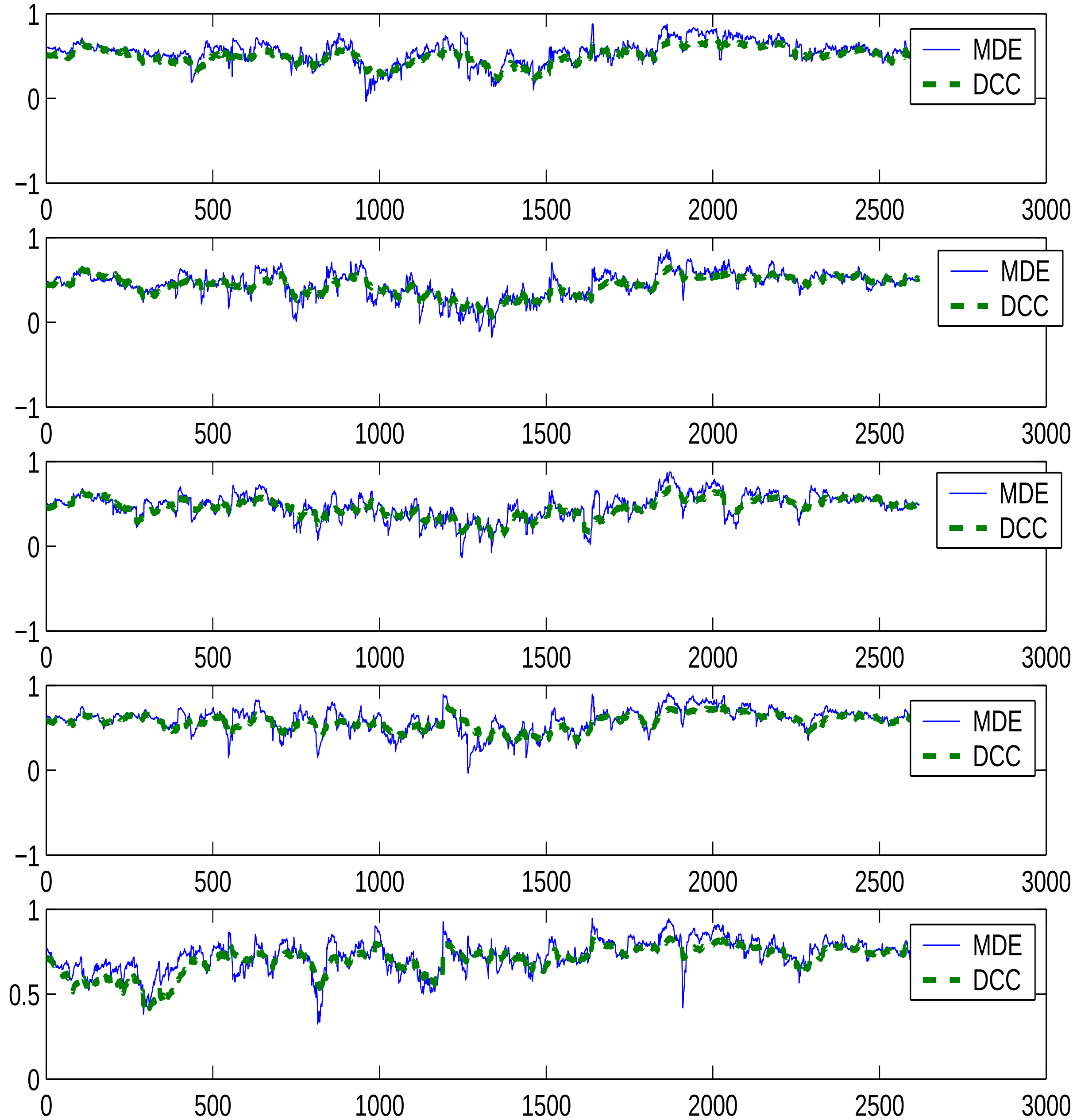

Figure 3: Comparison covariance MDE and DCC estimator - first five bivariate correlations in a scalar version of both estimators for all nine stocks (bivariate correlations of Allianz with Bayerische Hypo Bank, Bayer, BMW, Deutsche Bank and Munich Re)

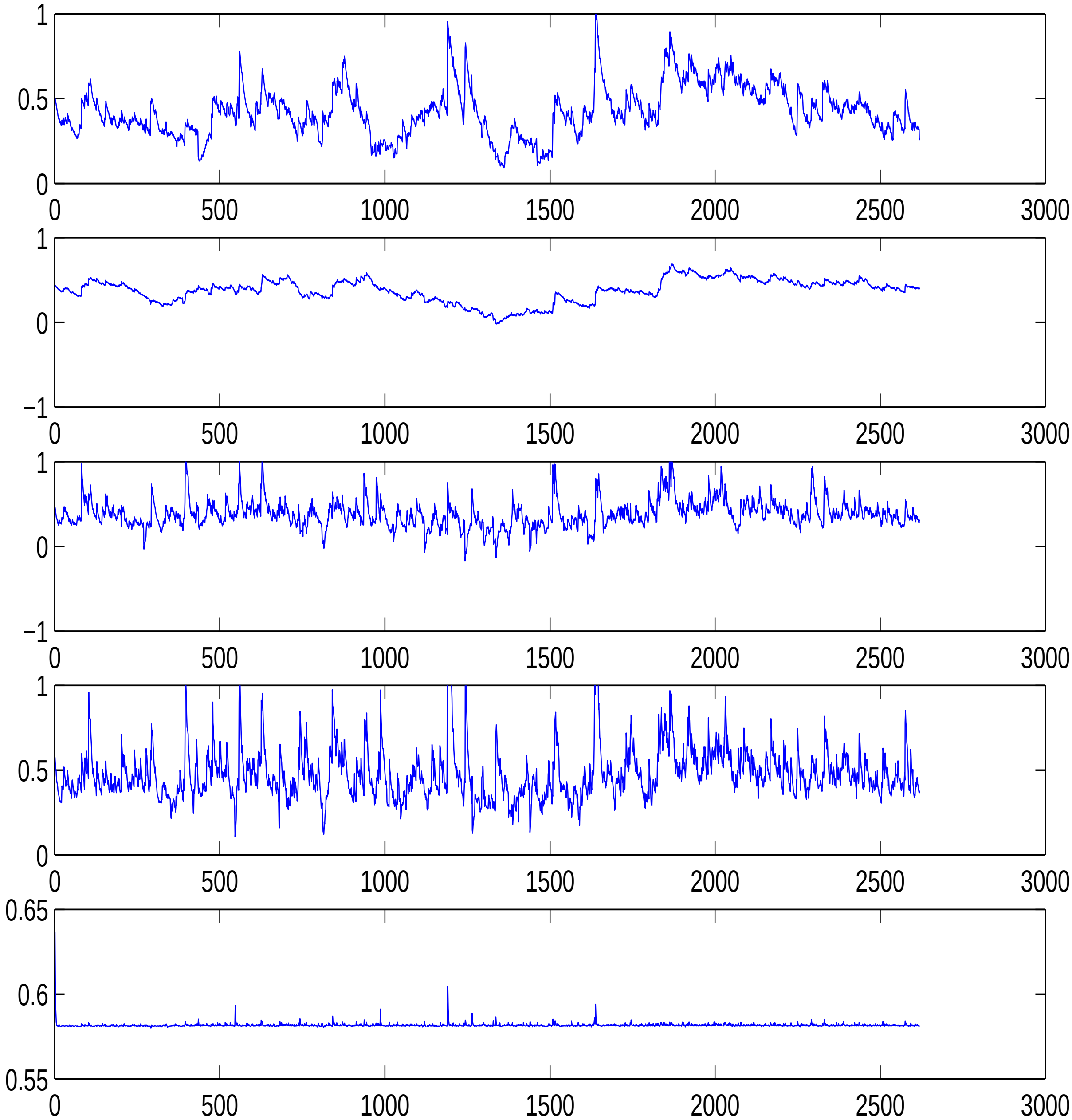

Figure 4: Correlation MDE - first five bivariate correlations in a non-scalar version (bivariate correlations of Allianz with Bayerische Hypo Bank, Bayer, BMW, Deutsche Bank and Munich Re)

values larger than one in the time period between $T = 1500$ and $T = 2000$. Violations also occur for the third and the fourth bivariate correlation estimates. As was outlined above, these violations could possibly be eliminated by augmenting the correlation model or the univariate GARCH processes in the first stage. However, this is beyond the scope of this (preliminary) version of the paper.

5 Conclusions

This paper aims to introduce a new model to estimate dynamic correlations directly and not as the ratio of the covariance and the variances. The minimum distance estimator is an alternative to the maximum likelihood estimation and avoids to employ restrictions to guarantee positive-definite covariance matrices. The dynamic correlations model estimates the time-varying correlations more flexibly and tests the positive-definiteness a posteriori. Interestingly, this property is rarely violated and never so in our empirical example for three, four, five and nine stocks in the scalar dynamic correlation model.

Future research should analyze the properties of the estimator, especially its efficiency and analyze augmented specifications of the basic model, e.g. asymmetric terms and thresholds in the covariance and correlation equations.

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Abstract

The crucial problem in estimating dynamic conditional correlation models is the need to guarantee a positive-definite covariance matrix. In order to avoid any violation of this property, many estimators impose strong restrictions on the model. In addition, many models do not parameterize the correlations directly but the covariance. This paper avoids this problem in proposing a minimum distance estimator (MDE) to estimate dynamic conditional correlations or multivariate GARCH models. The model allows full flexibility in the estimation. Violations of the positive-definiteness of the covariance matrices are part of the specification tests. A simulation study shows the performance of the estimator and the empirical section compares estimates of the MDE with the DCC estimator of Engle (2002). This paper does not only demonstrate an alternative estimator but also outlines how this model can be used to analyze the influence of the restrictions on the estimates in multivariate GARCH models.

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