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An Overview of Portfolio Insurances: CPPI and CPDO

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1 Introduction

Financial institutions try to protect their portfolios against failure events and therefore they often invest in derivative instruments. Derivative instruments are a fast growing market in which new products such as Constant Proportion Portfolio Insurance (CPPI) and Constant Proportion Debt Obligation (CPDO) are created. Although these two recently developed instruments function in different ways when deciding on their investment strategy, both investment funds attempt to provide a portfolio insurance. More precisely, their strategy is to invest only a part of the capital in a risky asset and to invest the remainder in a safe way. The total value of the portfolio at each time step will influence the position taken in the risky asset. The decisions on the risk position at each time step aim to allow the investor of a CPPI or CPDO to recover, at maturity, a given percentage of their initial capital. It could happen that the promised return is not achieved. In this case, for the CPPI structure, the bank will have to cover the losses at maturity while for the CPDO structure, the CPDO will unwind and the investor will not receive the promised amount at maturity but only the remainder amount at the time of unwinding. CPPI clearly safeguards a given percentage of the invested capital for the investor, while for CPDO the investor appears to be taking a risk. In the past this risk has always been seen as very small and CPDO has been sold as very safe. Here we will study those two products in depth in order to answer the question whether CPDO and CPPI are really as safe and attractive as they seem.

We will introduce the concept of Credit Risk in the first section. This is the risk that, after agreeing on a certain contract, one of the involved parties will not fulfil its financial obligations (such as paying a premium). Often the quality and price of financial products will heavily depend on this risk. Different ways to model this risk are presented and can be used to price financial products. Next, a financial instrument called credit default swap (CDS) is introduced. This is an instrument which tries to provide a protection for credit risk by transferring it. In exchange for a predefined cost, also called "spread", a second party will cover the losses one might suffer due to credit risk. The cost for such an insurance will clearly depend on the size of the risk and the impact of the possible losses. This cost and hence the price can be determined using the above mentioned models for credit risk. There are no obligations to perform credit default swaps so their changes in credit spread can be used for speculations, for example: CDS spreads can be used as risky assets in investment structures such as CPPI and CPDO.

In the next section two different types of insurance portfolios, CPPI and OBPI (option based portfolio insurance), are compared. They are capital guarantee derivative securities

that use a dynamic trading strategy in order to incorporate the performance of a certain underlying product such as a simple stock or a CDS. They were introduced more than 10 years ago and are frequently used. Here we will study the CPPI structure in detail including a short discussion on available papers on CPPI. In order to be able to test its functionality under different conditions or constraints the OBPI is introduced briefly as an alternative portfolio insurance.

One of the recent new products based on the idea of a CPPI is the CPDO or constant proportion debt obligations, which is discussed in section 4. CPDOs are used for credit portfolios comprising exposures to credit indices such as iTraxx and CDX. The CPDO structures borrow many features from the CPPI, such as the constant proportion. The main goal of a CPDO is to produce a high-yielding product and this is achieved through a high degree of leverage. Contrary to the CPPI, leverage will be increased when the net asset value of the portfolio decreases and descends below the target amount, but leverage will be decreased when the net asset value of the portfolio increases and approaches the target. Once this target amount is reached the CPDO will completely de-leverage. In this section we will not only focus on the dynamics of the structure but we will also give a short overview of the research concentrating on this topic and discuss the question of the "safeness" of this structure as has been highlighted recently in the news.

We will conclude by discussing in more depth the differences and similarities of CPPI and CPDO. This should give an even better insight into the structure of both financial instruments.

2 Credit Risk and Credit Default Swaps

This section is intended to give a short introduction to the main financial concepts which will play a role in this work. First the concept of credit risk is introduced and different ways to model this risk are discussed. Next, a short introduction on credit derivatives, i.e. more precise credit default swaps, is provided.

2.1 What is Credit Risk?

Credit risk refers to the risk that a specified reference identity does not meet its credit obligations within a specified time horizon T (called maturity). In other words, whenever two or more parties sign an agreement there is a risk that one of them will not meet its obligations. A simple example is the case where a single person signs a loan with a bank. Here, it can happen that the person does not repay his debt according to the agreement. In such a case, where one of two parties can no longer meet its obligation, we say that

default will have occurred. The risk that such an event will happen is called credit risk. The risk will always be spread over a certain time length.

Taking a more global perspective, in finance we do not only deal with the situation of a person and a loan but it is always possible to characterize credit risk in terms of the following components: the obligor, the set of criteria defining the default, and a time interval over which the risk is spread. When, for instance, we talk about bonds, their default can be defined in several ways. It could, for example, be bankruptcy but it might also be a rating downgrade of the company or failure to pay an obligation (such as a coupon). But it could also concern the value of a firm - here a firms value is linked to the value of its financial assets. In general one will look at the asset value $V = V_t, 0 \leq t \leq T$ and default will be defined as a boundary condition on the asset value. Here we could say that default occurs if the value falls below a certain fixed level L within the time horizon. Figure 1 presents two possible paths for the firm value over time $[0, 10]$ modelled through Black-Scholes where $\mu = 0.05$, $\sigma = 0.4$ and $S_0 = 100$ as explained in Subsection 2.1.1. In the case of the blue line, the value of the firm will fall below just before $t = 5$, the lower bound, which is fixed at 20 and hence will default. In the case that the firm value follows the red path no default will occur before $T = 10$.

As one will try to protect oneself from such defaults the size of the risk will also have an effect on prices and hence the techniques for estimating the probability of default of a reference entity within time T will be very important.

In the next subsection we will briefly explain some of the possible ways in which credit risk can be modelled.

2.1.1 How to model Credit Risk?

Developing new models for credit risk is an important topic in the field of finance. This is linked to the fact that in the last couple of years the volume of instruments linked to credit risk traded on the market has increased exponentially. Besides the increase in investments there has also been interest due to the Basel II Accord which encourages financial institutions to develop methods for assessing their risk exposure.[1] Credit risk models are usually classified into two categories: intensity-based models and structural models.

[1] The Basel II Accord, issued by Basel Committee on Banking Supervision in 2004, aims to secure the financial market. The accord sets up capital requirements for financial institutions to ensure that a bank holds capital reserves appropriate to the risk the bank exposes itself to through its lending and investment practices.

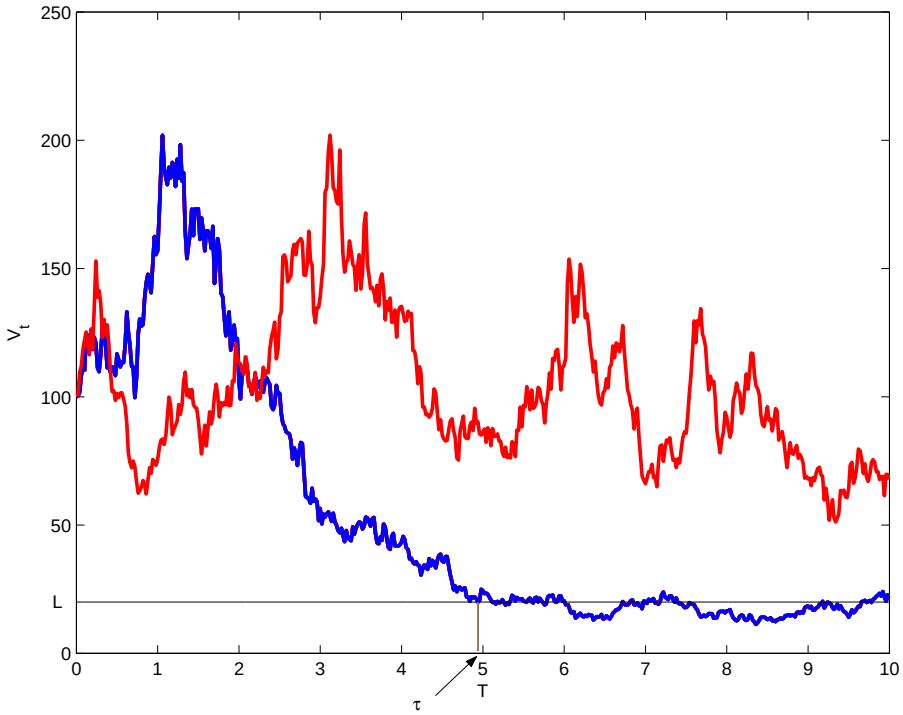

Fig. 1: *Two possible paths for a firms asset value over time with* $T = 10$. *When the blue line occurs the obligor will default (when* $V_t < L$*) in the other case the obligor will survive.*

Intensity-based models, also known as hazard rate or reduced-form models, focus directly on modelling the default probability. The main idea of these models lies in the fact that at any moment in time (as long as the contract is running) there is a probability that an obligor might default. Default is hence defined at the first jump of a counting process with a certain intensity. In practice, the models assume that the intensities of the default times follow a certain process (stochastic or deterministic) and under those conditions the underlying default model can be constructed. This intensity of the process depends heavily on the firms overall health and on the situation of the market.

The structural models, also known as firm value models, link default events to the value of the financial assets of the firm, such as in the example presented above. Credit risk will hence depend on the model used for the value of the financial assets of the firm and the criteria used for a default. In this paper we will almost always use this approach when modelling credit default.

A common way to model the time evolution of assets uses the following diffusion process:

$$dS_t = S_t(\mu dt + \sigma dW_t), \qquad S_0 > 0, \tag{1}$$

where W_t is a standard Brownian motion and μ and σ the so-called drift and volatility factors. A standard Brownian motion is defined as follows:

Definition 1. ***Brownian motion:*** *A stochastic process $W = \{W_t, t \geq 0\}$ defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ is a Brownian motion (or Wiener Process) if the following conditions hold:*

1. *$W_0 = 0$ almost surely,*
2. *the process has stationary,*
3. *the process has independent increments,*
4. *for $s < t$ the random variable $W_t - W_s$ has a normal distribution $\mathcal{N}(0, (t-s))$.*

From the above it follows that the path of a Brownian motion will be continuous. The can easily be simulated when one discretizes the time using very small steps Δt. The value of a Brownian motion at time $\{n\Delta t, n = 1, 2, \ldots\}$ is obtained by sampling a series of independent standard normal random numbers $\{\nu_n, n = 1, 2, \ldots\}$ and setting:

$$W_0 = 0, \text{ and } W_{n\Delta t} = W_{(n-1)\Delta t} + \sqrt{\Delta t}\nu_t.$$

Using the diffusion processes from 1 and the above way to model the Brownian motion, asset values over time can be modelled through

$$S_t = S_0 \exp((\mu - \sigma^2/2)t + \sigma W_t). \tag{2}$$

The above way to model price changes of financial products is also referred to as the Black-Scholes model. The main advantages of using the Black-Scholes formula for modelling are that it is easy to understand its background and modelling price changes of assets and derivatives is not time-consuming. A drawback is that it assumes normality of the log returns of the financial assets, which is often not true in reality. Moreover, the Black-Scholes model does not capture the possibility of sudden jumps which do occur in real live and often cause extra credit risk. Hence a more flexible stochastic process is needed to model reality in a better way.

It would be good to keep some properties of the Brownian motion such as independence and stationarity of the increments but to drop the constraints of normality and continuity of the paths. To create such a process we will restrict ourselves to the group of infinitely divisible distributions.[2] For each infinitely divisible distribution (with characteristic function $\phi(u)$) a stochastic process can be defined which starts at zero and has independent and stationary increments such that the distribution of the increments over $[s, s+t], s, t \geq 0$ has $(\phi(u))^t$ as characteristic function. Such processes are called Lévy processes, in honour of Paul Lévy, a pioneer of the theory.

Definition 2. ***Lévy process:*** *A cadlag*[3] *stochastic process* $X = \{X_t, t \geq 0\}$ *defined on a probability space* $(\Omega, \mathcal{F}, \mathbb{P})$ *is a Lévy process if the following conditions hold:*

1. X_t *is a continuous process* $\mathbb{P}$*-almost surely:*

$$\forall \varepsilon > 0 \quad \lim_{h \to 0} \mathbb{P}(|X_{t+h} - X_t| \geq \varepsilon) = 0,$$

2. $X_0 = 0$,

3. *the process has stationary increments,*

4. *the process has independent increments.*

[2] Suppose $\phi(u)$ is the characteristic function of a random variable X. If for every positive integer n, $\phi(u)$ is also the n-th power of a characteristic function, we say that the distribution is infinitely divisible.

[3] A function $f : [0, T] \to \mathbb{R}^d$ is said to be cadlag if it is right-continuous with left limit. the name derives from the French *continue à droit et limites à gauche*.

Lévy processes became very popular and are still more and more used in practice. An in-depth study of Lévy processes in finance can be found in Schoutens (2003). Examples of Lévy processes are the (Compound) Poisson process, the Gamma process, the inverse Gaussian process and the Variance Gamma process (VG).

We will describe the Variance Gamma process, introduced in the financial literature by Madan and Seneta (1990), a bit more in de detail as it will be used in the further of this paper. We start from the characteristic function of Variance Gamma (VG(σ, ν, θ)) distribution which is defined as follows:

$$\phi_{VG}(u; \sigma, \nu, \theta) = (1 - iu\theta\nu + \frac{1}{2}\sigma^2\nu u^2)^{-1/\nu}.$$

This distribution is infinitely divisible and we can define the VG process $X^{(VG)} = \{X_t^{(VG)}, t \geq 0\}$ with parameters $\sigma, \nu > 0$ and θ as a process which starts at zero, has stationary and independent increments and for which the increments $X_{s+t}^{(VG)} - X_s^{(VG)}$ follow a $VG(\sigma\sqrt{t}, \nu/t, t\theta)$ distribution of over the time $[s, t+s]$.

Another way of constructing the VG process is by the technique of time changing. Here we will start from a Gamma process. Recall that the density function of a Gamma(a, b) distribution is given by

$$f_{Gamma}(x; a, b) = \frac{b^a}{\Gamma(a)} x^{a-1} \exp(-xb), \quad x > 0,$$

where $\Gamma(.)$ is the Gamma function. It can be shown that the distribution is infinitely divisible and hence using the Gamma distribution one can build a process with independent and stationary Gamma increments. If we define $G = \{G_t, t \geq 0\}$ as a Gamma process with parameters $a = b = 1/\nu$, we have that G_t will follow a Gamma(at, b) distribution and $E(G_t) = t$. The VG (σ, ν, θ) process can also be defined as

$$X_t^{(VG)} = \theta G_t + \sigma W_{G_t}, t \geq 0, \tag{3}$$

where $W = \{W_t, t \geq 0\}$ is a standard Brownian motion independent from the Gamma process. For simulation reasons, a sample path of the VG processes can thus be obtained by sampling a standard Brownian motion and a Gamma process. The Gamma process can easily, like the Brownian motion, be simulated at time points $\{n\Delta t, n = 1, 2, \ldots\}$ with Δt small. First generate independent Gamma $(a\Delta t, b)$ random numbers $\{g_n, n = 1, 2, \ldots\}$. Then the Gamma process can be constructed by

$$G_0 = 0, \text{ and } G_{n\Delta t} = G_{(n-1)\Delta t} + g_n, \quad n \geq 1.$$

Similarly to Equation 2 for the Brownian motion, the value over time of an asset using the dynamics of a Variance Gamma process can now be modelled by:

$$S_t = S_0 \exp(\omega t + \theta G_t + \sigma W_{G_t}), \tag{4}$$

with $\omega = \nu^{-1} \log(1 - \sigma^2 \nu/2 - \theta\nu)$.

Both models will make it possible to generate a sample path for the price changes of financial assets over time and hence to model the connected credit risk.

In the next section we will define an important group of credit derivatives which are often used in practice. Generating a sample path of their prices can be done using one of the above models for the underlying asset(s).

2.2 Credit Default Swaps (CDS)

Credit Default Swaps (CDS) are very simple credit derivatives and have a big share in the market of credit derivatives. Credit derivatives can be defined as the group of all derivatives whose payoffs are affected by the default of a specified reference entity (or a basket of entities). They are often used to hedge, transfer or manage risk and can hence be considered as an insurance against default. The main idea of credit derivatives is that credit risk is transferred without reallocating the ownership of the underlying asset(s). This way they provide a certain protection against decreasing solvency or default of the underlying asset(s).

CDS in particular are designed to isolate the risk of default on a credit obligation. The protection buyer transfers the credit risk of a reference identity to the protection seller for a fixed time T. In exchange for this shift of risk, the protection buyer will make predetermined payments to the protection seller. These payments will occur in a continuous way until the end of the contract (the time of maturity) T unless a default occurs before the time to maturity. If default of the reference entity occurs, the protection seller will cover the losses (or part of the losses) of the protection buyer due to the default of the underlying entity and the contract will be terminated. The yearly rate paid by the protection buyer to enter a CDS contract against failure is called the CDS spread.[4] The amount of the spread will reflect the riskiness of the underlying credit, if the probability of default increases also the cost of the CDS (the spread) will increase. There is no requirement to actually hold any asset or suffer a loss, hence credit default swaps can also be used to speculate on changes in credit spread.

For a simple example, Figure 2 presents the cash flows for two possible scenarios (default at time $t = 7$ or no default). Here we consider the case where a person owns a zero-coupon defaultable bond of a company with a face value $F = 10.000$ Euro and

[4] Spreads are often quantified in bp; bp stands for "basispoint" and is equal to 0.01%.

maturity $T = 10$ years. Suppose that this person would like to cover himself against the possible default of the bond. He can buy this protection by entering into a CDS contract. A possible situation would be that the contract requests an annual payment of an amount of $400bp$ from the protection buyer to be protected against default. In return, the protection seller will cover the loss which might result from defaulting. The amount of the loss will be equal to the difference between F and the recovery value after default. We hence take into account that in case of default the total amount will not automatically be lost but that the value might be partially recovered. The concept of recovery can be understood through the following example. In the case that a company goes bankrupt, there are creditors claiming against the assets of the company, and the owner of the bond is one of those creditors. The assets are sold by a liquidator and the profits are used to meet the claims as far as possible. Historically values of the recovery rate fall between 20% and 50%. For the current example we assume that the recovery rate will be equal to $R = 40\%$.

The annual amount paid by the protection buyer in this example is hence equal to 400bp·10.000 = 400 Euro and the payment of the protection seller in case of default will be $F(1 - R) = 6.000$ Euro. In Figure 2 it is assumed that in the second scenario the bond defaulted at the beginning of the seventh year.

In practice, CDS are not only used to reallocate the risk of a single asset but a basket of assets might be considered. A credit default swap index is a credit derivative used to hedge credit risk or to take a position on a basket of credit entities. There are currently two main families of CDS indices: CDX en iTraxx. CDX indices contain North American and Emerging Market companies and are administered by CDS Index Company (CDSIndexCo) and marketed by Markit Group, and iTraxx contain companies from the rest of the world and are managed by the International Index Company (IIC).

The most widely traded of the indices is the iTraxx Europe index composed of the most liquid 125 CDS referencing European investment grade credits, subject to certain sector rules as determined by the IIC and also as determined by the SEC. There are also significant volumes, in nominal values, of trading in the HiVol and Crossover indices. HiVol is a subset of the main index consisting of what are seen as the most risky 30 constituents at the time the index is constructed. Crossover is constructed in a similar way but is composed of 45 sub-investment grade credits. A new series of CDS indices is issued every six months by Markit and IIC.

As there exist no requirements to hold CDSs and their indexes, they are not only used as insurance against risk but are also traded on the market in a speculative way. As a

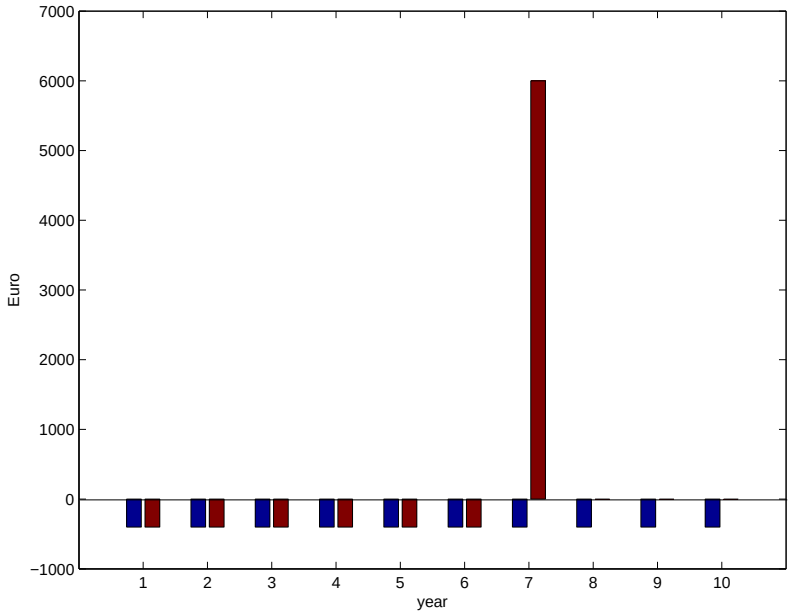

Fig. 2: *Cash flows from the protection buyer for a 10 year CDS. Blue: cash flows in case no default occurs. Red: cash flows in case there is a default at time $t = 7$.*

consequence pricing of the CDSs is a popular subject for research (see e.g. Cariboni, 2007).

3 Portfolio Insurances

Portfolio insurances are capital guarantee derivative securities that embed a dynamic trading strategy in order to make a contribution to the performance of a certain underlying assets. Two different types of portfolio insurances are considered here. First, the constant proportion portfolio insurance (e.g. Overhaus et al., 2007) and, second, the option-based portfolio insurance. Both invest partially in a risk-free way and combine this with a risky asset.

3.1 Constant Proportion Portfolio Insurance (CPPI)

The family of constant proportion portfolio insurance consists of investments for which the amount necessary for guaranteeing a repayment of a fixed amount N at maturity T is invested in a risk-free way, typically a bond, B, and only the exceeding amount will be invested in one or more risky assets, S_i. This way an investor can limit its downside risks and maintain some upside potential. This type of portfolio insurance has first been

introduced by Black and Jones (1987) and Perold (1986).

The product manager will take larger risks when the market is performing well. But if the market is going down he will reduce the risk rapidly. The following factors play a key role in the risk strategies an investor will take:

- Price: The current value of the CPPI. The value at time $t \in [0, T]$ will be denoted as V_t.
- Floor: The reference level to which the CPPI is compared. This level will guarantee the possibility of repaying the fixed amount N at maturity T, hence it could be seen as the present value of N at maturity. Typically this is a zero-coupon bond and its price at time t will be denoted as B_t.
- Cushion: The cushion is defined as the difference between the price and the floor,

$$\text{Cushion} = \text{Price} - \text{Floor}.$$

- Cushion % = Cushion/Price.
- Multiplier: The multiplier is a fixed value which represents the amount of leverage an investor is willing to take.
- Investment level: is the percentage invested in the risky asset portfolio; this also known as the exposure and is for each step fixed at:

$$e = \text{Multiplier} \times \text{Cushion}\%.$$

- "gap" risk: is the probability that the CPPI value will fall under the Floor, see e.g. Cont and Tankov (2007).

The level of risk an investor will take is equal to the investment level as long as the value of the CPPI exceeds the floor. For any time t the future investment decision will be made according to the following rule:

- if $V_t \leq \text{Floor} = B_t$, we will invest the complete portfolio in a into the zero-coupon bond,
- if $V_t > \text{Floor}$, we will invest an amount equal to e in the risky asset portfolio.

It can easily be shown that under the assumption that underlying stock will follow a Black-Scholes model with continuous trading, there is no risk of going below the floor and that the expected return at maturity of the CPPI is equal to (e.g. Cont and Tankov, 2007)

$$E(V_T) = N + (V_0 - Ne^{-rT}) \exp(rT + m(\mu - r)T).$$

In practice, however, it is known that the probability of going below the floor is non-zero. It might, for instance, happen that during a sudden downside move, the fund manager might not be able to adjust the portfolio in time, which then crashes below zero. In the case of an event where the actual portfolio value falls under the floor, at maturity the manager will have to cover the difference between the portfolio value and the guaranteed amount N. It is therefore of importance for the issuer of a CPPI note to be able to quantify this risk, also called "gap risk."

We will present an example of a possible cash flow for a CPPI with maturity T equal to 10 years. For the sake of simplicity we will consider only one risky asset with prices S_t and a risk-free asset, a zero-coupon bond B_t with a constant interest rate $r = 5\%$. We also assume that the initial price of the asset is equal to $S_0 = 100$ and the prices over time will be modelled using a Variance Gamma model as presented in Equation 4 with parameters $\sigma = 0.5$, $\nu = 0.25$ and $\theta = 0.026$. For the CPPI process the leverage or multiplier is fixed at 2.5 and the starting capital is 100. We also consider that the CPPI at maturity repays the investor with at least the initial capital. Figure 3 and Figure 4 present two examples of possible scenarios for the simple CPPI. In the first example the value of the CPPI will always stay above the floor, while in the second example at time τ a sudden drop of the risky asset will result in a CPPI value below the floor which is the gap risk.

In this example, as the repayment at maturity of the initial value should be insured, the floor will be $100 \exp(-r(10-t))$ at each time t. For each step the value of the cushion is calculated and the portfolio is re-balanced according to the risk exposure. The re-balancing is such that the bigger the difference between the CPPI value and the floor, the higher the cushion value and the more risk one will take. The process will stop once a drop of the asset value occurs of such a level that the CPPI value falls below the floor. If such a drop happens the product manager will put the risk exposure to zero and only invest in a risk-free way until maturity.

3.1.1 Recent Developments for CPPI

CPPIs receive a lot of attention, not only from banks but also from academia. Three main topics of research can be identified: limiting the risk, building insurances for this risk and the behaviour of CPPI.

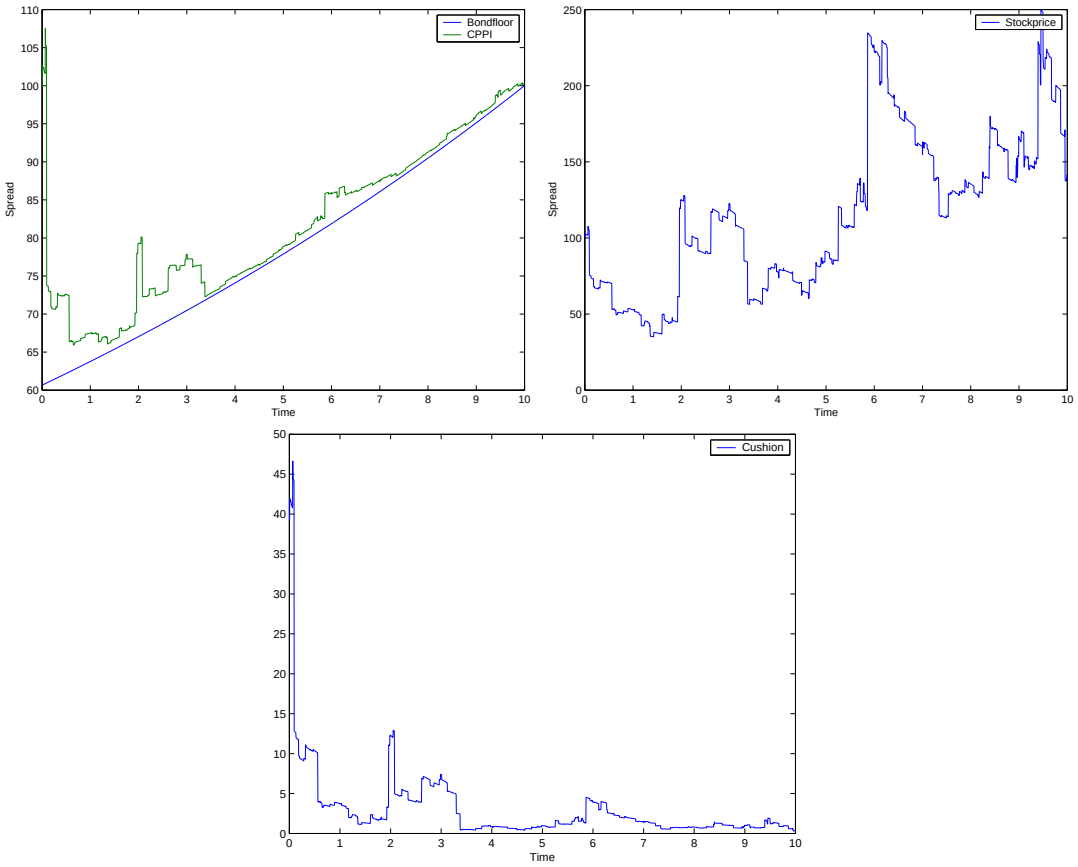

Fig. 3: *top left: CPPI performance, top right: value of the risky asset, bottom: cushion.*

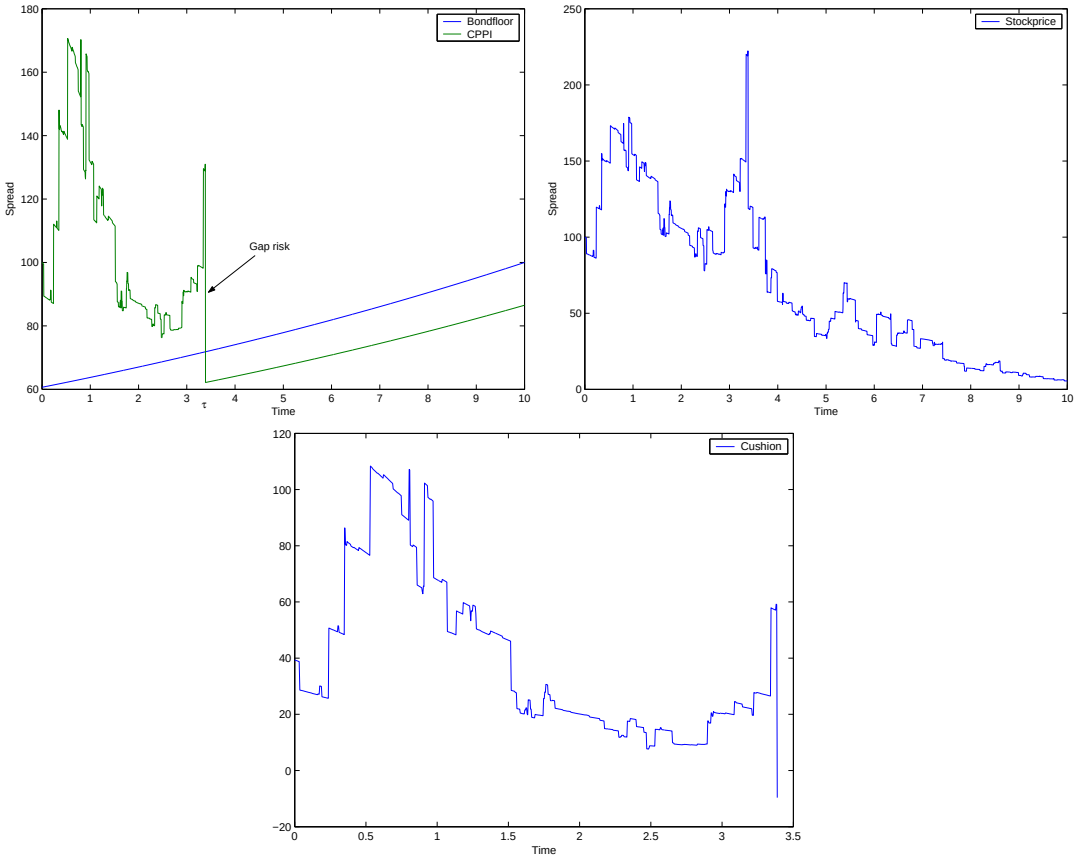

Fig. 4: *top left: CPPI performance, top right: value of the risky asset, bottom: cushion.*

The first group of papers try to measure the risk factors involved. For example, in Bertrand and Prigent (2002) an upper bound for the multiplier m is sought in such a way that the investment in the risky portfolio is maximized under the condition that the gap risk must stay under a certain limit. While Brun and Prigneaux (2007) presents an extended way to calculate the VaR and GVaR of the CPPI portfolio.

The next group of papers concentrate on extending the CPPI in such a way that an insurance against the small but existing gap risk is build in. The price and size of such an extra insurance will depend on the probability of hitting the floor and hence ways to quantify this risk are discussed in this group. Examples are Prigent and Tahar (2005) and Cont and Tankov (2007).

The last group of papers study in detail the behaviour of CPPI strategies under specific conditions for the underlying portfolio, such as Bertrand and Prigent (2003) and Garcia et al. (2007). In this last article, A dynamic Lèvy model, more precise a Multivariate Variance Gamma (MVG) model, is set up for a series of correlated spreads. As this jump diffusion model can generate spreads in a very fast way, it has been applied in order to price different exotic structures such as the CPPI. In this paper it is considered that the underlying risky asset of the CPPI fund is an index or a basket of indices.

3.2 Option Based Portfolio Insurance (OBPI)

Besides the CPPI this strategy of insuring a pay-off of a portfolio is also popular. The OBPI, introduced by Leland and Rubinstein (1976), consists essentially in buying simultaneously a risky asset S (usually a financial index such as the S&P) and a put option[5] written on it. Investing this way, independently of the value of S at maturity date T, the OBPI portfolio value will always be greater than the strike K of the put. Hence a pay-off value of K can be guaranteed. It might seem that the goal of the OBPI method is to guarantee a fixed amount only at the terminal date but in fact it can be shown that the OBPI method allows one to get a portfolio insurance at any time.

Note that the OBPI has just one parameter, the strike K of the put while the CPPI method is based on the choice of two parameters: the initial floor F_0 and the multiplier m. The strike K will therefore play the same role as $F_0 e^{rT}$ in the CPPI model.

In this paper we will not discuss the OBPI further as the OBPI will only be used as a tool to discuss the CPPI performance.

[5] A European put option gives someone the right but not the obligation to sell an underlying stock (with value S_t at time t) at maturity time T at a fixed value K (called strike price) instead of the actual value.

4 A New Financial Instrument: Constant Proportion Debt Obligations (CDPO)

Constant Proportion Debt Obligations (CPDO) first appeared in August 2006 and are a variation on the CPPI structure. They are used for credit portfolios comprising exposures to credit indices such as CDX and iTraxx. The CPDO's risk exposure, just as with the CPPI, is determined using a constant proportion approach and rebalances its portfolio between the credit portfolio and a safe asset. The CPDO structure does this with the aim of producing a high-yielding "AAA" rated product.[6]

A CPDO funds itself through the issuance of long-term debt paying timely coupon and principal on the notes. The promised coupon is a spead above LIBOR. [7] The high coupon together with the high rating have made CPDOs very popular products.

First, we will try to explain the structure in detail. Next, a short overview of research currently carried out in practice is given. Besides research papers, CPDOs have also been discussed within the news and a summary of this is provided in the third section. The final section studies an example in more detail.

4.1 The Structure

Constant proportion debt obligations are structures which use, as suggested by their name, a constant proportion approach for their risk exposure and re-balance their portfolio at every time step between the credit portfolio and a safe asset. The CPDO structure takes leveraged exposure to a risky asset by selling protection on individual names or indices (CDS or indices on CDS). The risky exposure ensures that there is enough spread to meet the promised liabilities and also covers the costs and potential losses that the transaction will absorb.

Risk will be taking in function of the value of the CPDO. If the structure is not performing well, the structure will increase its risk exposure (up to a pre-defined maximum leverage level) in order to allow for recovery from the negative performance by increasing the income from the risky asset to rebuild the portfolio's value. The following factors play a key role in the risk strategies an investor will take:

[6] In practice rating is used to classify different financial products according to their underlying risk of default. Rating agencies such as Moody's and Standard & Poor's evaluate the creditworthiness of companies and related financial instruments. Ratings are denoted with letters: while AAA stands for very safe, a rating of B3 is not safe (it refers to a default probability of around 12%).

[7] LIBOR refers to the 3 month London Interbank Offered Rates and are often used as a reference rate for short term interest rates

- Net Asset Value (NAV): This is the current value of the CPDO. It will be the sum of the safe investment and the market value of the risky portfolio.

- Riskfree: The amount of the total CPDO value invested in a risk free way.

- PV(liabilities): The present value of the current liabilities will be the sum of all discounted coupons still to be paid and the discounted value of the final principal amount.

- Shortfall: The shortfall is defined as the difference between the present value of the liabilities and the net asset value,

$$\text{Shortfall} = PV(\text{liabilities}) - NAV.$$

- CDS premium: The Premium is the amount to be paid to the protection seller (such as a CDS or CDS index) in exchange for the insurance. An increasing value of the spread refers to an increased default probability of the underlying asset and hence it will be a negative sign as a default will lead to a decreasing NAV. Conversely, an increasing spread results in an higher income.

- PV(CDS premium): The present value of all future premium payments up to maturity.

- Leverage: The leverage refers to the degree of risk which will be taken at each time step. A maximum level is often fixed at 15× and is defined as

$$\text{Leverage} = \min(\beta \frac{\text{Shortfall}}{PV(\text{CDS premium})}, \max(\text{Leverage})), \tag{5}$$

  where β is a multiplier. Similar as in Standard&Poor's (2007)(see pg 18) in this document β will be fixed at 1/riskfree.

- Cash-in: In case the NAV is equal or exceeds the target value (PV(liabilities)) the necessary amount to cover all future liabilities is reached and hence the risky exposure and leverage will be put to zero. From this point onwards the NAV will be completely invested at the risk-free rate, with coupon and fees being paid until maturity.

- Cash-out: if there are substantial losses and the NAV falls below a certain threshold (often fixed at 10% of the initial investment), it will be said that cash-out has occurred. In such a situation the CPDO will unwind and the investor will receive the remaining value.

The investment strategy follows the following steps. At every time-step one should check the discounted value of the future obligations (coupon and principal payment), which is the amount which one tries to reach.

Next, as the CPDO exposes itself to risk by selling protection (CDSs or CDS indices), the market-to-market of this risky investment is checked and compared to the price paid on the previous time step for this investment. An increase in the spread of the risky CDS means that the underlying insurance has become more expensive and hence it will be more likely that the seller of the protection will have to cover a future loss. Compared to the previous time-step this can be considered as a loss for the protection seller, as he appears to be underpaid for the protection it provides, and a gain for the protection buyer.

Besides this gain or loss linked to the protection spread, the costs (or incomes) related to the protection also need to be taken into account. Those costs are referred to as the fee which is equal to the sum of all the CDS premium payments made by the protection buyer for the insurance within the time step, taking into consideration the possibility of a default event.

The total value of the CPDO (NAV) at time $t+1$ will hence be equal at the accumulated value of the cash, invested risk free at time t, plus the value of the risky asset at time $t+1$ augmented with the gains (or losses) made by investing and the fees collected (or paid) for the insurance in the time period [t,t+1]. From this value we should subtract the coupon payments which need to be made to the CPDO investor.

Based on the above value, and before going to the next step, a decision is made on the leverage and according to the leverage a new investment is made in the risky asset. If the NAV increases, the shortfall decreases, and hence the leverage will go down; while if the shortfall increases the leverage will go up in order to try to fix the previous negative performance.

The above steps are repeated at each time step unit cash-in or cash-out occurs. Cash-in occurs when the total value of the CPDO reaches or exceeds the current value of the future obligations. In this case, the seller of a CPDO is sure to be able to fulfil all its future obligations and will from then on only invest in a risk free way. The probability for a CPDO that such an event takes place will have a big impact on its rating.

Cash-out occurs when the value of the CPDO hits or shoots below a predefined lower bound which is fixed in the beginning. If this happens, the CPDO will unwind and the investor will receive the remaining proceed. Such an event could be called the gap risk and is comparable to the gap risk of a CPPI. The risk of a cash-out event cannot be excluded, since in case of under-performance more risk will be taken, which increases even more the

possibility of arriving below the lower bound when a downward jump occurs.

As an example we take a similar situation as in Section 3 for Figures 3 and 4. For the risky asset a CDS-index is considered with starting spread $S_0 = 100$, and the spreads follow the same Variance Gamma model as before. The risk-free interest rate is fixed at $r = 5\%$. For the CPDO structure the coupon payment is fixed at $r+2\%$ and the maximum leverage is equal to $15\times$. In this example a cash-out level of 10% of the initial investment is considered. Figures 5 and Figure 6 present two examples of possible scenarios of a CPDO. In the first example, the target level is reached before maturity at time t around 8 from that point onwards all cash will be invested in a risk-free way. For the second example, the CPDO value will drop below the cash-out level just before time $t = 5$, which is similar to what the CPPI case referred to as a gap. If such an event occurs the CPDO will unwind and the investor will receive all remaining cash.

In practice risk positions will almost always be taken into CDS indices (iTraxx and CDX). The benefits of this choice are summarized on slide 10 of ABN-AMRO (2007). However, questions about safety in the sense of the correctness of the high triple-A rating do remain as is discussed in the next two sections.

4.2 Some Results from the Literature

CPDOs are very attractive as they appear very safe, thus their effects are a popular topic for research. As the structure is rather new, up until now most of the reports still concentrate on the dynamics of the CPDO structure. In this section an overview of the most important publications is provided.

Standard&Poor's (2007) discuss principally their modelling framework and criteria for rating CPDO structures. A detailed explanation of the mechanism of the modelling algorithm is provided and it is assumed that the prices of the underlying credit portfolio are modelled in a Gaussian copula framework. The article does not contain any concrete rating examples and only provides the detailed path of one single run.

UBS is the leading global wealth manager and one of the largest global asset managers, hence the dynamics of the CPDO have been studied in depth by UBS. In their first research paper on CPDOs of 2006 they consider a CPDO which looks like a BBB+ asset (which means that it has a chance of being fully paid out of 96%). Based on this fact it is considered that the outcome can be compared with a mezzanine 6-9%. Using Monte Carlo simulations they study the time to cash-in. The spreads seem to be one of the factors which play a key role on the CPDO value. For this reason the effect of the spreads on the value is studied in more detail. It is finally concluded that in their opinion CPDOs

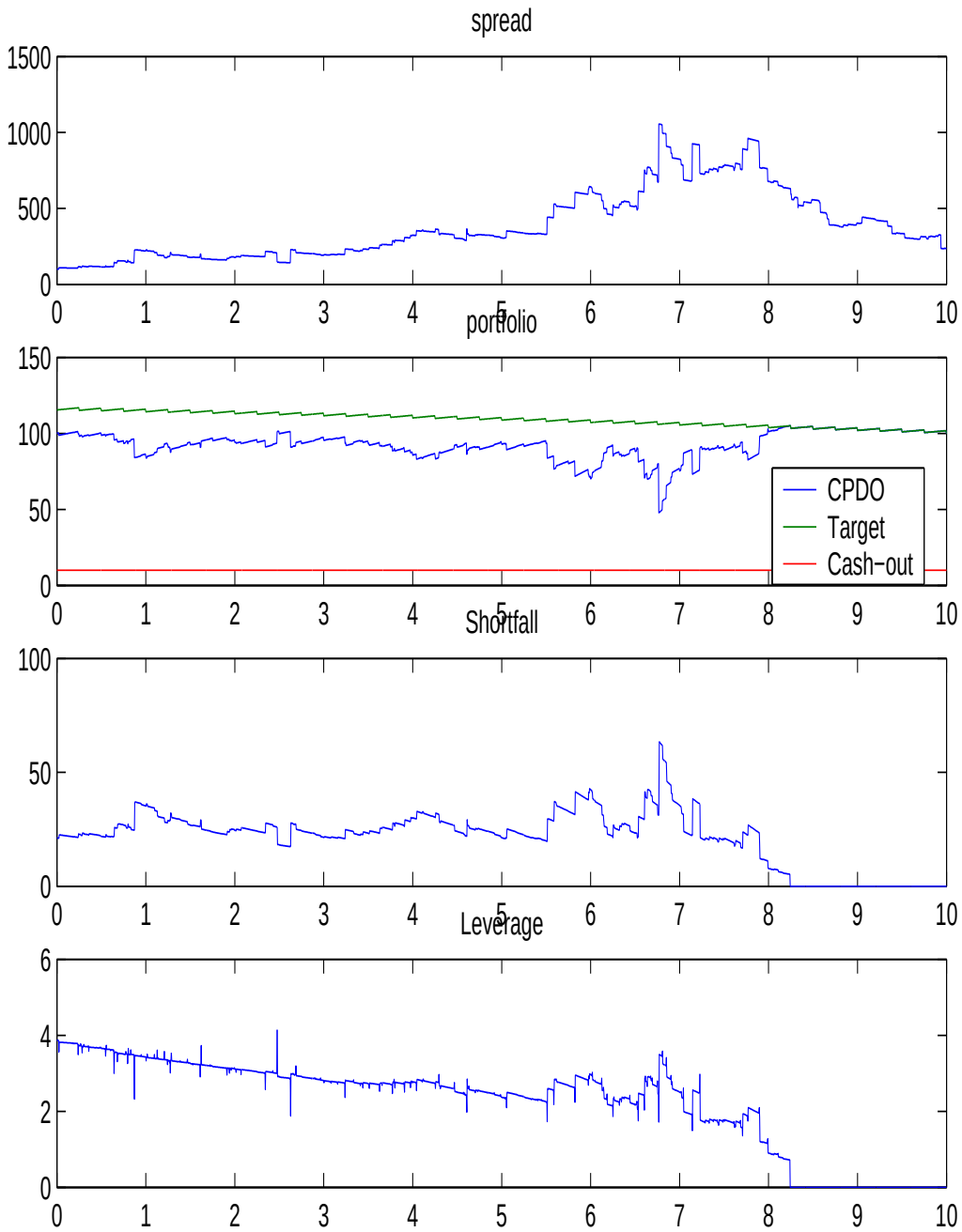

Fig. 5: *first graph: value of the risky asset (CDS index); second: the corresponding CPDO performance in case of a cash-in situation; third: shortfall at each time step; final: leverage taken at each time.*

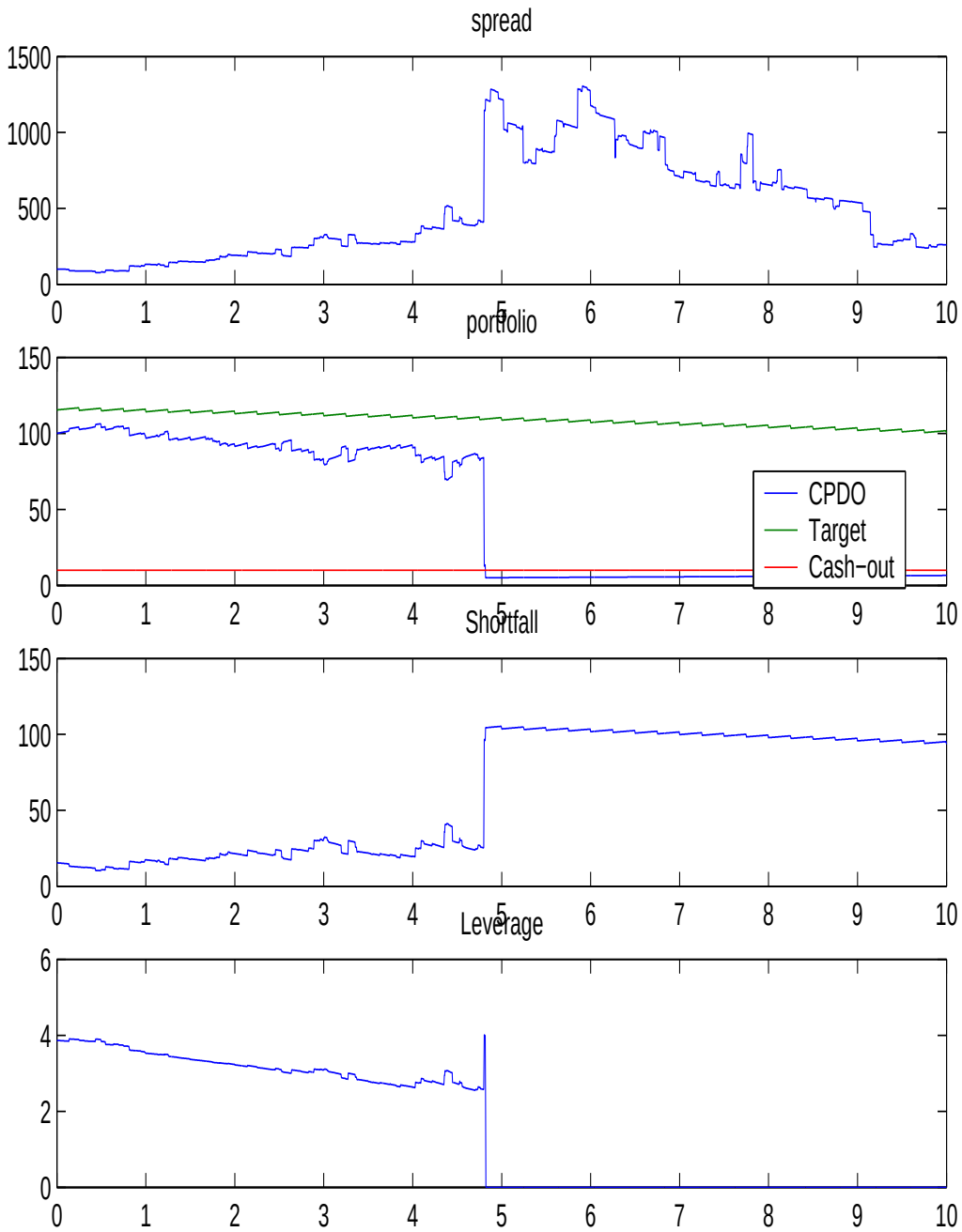

Fig. 6: *first graph: value of the risky asset (CDS index); second: the corresponding CPDO performance in case of a cash-out situation; third: shortfall at each time step; final:leverage taken at each time.*

are a welcome additional structure.

In February 2007 UBS CDO Insight published a CPDO Primer. The document describes the CPDO structure and its risks by explaining in detail the initial flows, the revenue and expense items, and the operating rules. Next, an analysis of the market risk and the time to cash-in has been performed. The authors refer to a figure, produced by Moody's, where the probability to cash-in appears to be 100%. Cashing-in almost always occurs here within 8 years and hence the AAA-rating for such CPDOs is defended. However, the authors admit that the analysis depends on many risk assumptions and may therefore be far from reality. As a last step the authors also perform a scenario analysis using Monte Carlo simulations in order to assess the reaction of the CPDO on the different risk factors. As a result it is concluded that the CPDO structure really does offer a certain protection. The level of the protection is, however, put up for discussion.

Another document on CPDOs was published by UBS in April 2007. Here a simplified model of the CPDO is introduced which has the virtue of reducing itself to a closed form formula. This simplification allows the authors to give an idea of the likelihood of a downgrade and the size of impact an active manager can have on a CPDO strategy. One of the simplification assumptions is that the model assumes normality of the returns of the credit investment. As result, cash-out is only observed in 0.31% of the cases which would lead to a Aa3 rating and an average cash-in time of 5.1 years is observed. Using this probability of cash-out and the expected price in case of cash-out and cash-in, a fair value can be given to the CPDO. Besides the calculation of the fair value, the simplified model also allows the authors to study the rating dynamics in depth.

The main difference in approach with the document published by Fitch (see Linden et al. 2007) is the way the credit spreads are modelled. In this paper, the authors claim that the choice of the model will have a significant impact on the probability of failure events in the CPDO model and hence the main focus of the article is put on the model and its corresponding parameters. Connected to this a sensitivity analysis is conducted and the robustness of the CPDO structure is tested.

ABM-AMRO has also published some documents regarding this new structure. In their presentation, ABN-AMRO (2007), they mainly describe the dynamics of the CPDO. They introduce the so-called surf step-up CPDO where the maximum leverage is determined based on the level of the index spreads: the higher the spread, the higher the maximum leverage. The presentation concludes with publicity for one of their products, namely Degas. The product offers tranches with have received Aaa and Aa2 ratings from Moody's.

As a final document we would like to mention the presentation Cont and Jessen (2008).

Here the dynamics of the CPDO are studied assuming a 1-factor top-down model for portfolio default intensity. The spreads will be linked to the time evolution of the default intensity of the in the underlying portfolio. The spreads will go up if the default intensity goes up. Next, these settings are used in a simulation study where it appears that if the probability of default within the underlying portfolio (within 10 years) is put equal to 2.5% the CPDO will cash-out with a probability of 0.7%. Furthermore a simple sensitivity analysis is performed.

As can be concluded, most of the research is still concentrated on understanding the dynamics of the CPDO in detail and on correct rating of the product. It should be noted that the above documents only give an overview of some of the reports published on CPDOs and should not be considered as complete, for example JP Morgan has also done research in the field of CPDOs (see e.g. Saltuk and Goulden 2007).

4.3 CPDOs in the Spotlight

Recently it has became clear that CPDOs are not as safe as is often thought. In real life cash-out events have occurred and these events have also received attention in the media. As a result the safety of CPDOs has been put up for discussion.

The article by Reuters (Richard Barley) of November 16th, 2007, discusses the fact that Moody's Investors Service downgraded its ratings on six CPDOs, one of which even to a "junk" rating of Ba2. The downgrading was done because of the continuing spread widening on the financial names underlying these CPDOs.

Soon after this article, on November 28th 2007, the first CPDO unwinding was announced. This unwinding shows the controversial credit product's potential for volatility, and moreover it has raised the question of whether the probability they will pay off is as high as the 99-plus percent implied in a triple-A rating.

And also more recently, on January 25th, 2008 Reuters (Karen Brettell and Jane Baird) published "UPDATE 1-More CPDOs liquidate, ratings cut-Moody's". In the article it is mentioned that Moody's Investors Service confirms that two more series of notes from structured deals backed by financial companies were liquidated after losing investors approximately 90 percent of their investment. Besides the two unwindings it also discusses a list of downgrading which have occurred.

The above mentioned articles only give a snapshot of the situation. A quick search on the web reveals that many more cases of downgrading and even unwinding have taken place in the last year.

5 Comparison between CPPI and CPDO

When CPDOs were created, they were considered as a variation of the CPPI. They borrow certain features such as a "constant proportion" approach to determining leverage and the re-balancing of the portfolio between the credit portfolio and the safe asset.

On the other hand, they are also very different. A CPPI will, at maturity, irrespective of the performance of the risky asset, receive the principal, together with any positive return generated from the risky asset. In case of loss, when the CPPI portfolio falls below the floor, the losses are covered by the seller of a CPPI so that the investor will still receive the principal. An investor can hence always be sure of receiving the principal and, in the case of good performance, even more. For the CPDO, on the other hand, a target value is aimed for and in the case of a well performing risky asset a cash-in event will occur and the investor will receive all promised coupon and principal payments. But when the risky asset does not perform well and a cash-out event occurs, the CPDO will unwind before maturity. In such a situation only the remaining amount will be paid out to the investor. Investors will want to know the amount of risk by investing in a CPDO and they will use its rating as an indication. This way, the rating of a CPDO becomes important and will have an effect on the price of a CPDO. In practice the risk of a cash-out event is often under-valued, which leads to a CPDO price higher than its correct value.

Besides the difference in outcome, different investment strategies are also used in order to realize the outcome. Initially, the CPDO value will be below the target value, while the CPPI manager tends to invest only the amount exceeding the floor which is needed to make the principal payment at maturity in a risky way. Once decisions on the risky exposure need to be taken the idea is that a CPDO investor will increase its risk as it is performing negatively while the CPPI will decrease its risk as it is not performing well and approaches the floor level. In other words, at each time step the CPPI investor takes risk exposure positions based on the amount of surplus the portfolio value has with respect to the floor value. The CPDO investor, on the other hand, will at each time take risk exposure proportional to the amount the CPDO portfolio is lacking in order to reach the target value. Once the CPDO value reaches the target value the manager will stop investing in a risky way as there is sufficient capital to pay out all future liabilities and there is no more need to create capital in a risky way. A CPPI manager will try to optimize its profit but will stop taking risk at the moment that the CPPI value touches the floor, as he is afraid to fall below by taking more risk.

So it can be concluded that besides the joined use of the re-balancing and constant proportion the two products are very different.

6 Conclusions

As the market of structured credit products keeps on growing, also the request of protection mechanisms in structured credit transactions stays high and hence also here a continuous evolution can be observed. CPPI and CPDO are recently developed products which aim to provide a protection.

CPPIs first came into use around 10 years ago and promise a pre-defined principal payment at maturity. A constant proportion rule is applied to decide the investment strategy. at every time step the investment in the underlying risky asset and the safe asset is re-balanced in order to optimize the profit.

CPDOs were only introduced in 2006 and are intended to be safe, high-yielding instruments. A similar constant proportion rule is used for their investment strategies and they will invest in a risky asset by selling protection (such as CDX and iTraxx). Similarly to the CPPI, re-balancing will be done at every time step until the targeted value has been reached.

The aim of this document was to create an in depth view of the dynamics and risks linked to both products. Hence, first we have tried to explain step by step how they both function and how investment decisions are made.

Understanding the dynamics well helps to identify remaining "safety gaps" and allows a person to get an idea about the size and possibility of experiencing such a gap. For the CPPI, the possibility that the value of the total portfolio will fall below the floor exists and will create a loss, while for the CPDO a loss occurs when a cash-out event occurs.

In both cases there is a strong interest in quantifying this risk. For CPPIs many existing research papers discuss this risk and for this paper we concentrated on the existing literature. In a first group of papers the researchers concentrate on quantifying the risk using specific conditions. Next, some propose ways to limit the multiplier factor in order to limit the risk, while other papers suggest the possibility of taking out an insurance on this risk. But also for those new developments a good and robust way of quantifying the risk is necessary and hence presented. It could be concluded that the gap-risk for CPPIs should not be neglected but safety nets can be used to avoid suffering from it.

As CPDOs are still very new, the field of research is still limited, and as they have only been used on the market for a couple of years, their performance in the real world has only been observed over a short time. As recent experience has shown, in real life CPDOs do not seem to be as safe as they were expected to be. Clearly there is still a strong need to quantify the risk of cash-out events in a more realistic manner, and a great deal of research remains to be done in this field.

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Abstract
Derivative instruments attempt to protect a portfolio against failure events. Constant proportion portfolio insurance (CPPI) and constant proportion debt obligations (CPDO) strategies are recent innovations and have only been adopted in the credit market for the last couple of years. Since their introduction, CPPI strategies have been popular because they provide protection while at the same time they offer high yields. CPDOs were only introduced into the market in 2006 and can be considered as a variation of the CPPI with as main difference the fact that CPDOs do not provide principal protection. Both CPPI and CPDO strategies take investment positions in a risk-free bond and a risky portfolio (often one or more credit default swaps). At each step, the portfolio is rebalanced and the level of risk taken will depend on the distance between the current value of the portfolio and the necessary amount needed to full all the future obligations. In a first step the functioning of both products is studied in depth concluding with drawing some conclusions on their risky-ness.

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