Elaboration of reliability methods for ageing assessment of NPP components.

(EC JRC Case Study on "Investigation of component age dependent reliability models")

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Abstract

The paper presents the results of a case study on "Investigation of component age dependent reliability models" implemented by INPE and JRC IE in the frame of EC JRC Ageing PSA Network Task 4 activities. Several cases of Generalized Linear Model were applied and investigated for the cases of continues and discrete data. The Fisher Chi-2 minimization approach was used for goodness of fit test and parameters elaboration. Finally, uncertainty analysis was done for parameters estimation and model extrapolations. The results were analyzed and compared with other approaches.

1. Task specification

The goal of the study is a demonstration of methods to build up and assess the component age-dependent reliability models.
The following tasks were performed :
- verification of models validity,
- parameters estimation,
- characterisation of uncertainties of estimated parameters and hole model,
- assessment on possible extrapolation and uncertainties of extrapolation.

2. Initial data sets

To demonstrate the method applicability and compare the results with other case studies JRC proposes to use two data sets :

- Data set 1, is a binned data on failure rates estimated at the bins. These data characterise component failure modes as fail to function, fail to run etc. The data correspond to the continuously distributed times to failure,
- Data set 2, is the failures and demands data, which represent failures on demand. From this data set, the binned data on failure probability on demand per bin could be derived.

All data in the data sets are "virtual". However, the statistic, which is provided for the case study is quite close to the real operating experience data collected on the French or German NPPs. In particular, data include large samples that represent of components from the same technological group.

Binned data (data set 1).
The failure rates were calculated on equal one-year intervals, sequence of which represents the time in operation or age of the component. This data has two particularities :

- there are some intervals without failures, consequently, failure rates are estimated as equal to 0,
- the cumulated operating time is different from one interval to another, this leads to the differences in confidence intervals for failure rates.

These particularities were taken into account during data analysis.

Failure on demand data (data set 2).
The data are censored by interval, e.g. the times in operation are truncated by right and by left ends.
For these data time in operation means number of demands, i.e. it is supposed to be known the number of demands before failure, number of demands on the beginning and on the end of observation (left and right censoring).

The components in the sample haven't the same date of putting in service, and as a consequence haven't the same age on the date of the beginning and of the end of observation.
In addition, components installed in different systems at the same unit type could have a different number of demands per year.
The data were regrouped and processed to obtain binned data sets similar to data set 1. In this case the estimated parameter is failure probability per demand.

3. Models and approach.

3.1 Models applied in case of data set 1.

For continuous time to failure (failure rate) variable it was proposed to apply following statistical models :

1. Constant failure rate : $\varphi\left(\vec{\theta};t\right)$=Const;
2. Linear failure rate : $\varphi\left(\vec{\theta};t\right)=\theta_1+\theta_2 t$;
3. Log-linear or exponential failure rate : $\ln\varphi\left(\vec{\theta};t\right)=\theta_1+\theta_2 t$;

Nota : for this model all calculations were done supposing $\ln\varphi\left(\vec{\theta};t\right)=\ln\left(\theta_1\right)+\theta_2 t$. Estimated interception parameter "a" presented in the results, corresponds to θ_1 and not to $\theta_1^*=\ln\theta_1$. In these terms failure rate function is $\varphi\left(\vec{\theta};t\right)$ = $\theta_1 \exp\left(\theta_2 t\right)$.

4. Power-low (Weibull) failure rate model : $\varphi\left(\vec{\theta};t\right)=\theta_1 t^{\theta_2}$

For models 2-4 the fact that parameter $\theta_2 > 0$ means positive trend in time, i.e. component failure rate increases with age of the component.

3.2 Models applied in case of data set 2

For discrete failures per demand the following models were applied :

1. Constant: $\varphi\left(\vec{\theta};t\right)$=Const;
2. Logit: $\varphi\left(\vec{\theta};t\right)=\dfrac{\exp\left(\theta_1+\theta_2 t\right)}{1+\exp\left(\theta_1+\theta_2 t\right)}$;
3. Probit: $\varphi\left(\vec{\theta};t\right)=\Phi\left(\theta_1+\theta_2 t\right)$;
4. Exponential: $\varphi\left(\vec{\theta};t\right)=\exp\left(\theta_1+\theta_2 t\right)$,

Here $\Phi\left(x\right)=\dfrac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{x}\exp\left(-\dfrac{u^2}{2}\right)du$ - is a normal distribution function $N\left(0;1\right)$.

3.3 Proposed approach

The applied approach is the same for continuously and discreetly distributed data. The difference is only with interpretation of "time" which is a time in operation for continuous functions and number of demands for discrete functions.

To choose the model, which better fits with observed data, first, the goodness of fit test was performed using Fisher statistic, then confidence limits for model parameters $\vec{\theta}=(\theta_1;\theta_2)$ and for resulting function $\varphi(\vec{\theta};t)$ were constructed.

3.4 Goodness of fit test and parameters estimation

The hypothesis of a parametric model form describing the behaviour of a failure rate parameter in time t is tested with the help of Fisher's criterion χ^2, the statistic of which is:

$$\chi^2(\vec{\theta})=\sum_{i=1}^{s}\frac{\left[\nu(\Delta_i)-\varphi(\vec{\theta};t_i)T_i\right]^2}{\varphi(\vec{\theta};t_i)T_i}, \qquad (1)$$

where $\varphi(\vec{\theta};t)$ is one of the four functions proposed to describe the failure rate $\lambda(t)$.

Here

$\Delta_1,\Delta_2,...,\Delta_s$ is the selected X-axis division,

$\nu(\Delta_i)$ is the number of failures per interval Δ_i,

T_i is the cumulated operating time of all components been in operation within the interval Δ_i.

The hypothesis to be tested is presented as follows :

$$H_0:\exists\vec{\theta}:\quad \lambda_i=\varphi(\vec{\theta};t_i), \qquad (2)$$

where λ_i is the averaged failure rate per interval Δ_i.

To calculate the statistic, an unknown value of $\vec{\theta}$ is substituted by an estimate $\hat{\vec{\theta}}$ obtained using the method of minimum χ^2:

$$\hat{\vec{\theta}}=\arg\min_{\vec{\theta}}\chi^2(\vec{\theta}). \qquad (3)$$

The criterion for testing a hypothesis of conformity is a simple comparison of p - value and a chosen confidence level value α. A p - value is calculated from:

$$p=\int_z^{\infty} f_{\chi^2_{s-r}}(t)\,dt, \qquad (4)$$

where

$z=\chi^2(\hat{\vec{\theta}})$;

$f_{\chi^2_{s-r}}(t)$ is the density of distribution χ^2 with $s-r$ degrees of freedom,

s is the number of group intervals Δ_i (where $\nu(\Delta_i)$ should differ from 0),

r is the number of parameters estimated. For constant failure rate model (model 1 in 3.1 and 3.2) $r=1$, for other proposed models (2 – 4 in 3.1 and 3.2) $r=2$.

The hypothesis (2) is accepted in case, if $p>\alpha$, otherwise it is rejected. Besides, if the hypothesis (2) is accepted for several models, preference is to be given to the model with a greater p - value.

The method is described in various statistical books, for example in references [2-5].

The detailed procedure of parameters estimation and model verification by using EXEL software is developed.

3.5 Parameters uncertainties

In case of two parameters model (models 2-4 in 3.1 and 3.2) the task of definition of confidence intervals for each of parameter is transformed in a task of definition of confidence areas.
When constructing the confidence areas the following statistic is used :

$$\chi^2\left(\vec{\theta}\right)=\sum_{i=1}^{s}\frac{\left[\nu\left(\Delta_i\right)-\varphi\left(\vec{\theta};t_i\right)\cdot T_i\right]^2}{\varphi\left(\vec{\theta};t_i\right)\cdot T_i}.$$

Let $1-\varepsilon$ be the confidence level of a confidence area. Solving the equation

$$\varepsilon=\int_{\mu_\varepsilon}^{\infty} f_{\chi_s^2}\left(t\right)dt$$

with a given ε value, determined is the parameter μ_ε.
Then a transcendental inequality is solved by numerical methods:

$$\chi^2\left(\vec{\theta}\right)=\sum_{i=1}^{s}T_i\frac{\left[\frac{\nu\left(\Delta_i\right)}{T_i}-\varphi\left(\vec{\theta};t_i\right)\right]^2}{\varphi\left(\vec{\theta};t_i\right)}\le\mu_\varepsilon. \tag{5}$$

To construct the ellipsoids of concentration (confidence areas for $\vec{\theta}$), Compaq Visual Fortran Professional with a Graphor graphic package, or MatLab can be applied. Isolines are easily plotted in these packages.

3.6 Model uncertainties and extrapolations

The following approach is applied to construct the confidence interval for a trend line. To construct the upper limit at moment t the extreme problem is solved

$$\varphi\left(\vec{\theta};t\right)\to\max_{\vec{\theta}}, \tag{6}$$

with the restriction $$\chi^2\left(\vec{\theta}\right)=\sum_{i=1}^{s}T_i\frac{\left[\frac{\nu\left(\Delta_i\right)}{T_i}-\varphi\left(\vec{\theta};t_i\right)\right]^2}{\varphi\left(\vec{\theta};t_i\right)}\le\mu_\varepsilon.$$

To construct the lower limit at time t the extreme problem is solved

$$\varphi\left(\vec{\theta};t\right)\to\min_{\vec{\theta}}, \tag{7}$$

with the restriction $$\chi^2\left(\vec{\theta}\right)=\sum_{i=1}^{s}T_i\frac{\left[\frac{\nu\left(\Delta_i\right)}{T_i}-\varphi\left(\vec{\theta};t_i\right)\right]^2}{\varphi\left(\vec{\theta};t_i\right)}\le\mu_\varepsilon.$$

As $\varphi\left(\vec{\theta};t\right)$ have no local extreme points, restrictions of the inequality type can be substituted by the following equality :

$$\chi^2\left(\vec{\theta}\right)=\sum_{i=1}^{s}T_i\frac{\left[\frac{\nu\left(\Delta_i\right)}{T_i}-\varphi\left(\vec{\theta};t_i\right)\right]^2}{\varphi\left(\vec{\theta};t_i\right)}=\mu_\varepsilon ,$$

since the solution will be inside of confidence ellipse area.

4. Results of calculations

4.1 Presentation of the results

Data set 1.

In case of continuous distributions the results of goodness-of-fit test (fitted model parameters $\vec{\theta}=\left(\theta_1;\theta_2\right)$ and p-values) are presented in a Table 1.

Example of graphical interpretation of fitted models and data uncertainties are provided at Figures 1-3.

Table 1. Summary of parameters estimation for data set 1.

Component group	Parameters	Models				Comments
		Constant	Linear	Log-linear	Weibull	
#3	θ_1	0.030	0.012	0.015	0.013	No model fit with the data
	θ_2		0.0017	0.0637	0.0017	
	p-value	0.002	0.006	0.006	0.003	
#6	θ_1	0.023	0.014	0.013	0.014	No model fit with the data
	θ_2		0.0010	0.0539	0.2179	
	p-value	0	0	0	0	
#6.1	θ_1	0.029	0.017	0.018	0.016	?
	θ_2		0.0011	0.0415	0.2697	
	p-value	0.006	0.014	0.015	0.012	
#7	θ_1	0.019	0.010	0.011	0.009	Log-linear fits the best (slow ageing)
	θ_2		0.0010	0.0546	0.3414	
	p-value	0.019	0.542	0.567	0.360	
#7.1	θ_1	0.019	0.004	0.007	0.003	Log-linear fits the best (slow ageing)
	θ_2		0.0012	0.0792	0.7255	
	p-value	0.041	0.429	0.492	0.365	
#8.1	θ_1	0.015	0.011	0.012	0.007	?
	θ_2		0.0004	0.0161	0.3073	
	p-value	0.057	0.051	0.046	0.100	
#11.1	θ_1	0.021	0.009	0.012	0.006	Weibull fits the best (slow ageing)
	θ_2		0.0012	0.0503	0.5482	
	p-value	0.203	0.278	0.271	0.303	
#13.3	θ_1	0.003	0.001	0.002	0.001	All models fit the data, (slow ageing)
	θ_2		0.0002	0.0426	0.4667	
	p-value	0.748	0.762	0.728	0.793	
#14.1	θ_1	0.00028	0.00035	0.00034	0.00028	All models fit the data (no ageing)
	θ_2		-0.00001	-0.02360	0.00001	
	p-value	0.967	0.938	0.934	0.926	
#16.2	θ_1	0,000	0,000	0,000	0,001	All models fit the data (no ageing)

Component group	Parameters	Models				Comments
		Constant	Linear	Log-linear	Weibull	
	θ_2		0,0000	-0,0670	-0,3802	
	p-value	0,923	**0,995**	0,987	0,948	
#17.1	θ_1	0.002	0.003	0.003	0.003	All models fit the data (no ageing)
	θ_2		-0.0001	-0.0880	-0.3929	
	p-value	0.912	0.976	0.962	0.920	
#19.1	θ_1	0.039	0.061	0.071	0.081	Weibull fits the best (no ageing)
	θ_2		-0.0030	-0.0914	-0.4775	
	p-value	0.493	0.669	0.704	0.873	
#30.1	θ_1	0.045	0.073	0.158	0.311	?
	θ_2		-0.0023	-0.0992	-0.7772	
	p-value	0.023	0.102	0.108	0.045	
#32.2	θ_1	0.005	0.004	0.004	0.004	All models fit the data, but const. fits the best (?)
	θ_2		0.0002	0.0333	0.1223	
	p-value	0.543	0.534	0.532	0.489	
#34.1	θ_1	0,025	0,021	0,020	0,022	
	θ_2		0,0003	0,0148	0,0379	
	p-value	**0,807**	0,773	0,778	0,758	
#35.1	θ_1	0,055	0,024	0,024	0,024	
	θ_2		0,0027	0,0704	0,3551	
	p-value	0,008	0,079	**0,148**	0,028	
#36.2	θ_1	0,094	0,122	0,121	0,120	
	θ_2		-0,0023	-0,0213	-0,1020	
	p-value	0,229	**0,242**	0,232	0,194	
#38.1	θ_1	0,006	0,003	0,003	0,003	
	θ_2		0,0002	0,0487	0,2676	
	p-value	0,437	0,495	**0,519**	0,424	
#39.1	θ_1	0,067	0,006	0,013	0,005	
	θ_2		0,0045	0,1099	0,9965	
	p-value	0,001	0,023	**0,071**	0,021	
#43.1@	θ_1	0,020	0,009	0,010	0,008	
	θ_2		0,0013	0,0761	0,4249	
	p-value	0,893	0,906	**0,911**	0,890	
#44.1	θ_1	0,001	0,002	0,002	0,002	
	θ_2		-0,0001	-0,1251	-0,4689	
	p-value	0,298	**0,761**	0,663	0,360	
#45@	θ_1	0,008	0,011	0,013	0,017	
	θ_2		-0,0003	-0,0522	-0,4009	
	p-value	0,862	0,923	0,946	**0,954**	

Component group	Parameters	Models				Comments
		Constant	Linear	Log-linear	Weibull	
#47.1	θ_1	0,006	0,003	0,002	0,004	
	θ_2		0,0003	0,0817	0,2051	
	p-value	0,112	0,150	**0,214**	0,085	
#48.2	θ_1	0,001	0,001	0,001	0,003	
	θ_2		0,0000	-0,0433	-0,5343	
	p-value	0,189	0,112	0,184	**0,362**	
#48.3	θ_1	0,001	0,002	0,002	0,004	
	θ_2		-0,0001	-0,0716	-0,6499	
	p-value	0,045	0,074	0,116	**0,543**	
#49.5	θ_1	0,001	0,001	0,001	0,001	
	θ_2		0,0000	-0,0176	0,0592	
	p-value	**0,810**	0,728	0,721	0,714	
#50	θ_1	0,011	0,010	0,010	0,010	
	θ_2		0,0001	0,0125	0,0693	
	p-value	**0,828**	0,727	0,727	0,725	
#55	θ_1	0,005	0,006	0,006	0,004	
	θ_2		-0,0001	-0,0073	0,1833	
	p-value	**0,422**	0,294	0,293	0,305	
#56	θ_1	0,021	0,029	0,026	0,015	
	θ_2		-0,0007	-0,0164	0,1504	
	p-value	**0,176**	0,137	0,131	0,130	
#56.1	θ_1	0,021	0,022	0,022	0,022	
	θ_2		-0,0001	-0,0058	-0,0206	
	p-value	**0,889**	0,853	0,853	0,851	
#57	θ_1	0,029	0,026	0,026	0,030	
	θ_2		0,0002	0,0087	-0,0167	
	p-value	**0,902**	0,864	0,865	0,861	
#58	θ_1	0,017	0,025	0,027	0,031	
	θ_2		-0,0007	-0,0499	-0,3202	
	p-value	0,666	0,694	0,702	**0,723**	
#59.1	θ_1	0,057	0,012	0,017	0,008	
	θ_2		0,0030	0,0782	0,7312	
	p-value	0,257	0,482	**0,561**	0,438	
#59.1@WR	θ_1	0,151	0,059	0,066	0,039	
	θ_2		0,0068	0,0575	0,5348	
	p-value	0,031	0,138	**0,187**	0,113	

Component group	Parameters	Models				Comments
		Constant	Linear	Log-linear	Weibull	
#62.2	θ_1	0,052	0,037	0,037	0,034	
	θ_2		0,0015	0,0326	0,1923	
	p-value	0,854	0,859	**0,864**	0,844	
#63.1	θ_1	0,002	0,003	0,003	0,004	
	θ_2		-0,0001	-0,0479	-0,3450	
	p-value	**0,778**	0,742	0,741	0,759	
#65	θ_1	0.046	0.075	0.104	0.116	Log-linear fits the best (no ageing)
	θ_2		-0.0027	-0.0830	-0.4365	
	p-value	0.005	0.458	0.539	0.073	

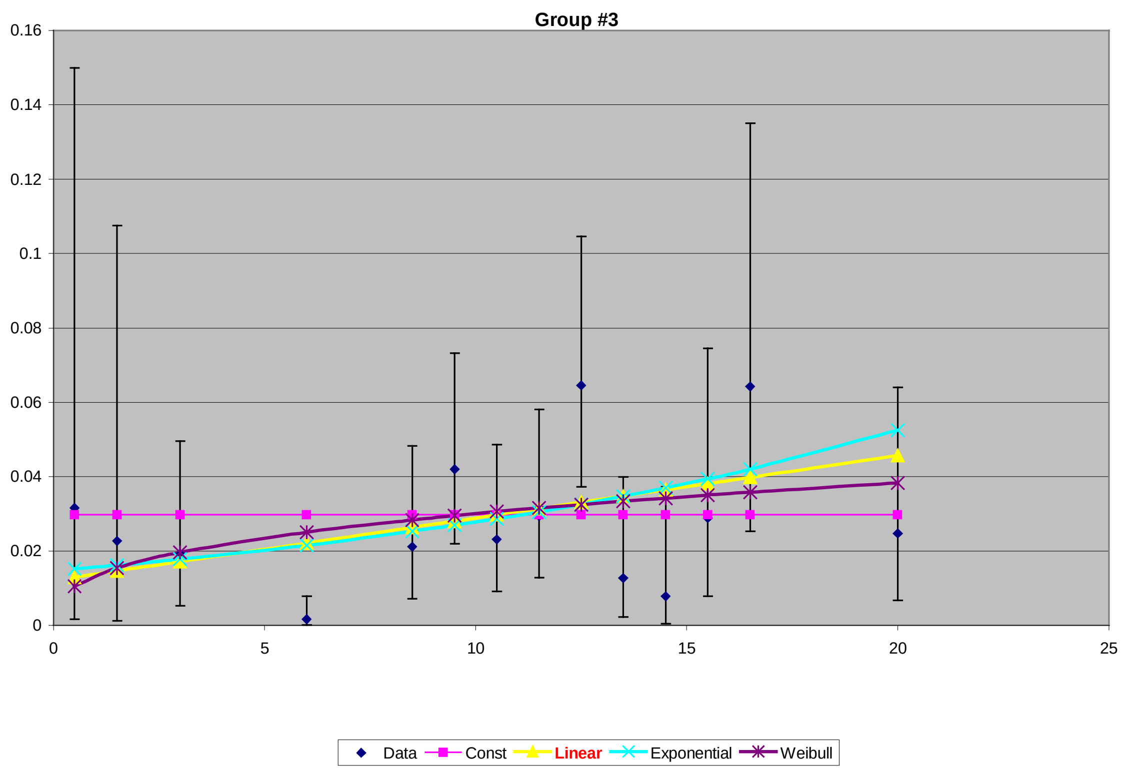

Figure 1. Component group #3. Fitted failure rates (1/y), as the functions of time in operation (y).

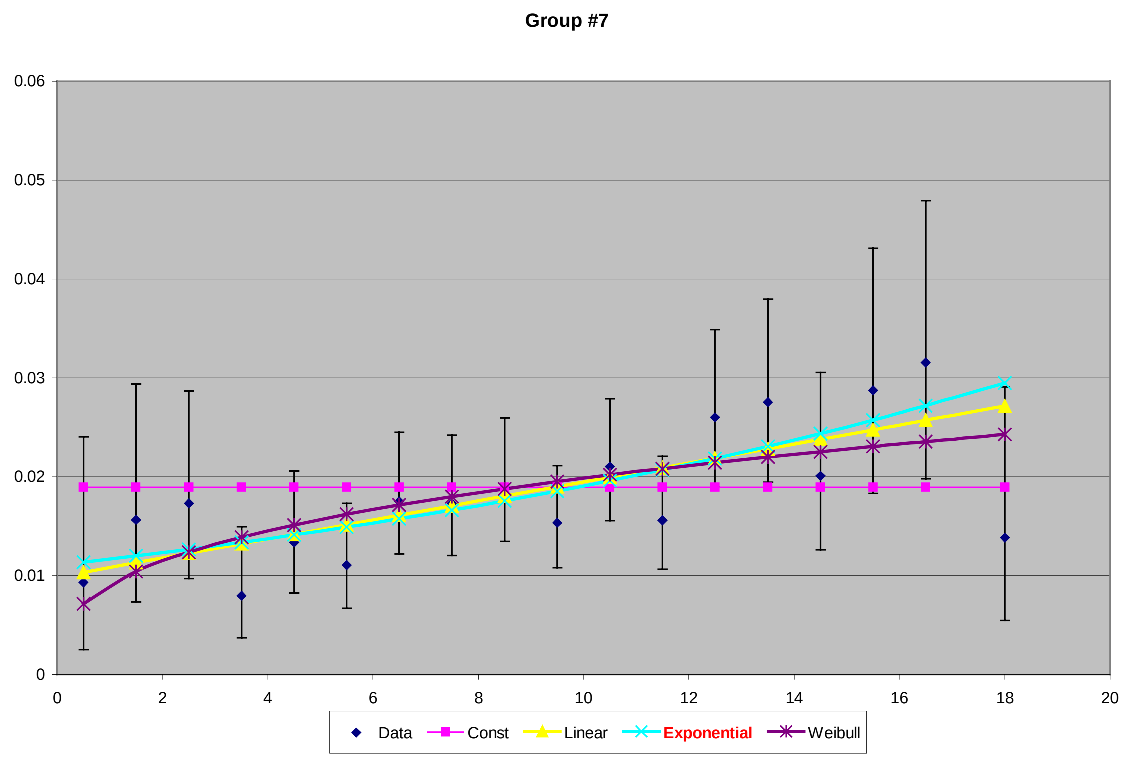

Figure 2. Component group #7. Fitted failure rates (1/y), as the functions of time in operation (y).

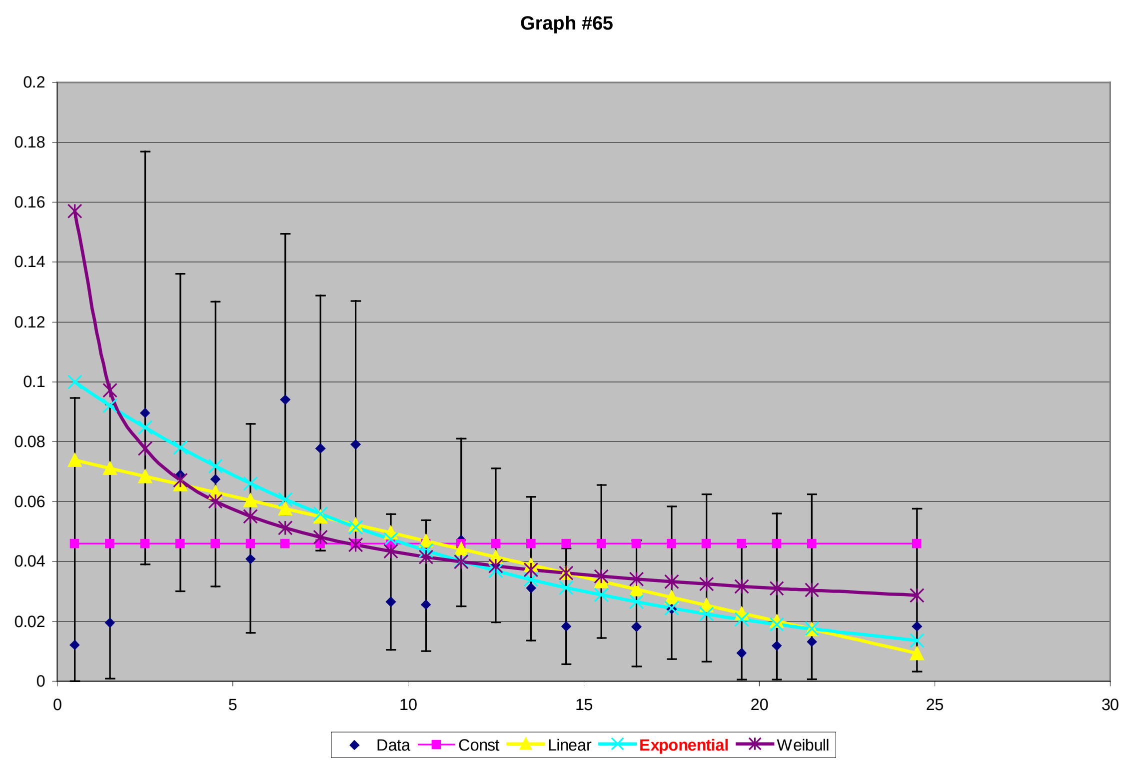

Figure 3. Component group #65. Fitted failure rates (1/y), as the functions of time in operation (y).

Data set 2.

For the discrete data the results of parameters estimation and goodness-of-fit test are presented in a Table 2. The graphical interpretation of uncertainties is given in Annex 6.
The presented cases are those where initial data contains more then 10 failures per component group.

Table 2. Summary of parameters estimation for data set 2.

Component group	Parameters	Models				Comments
		Constant	Logit	Probit	Exponential	
U_C	θ_1	0.003	-5.63	-2.69	-5.63	Constant model fits the best
	θ_2		-5.16E-04	-1.71E-04	-5.13E-04	
	p-value	**0.26**	0.15	0.15	0.15	
U_D	θ_1	0.004	-3.88	-2.08	-3.88	Decreasing trend (no ageing)
	θ_2		-8.11E-03	-2.78E-03	-8.11E-03	
	p-value	0.09	0.67	**0.68**	0.67	
U_F	θ_1	0.0018	-5.50	-2.66	-5.50	Decreasing trend (no ageing)
	θ_2		-1.19E-03	-3.60E-04	-1.19E-03	
	p-value	0.57	**0.89**	0.88	**0.89**	
ABC	θ_1	1.46E-05	-11.01	-4.15	-11.00	No model fits with the data
	θ_2		-2.38E-05	-5.30E-06	-2.38E-05	
	p-value	0.03	**0.05**	**0.05**	**0.05**	
DEF	θ_1	0.00013	-8.54	-3.55	-8.54	Decreasing trend (no ageing)
	θ_2		-0.00084	-0.00021	-0.00084	
	p-value	0.53	**0.86**	**0.86**	**0.86**	

4.2. Results analysis and interpretation

4.2.1. Identification of component susceptible to ageing

Analysis of results could be performed in three stages :

- on the first stage, the component groups for which one or more proposed models fit well with the data could be selected. It was decided to consider all models where p-value is more then 0,1.
- secondly, component groups for which best fitted model shows negative “ageing” parameter ($\theta_2 < 0$) could be ignored for following assessment,
- then, component groups with positive ageing trends could be identified by comparing the “ageing” parameter (θ_2) and its confidence intervals with zero. In case if the lower bound of 90% confidence interval for “ageing” parameter θ_2 is above 0, the ageing trend could be assumed.

The following paragraphs present the results of such screening.

Data set 1.

The results of the screening show that from 37 component groups from Data Set 1 the positive ageing trend could be assumed for 10 component groups listed below :

- #7 (best fitted model is log-linear with θ_2 = 0.055),
- #7.1 (best fitted model is log-linear with θ_2 = 0.079),
- #8.1 (best fitted model is Weibull, p = 0.1, with θ_2 = 0.31),
- #11.1 (best fitted model is Weibull with θ_2 = 0.55),
- #13.3 (best fitted model is Weibull with θ_2 = 0.47),
- #35.1 (best fitted model is log-linear, p=0.15, with θ_2 = 0.07),

- #38.1 (best fitted model is log-linear with θ_2 = 0.049),
- #47.1 (best fitted model is log-linear with θ_2 = 0.082),
- #59.1 (best fitted model is log-linear with θ_2 = 0.078),
- #59.1@WR (best fitted model is log-linear with θ_2 = 0.057).

Figure 4 and 5 present the areas of uncertainties for estimated model parameters in cases of component groups #7.1 and #11.1.

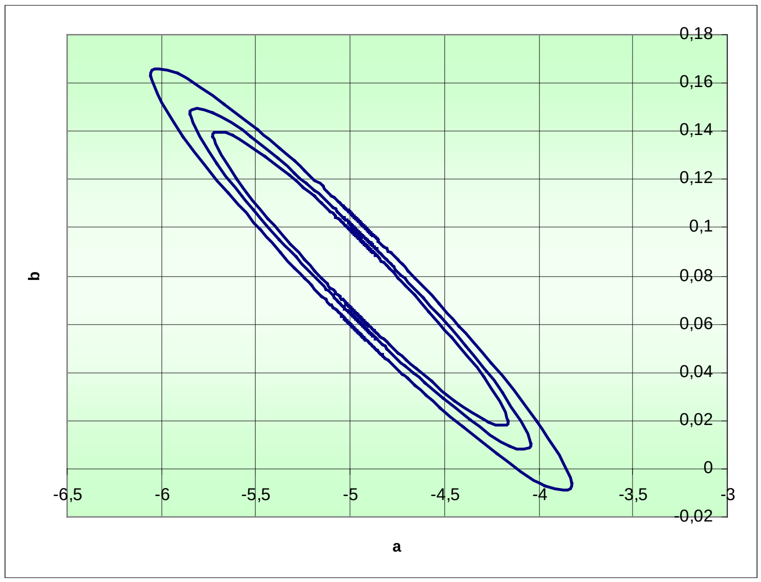

Figure 4. Component group #7.1, log-linear model parameters uncertainties (90, 95 and 99% confidence areas). "a" = θ_1^* = ln 0.007 = -4.96, "b" = θ_2 = 0.79.

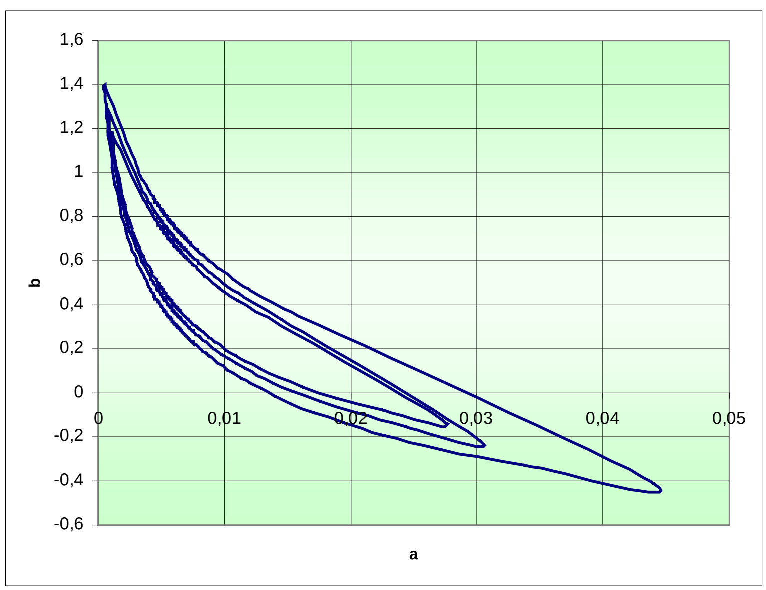

Figure 5. Component group #11.1, Weibull model parameters uncertainties (90, 95 and 99% confidence areas). "a" = θ_1 = 0.006, "b" = θ_2 = 0.55.

For 2 component groups log-liner model with positive ageing parameter was identified as well fitted, but the value of 90% low bound of “ageing” parameter is below zero.

- #43.1 (best fitted model is log-linear with $\theta_2 = 0.076$),
- #62.2 (best fitted model is log-linear with $\theta_2 = 0.033$).

For following 10 component groups the best fitted model is constant : #14.1, #32.2, #34.1, #49.5, #50, #55, #56, #56.1, #57, #63.1.

For the rest 16 component groups the situations are as following : even no model fits with the data, i.e. p-value is very small (for example, component groups #3, #6, #6.1, etc.), or negative “ageing” parameter are obtained (see for example, #17.1, #19.1, #30.1, etc.).

For better understanding of obtained results and importance of ageing trends, relative increasing in failure rate in time with regard to constant failure rate are presented in Table 3.

Table 3. Failure rate increasing.

Component group	Best fitted model	Parameters : θ_1 θ_2	φ=c	$\varphi(\theta, 10) / \varphi$=c	$\varphi(\theta, 20) / \varphi$=c	$\varphi(\theta, 30) / \varphi$=c
#7	log-linear	0.011 0.0546	0.019	0.58	1.73	2.98
#7.1	log-linear	0.007 0.0792	0.019	0.37	1.80	3.96
#8.1	Weibull	0.007 0.3073	0.015	0.95	1.17	1.33
#11.1	Weibull	0.006 0.5482	0.021	1.01	1.48	1.84
#13.3	Weibull	0.001 0.4667	0.003	0.98	1.35	1.63
#35.1	log-linear	0.024 0.0704	0.055	0.44	1.78	3.61
#59.1@	log-linear	0.066 0.0575	0.151	0.44	1.38	2.45

These figures show that application of constant failure rate model could provide underestimated unavailability values in case of aged NPPs. The interception point of constant and time-dependent failure rates corresponds to the plant ages between 10 and 20 years. Taking into account the delay between data collection, parameters estimation and PSA update it could lead to underestimation in final PSA results.
In presented data examples the data collection covers the ages window between 0 and 20 years in operation. Now, if 10 years periodicity of PSA update will be assumed and for the 30-years examination this data set will be applied, the underestimate of failure rates could rise up to the factor 4 (see $\varphi(\theta, 30) / \varphi$=c for component group #7.1, for example).
Of course, it’s true in case if the trend will continue in time.

<u>Data set 2.</u>
There are no component groups in this data set which show increasing trend of failure rate. The following analysis does not include the examples from data set 2, but the main conclusions of the analysis provided in chapter 4.2.7 could be valid for discrete data as well.

4.2.2. Comparison with results of non-parametric inversion test

A non-parametric inversion test was performed for most of component groups. As a result increasing failure rates were identified for component groups : #3, #6, #6.1, #7, #11.1, #39.1, #43.1, #45, #47.1, #50.1, #56, #58, #62.2.
For component groups #7, #7.1, #11.1, #43.1, #47.1 and #62.2 conclusions of inversion test were confirmed by parametrical modeling. The results of goodness of fit test for component groups #3, #6, #6.1 and #39.1 show that no model fit with the data. For the rest cases parametrical models do not confirm the ageing trend.

From the other hand, inversion test does not identify ageing trends in case of #8.1, #13.3, #35.1, #38.1, #59.1 and #59.1@. This again shows the weakness of non-parametrical tests and the necessity to apply different methods for ageing detection.

4.2.3. Impact of burn-in failures

Visual examination of data permits to suppose existence of burn-in failures for certain component groups, for example : #3, #6, #6.1, #7.1, #32.2, #35.1, #38.1, 39.1, #45@, #48.2, #48.3. Figures 6 and 7 presents the examples of graphs used for visual examination in cases of components #3 and #7.1.

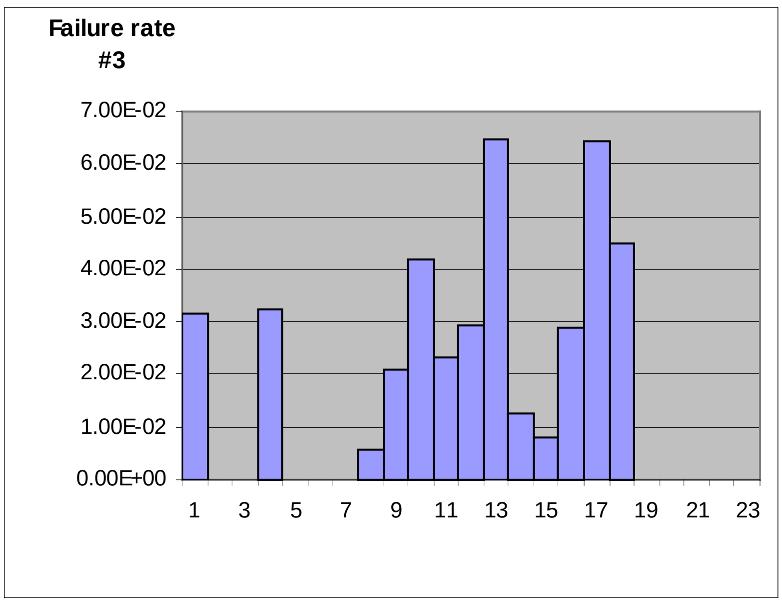

Figure 6. Component group #3. Failure rate distribution in time bins.

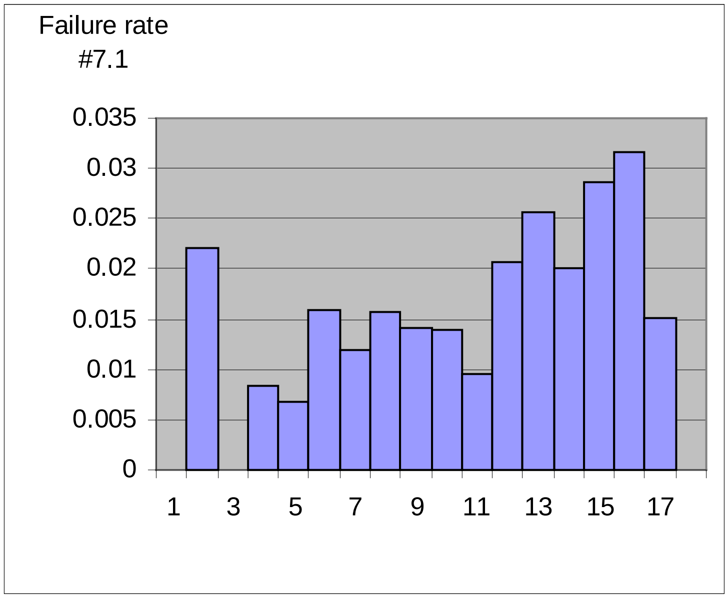

Figure 7. Component group #7.1. Failure rate distribution in time bins.

Additional examination was done for these component groups by excluding first intervals from data sets.
The results for groups #3, #6, #6.1 show an increase of "ageing" parameter, but p-value still resides very low.
Results of calculation for other component groups are presented in Table 4.

Table 4. Impact of burn-in failures.

Component group	Best fitted model	Parameters: θ_1 θ_2 p-value	φ=c	φ (θ, 10) / φ=c	φ (θ, 20) / φ=c	φ (θ, 30) / φ=c
#7.1	log-linear	0.007 0.0792 0.49	0.019	0.81	1.80	3.96
#7.1 /burn-in failures excluded	linear	0.0014 0.0015 0.47	0.019	0.86	1.65	2.44
#32.2	const	0.005 0.54	0.005	1	1	1
#32.2 /burn-in failures excluded	Weibull	0.0018 0.49 0.65	0.005	1.11	1.56	1.91
#35.1	log-linear	0.024 0.0704 0.15	0.055	0.88	1.78	3.61
#35.1/burn-in failures excluded	log-linear	0.015 0.1 0.35	0.054	0.76	2.05	5.58

#38.1	log-linear	0.0032 0.049 0.52	0.006	0.53	1.42	2.32
#38.1/burn-in failures excluded	Weibull	0.00024 1.22 0.58	0.006	0.66	1.55	2.54
#39.1	log-linear	0.013 0.11 0.07	0.067	0.58	1.75	5.26
#39.1/burn-in failures excluded	Log-linear	0.0032 0.18 0.14	0.075	0.26	1.56	9.45
#45.@	Weibull	0.017 -0.40 0.954	0.078	0.87	0.66	0.56
#45@/burn-in failures excluded	Log-linear	0.00031 2.97E-04 0.999	0.0048	0.68	1.30	1.92
#48.2	Weibull	0.017 -0.40 0.36	0.078	0.87	0.66	0.56
#48.2/burn-in failures excluded	Const	0.00068 0.56	0.00068	1	1	1
#48.3	Weibull	0.0035 -0.65 0.54	0.0011	0.72	0.46	0.35
#48.3/burn-in failures excluded	Log-liner	0.00021 0.081 0.997	0.00064	0.74	1.66	3.74

Consideration of burn-in failures could improve the result of goodness of fit test, as for example in case of group #39.1 which was initially excluded from the screening because of very small p-value. Results of additional examination permits to conclude about the existence of ageing trend for this component group.

Consideration of burn-in failures (excluding them from data) could change the conclusion about existence or absence of ageing trend, as for example in case of groups #32.2, #45@, #48.2 and #48.3. Three of these groups (#32.2, #45@, #48.3) could be added to the list of components with identified ageing trend.

For group #32.2 the conclusion of first calculation was that failure rate is *constant* with significance level 0.54 but all others models also fitted quite well. Neglecting of burn-in failures leads to the conclusion that *Weibull* model fits the best (p-value = 0.64) but the constant failure rate still fits good with p-value = 0.51. Choice of constant failure rate model could lead to underestimation of unavailability for 30-years aged component by factor 1,9 ($\varphi(\theta, 30) / \varphi$=const.) in comparison with Weibull model.

In case of component group #48.3 where conclusion from the first examination is an existence of *decreasing trend* (i.e. reliability of component is increasing with time), the consideration of burn-in failures changed the conclusion to opposite one : i.e. existence of *increasing trend*.

In some cases the burn-in failures do not impact a lot to the time-dependent models extrapolations, so the calculated failure rate values are close to each other.

An example is the group #38.1. Here, analysis of complete data set provides best fitted log-linear model with significance level 0.52. Excluding burn-in failures from the analysis gives a conclusion that Weibull fits the best with significance level 0.58. Comparison of failure rate extrapolations up to the age of 30 years for both of these models with constant failure rate

(which is the same in both of the cases) provides about the same level of underestimation : 2.32 in case of complete sample and 2.54 in case where the burn-in failures neglected.
In one case, group #39.1, the excluding of burn-in failures has led to increasing in failure rate by order of magnitude in comparison with constant failure rate value.
All those examples show the importance of consideration of burn-in failures in the ageing assessment.

4.2.4. Comparison with other parametrical methods

The results of the calculations were compared with estimations by other parametrical methods in the frame of Ageing PSA Task Group 4 activities.
As the alternative methods the Bayesian analysis with non informative priors and Stochastic Expectation Maximization were chosen.

In case of Bayesian analysis the same sets of binned data were analyzed.
To check the validity of the model, it was used the posterior predictive distribution for the number of failures in each bin to compare observed and replicated chi-square statistics. The overlap probability, is referred to here as a Bayesian p-value.
Analysis was done by free-available software WinBUGS.
The calculations were performed for two component groups #3 and #7.1. The results of calculation are presented in Table 5.

Table 5. Comparative parameters estimation (frequentist vs Bayesian).

Component group	Parameters	Models				Comments
		Constant	Linear	Log-linear	Weibull	
#3 Chi-2 min.	θ_1	0.030	0.012	0.015	0.013	No model fit with the data
	θ_2		0.0017	0.0637	0.0017	
	p-value	0.002	0.006	0.006	0.003	
#3 Bayesian	θ_1	0.023	0.007	0.01	0.007	No model fit with the data
	θ_2		0.002	0.07	0.62	
	p-value	0.004	0.01	0.01	0.007	
#7.1 Chi-2 min.	θ_1	0.019	0.004	0.007	0.003	Log-linear fits the best (slow ageing)
	θ_2		0.0012	0.0792	0.7255	
	θ_2 90% conf. interval		(8.8E-4, 0.0017)	(0.059, 0.099)	(0.63, 0.82)	
	p-value	0.041	0.429	0.492	0.365	
#7.1 Bayesian	θ_1	0.017	0.004	0.007	0.003	Log-linear fits the best (slow ageing)
	θ_2		0.001	0.079	0.814	
	θ_2 90% conf. interval		(7.0E-4, 0.002)	(0.04, 0.12)	(0.41, 1.27)	
	p-value	0.046	0.41	0.47	0.33	

In case of component group #3 the Bayesian analysis leads to the same conclusion as a frequentist one that no model fit with the data (for all models the p-value is very small). That could be the reason of slight difference in parameters estimation.
Comparison of the results for group #7.1 shows that Bayesian approach with non informative priors provides numerical results similar (or very close) to frequentist analysis : the calculated model parameters for best fitted models (linear and non-linear) are the same and the p-values are very close to each other. The 90% confidence interval is a little bit more tight in case of frequentist analysis, but still comparative with figures provided by Bayesian estimation.

Stochastic Expectation Maximization (SEM) method was applied for the times to failure data, which were used to develop initial data sets.. The SEM algorithm was realized only for Weibull

model parameters estimation and has some limits from application point of view. The algorithm provides the point estimations only.
The comparison was done for three component groups : #8.1, #11.1 and 13.3. For these groups the goodness of fit test identified the Weibull as best fitted model. The results of the calculations are presented in Table 6.

Table 6. Comparative parameters estimation (Chi-2 min. vs SEM).

Component group	Best fitted model	Parameters : θ_1 θ_2	φ=c	$\varphi(\theta, 10) / \varphi$=c	$\varphi(\theta, 20) / \varphi$=c	$\varphi(\theta, 30) / \varphi$=c
#8.1 Chi-2	Weibull	0.007	0.015	0.95	1.17	1.33
		0.31				
#8.1 SEM	Weibull	0.0059	0.015	1.06	1.43	1.70
		0.43				
#11.1 Chi-2	Weibull	0.006	0.021	1.01	1.48	1.84
		0.55				
#11.1 SEM	Weibull	0.0023	0.021	0.91	1.72	2.50
		0.92				
#13.3 Chi-2	Weibull	0.001	0.003	0.98	1.35	1.63
		0.47				
#13.3 SEM	Weibull	0.00025	0.003	1.10	2.39	3.76
		1.12				

In all three cases the SEM provides more conservative estimation of “ageing” parameter. As a consequence, the extrapolated values of failure rates are much higher. For example in case of component group #13.3, the “ageing” parameter estimated by SEM more then twice higher of those estimated with Chi-2 minimization approach.
One possible explanation of this difference is that times to failure data are more informative that binned one. But more detailed investigation of this issue is necessary.

4.2.5. Uncertainties of extrapolation.

To apply developed time dependent reliability models in PSA it is necessary to perform some predictive estimation of failure rates. Uncertainties of predictive extrapolations and impact of the choice of the model to extrapolation results were analyzed in the frame of the study. The Annex 5 provides a graphical interpretation of parameters and models uncertainties.
Table 7 presents the results of relative increase of extrapolated failure rate with regards to constant failure rate model.

Table 7. Failure rate extrapolations with different time dependent models.

Component group	Fitted model	Parameters : θ_1 θ_2 p-value	φ=c	$\varphi(\theta, 10) / \varphi$=c	$\varphi(\theta, 20) / \varphi$=c	$\varphi(\theta, 30) / \varphi$=c
#7.1	linear	0.004	0.019	0.84	1.47	2.11
		0.0012				
		0.43				
#7.1	Log-linear	0.007	0.019	0.37	1.80	3.96
		0.0792				
		0.49				
#7.1	Weibull	0.003	0.019	0.85	1.41	1.89
		0.73				

		0.37				
#43.1	linear	0.009 0.0013 0.906	0.02	1.10	1.75	2.40
#43.1	Log-linear	0.01 0.076 **0.911**	0.02	1.07	2.29	4.89
#43.1	Weibull	0.008 0.43 0.89	0.02	1.08	1.45	1.73
#38.1	linear	-0.0010 0.0005 **0.58**	0.006	0.69	1.54	2.39
#38.1	Log-linear	0.00150 0.092 0.53	0.006	0.63	1.57	3.95
#38.1	Weibull	0.00024 1.22 0.58	0.006	0.66	1.55	2.54

Comparison of results of extrapolation leads to the conclusion that in all of the cases the most conservative estimation is provided by log-linear model.

This is an important observation. The p-values (used here as a criteria for choice of the model) are quite close in all presented cases, but the extrapolated up to the 30-years age failure rates are different. For example in case of component group #43.1 the difference in estimation using log-linear and linear model is more then of factor 2. If log-linear calculation is compared with result of Weibull model the difference rises up to the factor 2.8.

From the other side, log-linear model provides more uncertain extrapolations. This is shown in the Figures 8 - 13.

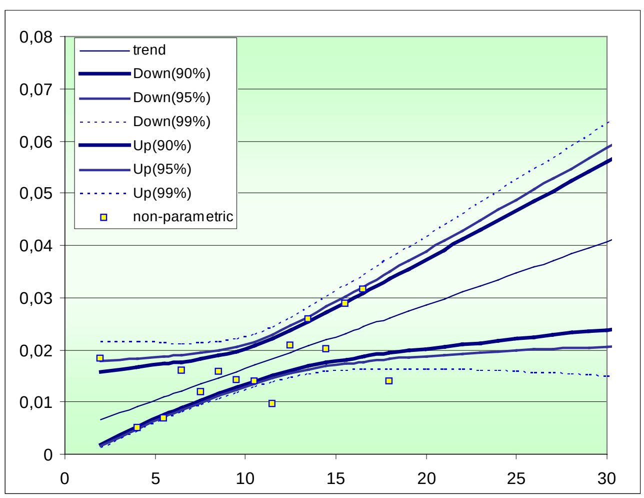

Figure 8. Component group #7.1 – linear extrapolation.

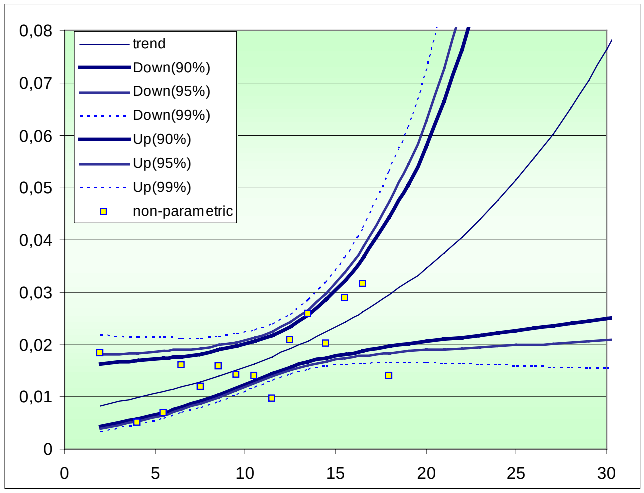

Figure 9. Component group #7.1 – log-linear extrapolation.

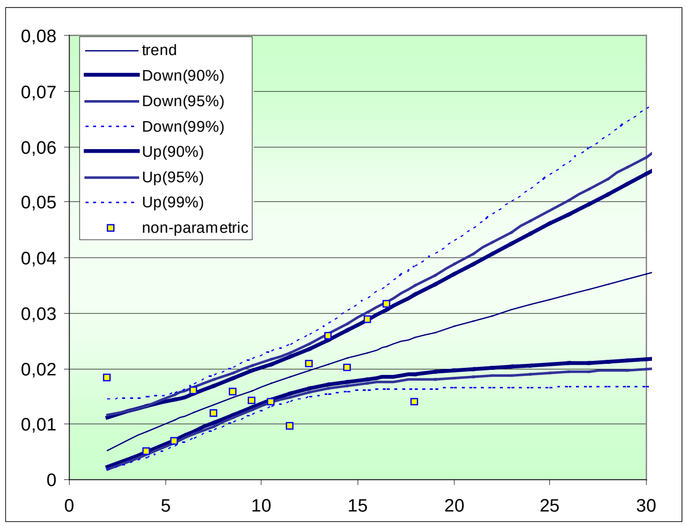

Figure 10. Component group #7.1 – Weibull extrapolation.

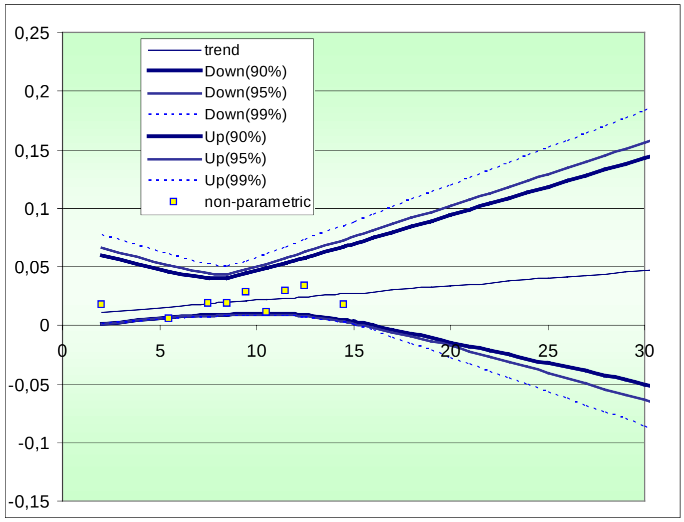

Figure 11. Component group #43.1 – linear extrapolation.

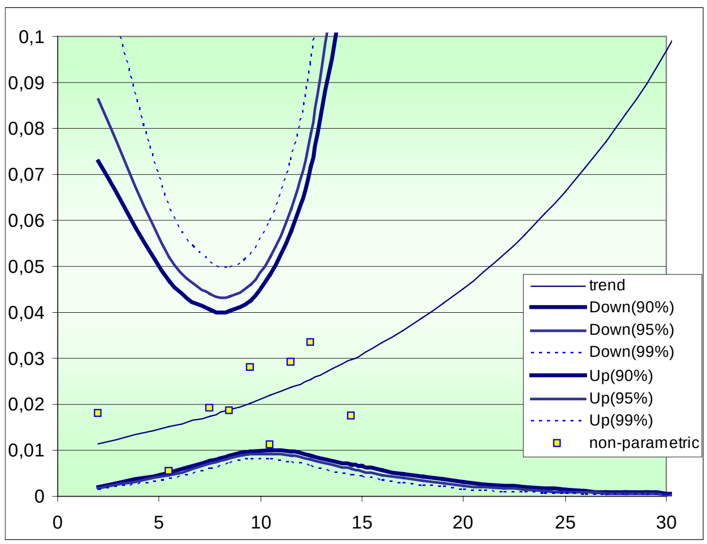

Figure 12. Component group #43.1 – log-linear extrapolation.

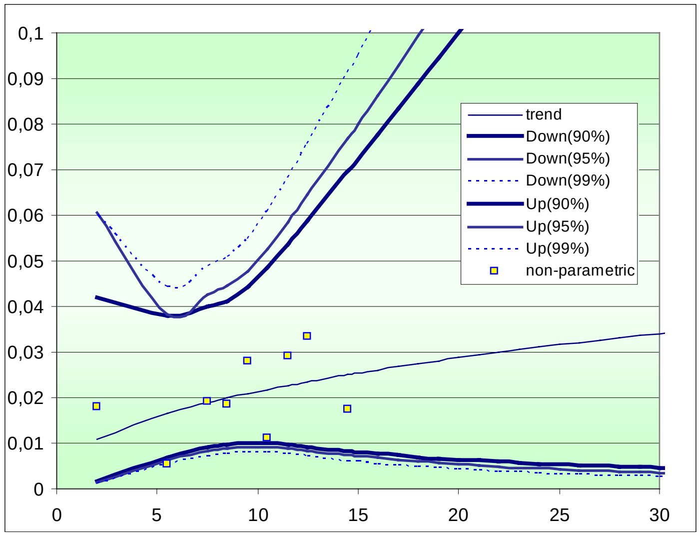

Figure 13. Component group #43.1 – Weibull extrapolation.

The following issues are open and have to be discussed :

- what model to chose for extrapolation if several time-dependent models fit well with the data,
- how to take into account the extrapolation uncertainties when introduce parameters into PSA model,
- and, what are the ways to reduce the uncertainties.

5. Conclusions and recommendations

1) Proposed approach permitted to identify the component groups with increasing failure rate and to choose best fitted reliability model. The following 10 component groups were identified as susceptible for ageing :

- #7 (best fitted model is log-linear with θ_2 = 0.055),
- #7.1 (best fitted model is log-linear with θ_2 = 0.079),
- #8.1 (best fitted model is Weibull, p = 0.1, with θ_2 = 0.31),
- #11.1 (best fitted model is Weibull with θ_2 = 0.55),
- #13.3 (best fitted model is Weibull with θ_2 = 0.47),
- #35.1 (best fitted model is log-linear, p=0.15, with θ_2 = 0.07),
- #38.1 (best fitted model is log-linear with θ_2 = 0.049),
- #47.1 (best fitted model is log-linear with θ_2 = 0.082),
- #59.1 (best fitted model is log-linear with θ_2 = 0.078),
- #59.1@WR (best fitted model is log-linear with θ_2 = 0.057).

2) In addition, for 2 component groups log-liner model with positive ageing parameter was identified as well fitted, but the value of 90% low bound of “ageing” parameter is below zero :

- #43.1 (best fitted model is log-linear with θ_2 = 0.076),
- #62.2 (best fitted model is log-linear with θ_2 = 0.033).

3) Examination of the impact of burn-in failures provided fore additional groups to the list of components susceptible for ageing :

- #32.2 (best fitted model is Weibull with θ_2 = 0.46),
- #39.1 (best fitted model is log-linear with θ_2 = 0.18),
- #45@ (best fitted model is log-linear with θ_2 = 0.0003),
- #48.3 (best fitted model is log-linear with θ_2 = 0.081).

For these gropes 90% confidence intervals for estimated parameters were not examined.

4) Consideration of burn-in failures could improve the result of goodness of fit test, as for example in case of group #39.1, and could change the conclusion about existence or absence of ageing trend, as for example in case of groups #32.2, #45@, #48.2 and #48.3.

5) The results of the calculations were compared with estimations by other parametrical methods : Bayesian analysis with non informative priors and Stochastic Expectation Maximization (SEM).
Bayesian analysis was performed with the same representation of data, i.e. binned data, when SEM calculations were done by using times to failure type data.
Bayesian approach with non informative priors provides numerical results similar (or very closes) to those obtained by frequentist analysis.

6) The SEM algorithm applied for times to failure type data provides more conservative estimation of "ageing" parameter. As a consequence, the extrapolated values of failure rates are much higher of those estimated with Chi-2 minimization approach.
One possible explanation of this difference is that times to failure data are more informative that binned one. But more detailed investigation of this issue is necessary.

7) The impact of the choice of the model to extrapolation results were analyzed. Comparison of results of extrapolation leads to the conclusion that in all of the examined cases the most conservative estimation is provided by log-linear model.
From the other side, log-linear model provides more uncertain extrapolations.

8) With regards to extrapolation of failure rate functions the following issues are open and have to be discussed :

- what model to chose for extrapolation if several time-dependent models fit well with the data,
- how to take into account the extrapolation uncertainties when introduce parameters into PSA model,
- and, what are the ways to reduce the uncertainties.

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