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# Evaluating options for shifting tax burden to top income earners

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Jorge Onrubia, Fidel Picos and María del  
Carmen Rodado

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**Contact information**

Name: Fidel Picos

E-mail: Fidel.PICOS@ec.europa.eu

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## **Abstract**

During the last decade, research on income inequality has paid special attention to top income earners. At the same time, top marginal tax rates on upper income earners have declined sharply in many OECD countries. Discussions are still open on the relationship between the increase of the income share of the richest and to what extent the tax burden should be shifted towards top income earners. In this paper we analyse these questions by building and computing a theoretical framework using the decomposition by income groups proposed by Lambert and Aronson (1993) and Alvaredo (2011). We show that for three types of revenue-neutral reforms based on Pfähler (1984) the redistributive effect is always higher than before the reform. When the size of the rich group is sufficiently small we also find that the best option is allocating tax changes proportionally to net income, and the worst doing it proportionally to tax liabilities.

# 1 Introduction<sup>1</sup>

During the last decade, research on income inequality has paid special attention to top income earners. The work of Piketty (2014) has encouraged public debate by showing how in developed countries wealth has become concentrated in a very small proportion of citizens (e.g. Atkinson *et al.* (2011) highlighted that the top percentile income share has more than doubled in the last decades). This trend is particularly noticeable in the United States, but it is also present in many other countries worldwide, including Southern European countries, like Spain. At the same time, top marginal tax rates on upper income earners have declined sharply in many OECD countries, particularly in Anglo-Saxon countries (Piketty *et al.*, 2014).

Given this scenario, a recurrent idea in the public debate is the possibility of shifting part of the tax burden from lower and middle income to high incomes. A change like this would imply both equity and efficiency effects. In terms of equity, one of the first relevant questions to analyse is the redistributive potential that the mentioned tax shift may have. Although intuitively this type of reforms has positive redistributive effects, actually the final effect would depend on how they are implemented. This means that we should analyse what is the most convenient way of allocating individually the tax increase to the “rich” and the corresponding tax decrease to the “poor”. As far as we know, there is only some empirical evidence based in simulation exercises (i.e. Gale *et al.*, 2015), but without a theoretical framework that incorporates the main underlying relations between tax progressivity, tax burden and income distribution.

## 2 Theoretical framework

### 2.1 Decomposing global PIT progressivity and redistributive effect by income level groups

The redistributive effect of the personal income tax (PIT) can be expressed with the Reynolds-Smolensky (1977) redistribution index as  $\Pi^{RS} = G_Y - G_{Y-T}$ , as the difference between gross (pre-tax) income inequality ( $G_Y$ ) and net (after tax) income inequality ( $G_{Y-T}$ ), both measured in terms of the Gini index.

In order to decompose the redistributive effect among different groups we can take as a starting point the expression proposed by Lambert and Aronson (1993) to split the Gini index for  $k$  groups of population:

$$G_Y = G_Y^B + \sum_k p_k s_{Y_k} G_{Y_k} + R \quad (1)$$

where  $G_Y^B$  is a between-groups component, that expresses the inequality between the  $k$  groups assuming that all individuals within the group hold the same (average) income  $\mu_{Y_k}$ , and  $\sum_k p_k s_{Y_k} G_{Y_k}$  is the within-groups component that is calculated as the sum of the inequality

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<sup>1</sup> An earlier version of this paper was published in FEDEA Working Papers Series (Documento de Trabajo 2015/12).

indices within each group weighted by their share in the total population ( $p_k = N_k/N$ ) and total income ( $s_{Y_k} = Y_k/Y$ ). Finally  $R$  represents an extra term to make the decomposition work when the subgroup income ranges overlap. If the subgroups are defined as  $k = 1, 2, \dots, K$  non-overlapping subpopulations of  $N_K$  individuals then  $R = 0$ .

As a specific case of Lambert and Aronson (1993), Alvaredo (2011) proposes a computational expression for the Gini index that takes into account the existence of two groups only differentiated by their income level. Equation (2) shows that expression for a partition between the first 99 centiles of individuals according to their gross income (simply “99” for notation) and the remaining 1% (“100”):

$$G_Y = (s_Y - p) + (1 - p)(1 - s_Y)G_Y^{99} + ps_YG_Y^{100} \quad (2)$$

where  $p$  represents the population share of group 100 (i.e. 0.01) and  $s_Y$  is the share of gross income held by that group. This means that  $1 - p$  is the population share of group 99 (0,99) and  $1 - s_Y$  the gross income share of that group.  $G_Y^{99}$  and  $G_Y^{100}$  are the Gini indices of gross income within each group respectively. Comparing expressions (1) and (2) the correspondence between the components “between” ( $G_Y^B = s_Y - p$ ) and “within” ( $\sum_k p_k s_{Y_k} G_{Y_k} = ps_Y G_Y^{100} + (1 - p)(1 - s_Y)G_Y^{99}$ ) is straightforward, while  $R$  is not present in (3) because there is no overlapping effect.

Using the Gini decomposition presented in Equation (2) and rearranging terms we can now write the Reynolds-Smolensky index as follows:

$$\Pi^{RS} = (s_Y - s_{Y-T}) + (1 - p)(1 - s_Y)\Pi_{RS}^{99} + ps_Y\Pi_{RS}^{100} - (s_Y - s_{Y-T})[(1 - p)G_{Y-T}^{99} + pG_{Y-T}^{100}] \quad (3)$$

where  $s_{Y-T}$  is the proportion of the net (after-tax) income accumulated by group 100 and  $\Pi_{RS}^{99}$  and  $\Pi_{RS}^{100}$  are the Reynold-Smolensky indices for groups 99 and 100 respectively, i.e. the difference between the Gini indices before and after taxes within each group. We assume that the applied tax has a structure  $T = t(y)$  where tax liability  $T$  only depends (positively) on income. This ensures that its application does not produce re-ranking, therefore each group includes the same observations before and after tax. Overall redistribution in (3) can be then understood as the sum of a ‘between effect’ ( $s_Y - s_{Y-T}$ ), two weighted within effects ( $(1 - p)(1 - s_Y)\Pi_{RS}^{99}$  and  $ps_Y\Pi_{RS}^{100}$ ) and an interaction term ( $-(s_Y - s_{Y-T})[(1 - p)G_{Y-T}^{99} + pG_{Y-T}^{100}]$ ).

Alternatively we can further develop (3) to embed the interaction term into the “within” terms, as follows<sup>2</sup>:

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<sup>2</sup> These new within effects are not expressed in terms of Reynold-Smolensky indices, but in terms of pseudo-Reynolds-Smolensky indices, since  $G_Y$  and  $G_{Y-T}$  are weighted respectively by the shares of gross income and net income of each group.

$$\Pi^{RS} = (s_Y - s_{Y-T}) + (1 - p)[(1 - s_Y)G_Y^{99} - (1 - s_{Y-T})G_{Y-T}^{99}] + p[s_Y G_Y^{100} - s_{Y-T} G_{Y-T}^{100}] \quad (4)$$

Following Kakwani (1977), the Reynolds-Smolensky index can be decomposed as a product of a progressivity measure (the Kakwani index,  $\Pi^K = G_T - G_Y$ ) and a relative measure of revenue (the net effective average tax rate,  $t/(1 - t)$ ). Therefore we can express overall redistribution  $\Pi^{RS}$  in Equation (3) in terms of the Kakwani decomposition for each group as follows:

$$\Pi^{RS} = (s_Y - s_{Y-T}) + (1 - p)(1 - s_Y) \frac{t^{99}}{1 - t^{99}} \Pi_K^{99} + p s_Y \frac{t^{100}}{1 - t^{100}} \Pi_K^{100} - (s_Y - s_{Y-T})[(1 - p)G_{Y-T}^{99} + pG_{Y-T}^{100}] \quad (5)$$

where  $\Pi_K^{99}$  and  $\Pi_K^{100}$  are the Kakwani indices of groups 99 and 100 respectively, and  $t^{99}/(1 - t^{99})$  and  $t^{100}/(1 - t^{100})$  are the net effective average tax rates of groups 99 and 100 respectively.

Alternatively, using Equation (2) to decompose the two components of the Kakwani index and rearranging terms we can also express the Reynolds-Smolensky index as follows:

$$\Pi^{RS} = \frac{t}{1 - t} \{(s_T - s_Y) + (1 - p)[G_T^{99}(1 - s_T) - G_Y^{99}(1 - s_Y)] + p(G_T^{100}s_T - G_Y^{100}s_Y)\} \quad (6)$$

where  $s_T$  is the share of tax revenue borne by group 100.

## 2.2 Linear reforms of progressive PITs

A reform of a progressive personal income tax  $T^{(1)}(y)$  that implies an increase or a reduction for all taxpayers can be treated as a linear transformation of the original tax. Calling  $\lambda$  to the relative reduction or increase, the new tax  $T^{(2)}(y)$  will raise a total revenue of  $T^{(2)} = (1 \pm \lambda)T^{(1)}$ .

Following Pfähler (1984), there is a set of linear reforms that are neutral in relation to different local progressivity measures<sup>3</sup>. There are three types of relevant reforms ( $j = \{a, b, c\}$ ) that allow the comparison of their redistributive impact, their revenue elasticity and their ranking in terms of social preference<sup>4</sup>:

- a. The reduction (increase) of each taxpayer's tax liability is a constant fraction  $\alpha$  of the original tax liability,  $T_a^{(2)}(y) = (1 \pm \alpha)T^{(1)}(y)$ , where  $\alpha = \lambda$ . For this reform the liability progression is kept constant, i.e.  $LP^{(1)}(y) = LP^{(2)}(y)$ ,  $\forall y > 0$ .
- b. The reduction (increase) of each taxpayer's tax liability is a constant fraction  $\beta$  of the original net income,  $T_b^{(2)}(y) = T^{(1)}(y) \pm \beta[y - T^{(1)}(y)]$ , where  $\beta = \lambda T^{(1)}/[Y - T^{(1)}]$ .

<sup>3</sup> Musgrave and Thin (1948) formally establish measures of progression along the income distribution: Average Rate Progression (ARP), Liability Progression (LP) and Residual Progression (RP). See Lambert (2001) for a detailed explanation..

<sup>4</sup> In the Appendix we summarize the main findings concerning the ranking of these tax reforms in redistributive terms.



For this reform the residual progression is kept constant, i.e.  $RP^{(1)}(y) = RP^{(2)}(y)$ ,  $\forall y > 0$ .

- c. The reduction (increase) of each taxpayer's tax liability is a constant fraction  $\zeta$  of gross income,  $T_c^{(2)}(y) = T^{(1)}(y) \pm \zeta y$ , where  $\zeta = \lambda T^{(1)}/Y$ . For this reform the average rate progression is kept constant, i.e.  $ARP^{(1)}(y) = ARP^{(2)}(y)$ ,  $\forall y > 0$ .

For the same revenue change it results that  $\zeta = \alpha \bar{t} = \beta(1 - \bar{t})$ , where  $\bar{t}$  is the average effective rate of the original tax,  $\bar{t} = T^{(1)}/Y$ .

According with Pfähler (1984) and the well-known identity  $L_Y \equiv (1 - \bar{t})L_{Y-T(Y)} + \bar{t}L_T$  in terms of Lorenz curves, we can sort the considered reforms a, b and c in terms of their global redistribution and progressivity effect. In redistributive terms, reducing the tax liabilities proportionally to net income (option b) keeps the redistributive effect of the original tax unchanged, while reductions proportional to gross income (option c) or tax liability (option a) lead to a reduction in redistribution (larger in the latter case than in the former). Formally:

$$L_{Y-T(Y)}^1 = L_{Y-T(Y)}^{2b} > L_{Y-T(Y)}^{2c} > L_{Y-T(Y)}^{2a} > L_Y \quad (7)$$

For tax increases (denoted with ') the ranking is the opposite:

$$L_{Y-T(Y)}^{2a'} > L_{Y-T(Y)}^{2c'} > L_{Y-T(Y)}^{2b'} = L_{Y-T(Y)}^1 > L_Y \quad (8)$$

In terms of progressivity, reducing tax liabilities proportionally to themselves (option a) keeps the progressivity of the original tax unchanged, while reductions proportional to gross income (option c) or net income (option b) lead to an increase in progressivity (larger in the latter case than in the former). Formally:

$$L_Y > L_{T(Y)}^1 = L_{T(Y)}^{2a} > L_{T(Y)}^{2c} > L_{T(Y)}^{2b} \quad (9)$$

For tax increases the ranking is again the opposite:

$$L_Y > L_{T(Y)}^{2b'} > L_{T(Y)}^{2c'} > L_{T(Y)}^{2a'} = L_{T(Y)}^1 \quad (10)$$

### 2.3 Shifting tax burden to top income earners through yield-equivalent linear PIT reforms

Keeping in mind the class of reforms explained above, we can define a generic revenue-neutral reform of the PIT so that the revenue obtained from the 99% poorest taxpayers (group 99) is reduced in a fraction  $\lambda$  which is now borne by the 1% richest taxpayers (group 100), i.e.  $T^{(2)} = (1 - \lambda)T^{99(1)} + T^{100(1)} + \lambda T^{99(1)}$ , where  $T^{99(1)}$  and  $T^{100(1)}$  express the total revenue paid by each group under the original tax. We can express the revenue shifted from group 99

to group 100 ( $\lambda T^{99(1)}$ ) as a fraction  $\ell$  of total revenue ( $\ell T^{(1)}$ ),  $T^{(2)} = (1 - s_T - \ell)T^{(1)} + (s_T + \ell)T^{(1)}$ .

Now the relevant question is how this shift of  $\ell$  from the “poor” to the “rich” changes the total redistributive effect of the PIT. The result will depend on the way we apply the reduction to the poor and the increase to the rich. This can be done in an infinite number of ways, but if we limit the reform to the three types of linear changes explained before, which we can implement it in nine different ways (the combination of  $a$ ,  $b$  y  $c$  in the tax reduction of group “99” and in the increase of group “100”). To evaluate these reforms in redistributive terms we apply the results of Equations (7) and (8) to Equation (3), and we see that:

- The ‘between effect’  $s_Y - s_{Y-T}$  is positive in the nine reforms, because the tax increase for group 100 makes its net income share ( $s_{Y-T}$ ) smaller and therefore  $s_Y - s_{Y-T}$  larger.
- Following Equation (7) the within effect of group 99 ( $(1 - p)(1 - s_Y)\Pi_{RS}^{99}$ ) will be unchanged (reform  $b$ ) or will decrease (reforms  $a$  and  $c$ ).
- Following Equation (8) the within effect of group 100 ( $p s_Y \Pi_{RS}^{100}$ ) will be unchanged (reform  $b$ ) or will increase (reforms  $a$  and  $c$ ).
- The change in the interaction term ( $-(s_Y - s_{Y-T})[(1 - p)G_{Y-T}^{99} + pG_{Y-T}^{100}]$ ) is always negative since all their terms ( $s_Y - s_{Y-T}$ ,  $(1 - p)G_{Y-T}^{99}$  and  $pG_{Y-T}^{100}$ ) are positive.

This implies an *a priori* ambiguous result, since positive and negative terms coexist in the nine possible combinations. However it is possible to obtain unambiguous conclusions for the three cases in which we apply the same type of reforms to both groups (99 and 100).

### Proposition 1

Any yield-equivalent reform of the progressive tax  $T = t(y)$  that reduces all tax liabilities in group 99 proportionally to their original tax liabilities ( $a$ ) and increases all tax liabilities in group 100 proportionally to their original tax liabilities ( $a'$ ) will increase the global redistributive effect.

*Proof*

According to Equations (9) and (10) we know that the Gini indices of the initial tax liabilities for both groups are not affected by the reform ( $G_T^{99(1)} = G_T^{99(2a)}$  and  $G_T^{100(1)} = G_T^{100(2a)}$ ). Expressing Equation (6) in terms of the fraction  $\ell$  of total revenue we can write the global redistributive effect of the new tax ( $\Pi^{RS(2aa')}$ ) as:

$$\Pi^{RS(2aa')} = \frac{t}{1-t} \left\{ (s_T + \ell - s_Y) + (1 - p) \left[ G_T^{99(1)} (1 - s_T - \ell) - G_Y^{99} (1 - s_Y) \right] + p \left( G_T^{100(1)} (s_T + \ell) - G_Y^{100} s_Y \right) \right\} \quad (11)$$

Isolating  $\Pi^{RS(1)}$  in (11) we obtain:

$$\Pi^{RS(2aa')} = \Pi^{RS(1)} + \frac{t}{1-t} \ell \left\{ 1 - (1-p)G_T^{99(1)} + pG_T^{100(1)} \right\} \quad (12)$$

It is straightforward to show that  $\Pi^{RS(2aa')} > \Pi^{RS(1)}$ , since  $\frac{t}{1-t} \ell > 0$ ,  $1 - (1-p)G_T^{99(1)} > 0$  and  $+pG_T^{100(1)} > 0$ .

### Proposition 2

Any yield-equivalent reform of the progressive tax  $T = t(y)$  that reduces all tax liabilities in group 99 proportionally to their original net income (b) and increases all tax liabilities in group 100 proportionally to their original net income (b') will increase the global redistributive effect.

*Proof*

According to Equations (7) and (8) we know that the initial Gini indices of net income for both groups are not affected by the reform ( $G_{Y-T}^{99(1)} = G_{Y-T}^{99(2b)}$  and  $G_{Y-T}^{100(1)} = G_{Y-T}^{100(2b')}$ ), and neither the corresponding Reynolds-Smolensky indices. We also know that the new liability share of group 100 is  $s_T + \ell$ , so we can express their new net income share as  $s_{Y-T} - \ell \frac{t}{1-t}$ . Therefore using Equation (4) we can write the global redistributive effect of the new tax ( $\Pi^{RS(2bb')}$ ) as:

$$\begin{aligned} \Pi^{RS(2bb')} = & \left( s_Y - s_{Y-T} + \ell \frac{t}{1-t} \right) + (1-p) \left[ (1-s_Y)G_Y^{99} - \left( 1 - s_{Y-T} + \ell \frac{t}{1-t} \right) G_{Y-T}^{99(1)} \right] + \\ & p \left[ s_Y G_Y^{100} - \left( s_{Y-T} - \ell \frac{t}{1-t} \right) G_{Y-T}^{100(1)} \right] \end{aligned} \quad (13)$$

Isolating  $\Pi^{RS(1)}$  in (13) we obtain:

$$\Pi^{RS(2bb')} = \Pi^{RS(1)} + \frac{t}{1-t} \ell \left\{ 1 - (1-p)G_{Y-T}^{99(1)} + pG_{Y-T}^{100(1)} \right\} \quad (14)$$

Since  $\frac{t}{1-t} \ell > 0$ ,  $1 - (1-p)G_{Y-T}^{99(1)} > 0$  and  $+pG_{Y-T}^{100(1)} > 0$ , the condition  $\Pi^{RS(2bb')} > \Pi^{RS(1)}$  is fulfilled.

### Proposition 3

Any yield-equivalent reform of the progressive tax  $T = t(y)$  that reduces all tax liabilities in group 99 proportionally to their gross income (c) and increases all tax liabilities in group 100 proportionally to their gross income (c') will increase the global redistributive effect.

*Proof*

For this type of reform the new tax liability for each taxpayer in group 100 is the original tax liability plus a fixed proportion of her gross income. Following Rietveld (1990) we can express the Gini index of the new tax liability as the weighted sum of the Gini index of the original tax

liability ( $G_T^{100(1)}$ ) plus the Gini index of the tax increase, which equals the Gini index of gross income ( $G_Y$ ):

$$G_T^{100(2c')} = \frac{s_T}{s_T + \ell} G_T^{100(1)} + \frac{\ell}{s_T + \ell} G_Y \quad (15)$$

Applying the same rule to group 99 we have:

$$G_T^{99(2c')} = \frac{(1-s_T)}{(1-s_T-\ell)} G_T^{99(1)} - \frac{\ell}{(1-s_T-\ell)} G_Y \quad (16)$$

Replacing Equations (15) and (16) in Equation (6) we get:

$$\begin{aligned} \Pi^{RS} = & \frac{t}{1-t} \left\{ (s_T + \ell - s_Y) + (1-p) \left[ \left( (1-s_T) G_T^{99(1)} - \ell G_Y \right) - G_Y^{99} (1-s_Y) \right] + \right. \\ & \left. p \left( \left( s_T G_T^{100(1)} + \ell G_Y \right) - G_Y^{100} s_Y \right) \right\} \end{aligned} \quad (17)$$

Isolating  $\Pi^{RS(1)}$  in (17) we obtain:

$$\Pi^{RS(2cc')} = \Pi^{RS(1)} + \frac{t}{1-t} \ell \{ 1 - (1-p) G_Y^{99} + p G_Y^{100} \} \quad (18)$$

Once more, it is straightforward to show that  $\Pi^{RS(2cc')} > \Pi^{RS(1)}$ , since  $\frac{t}{1-t} \ell > 0$ ,  $1 - (1-p) G_Y > 0$  and  $+p G_Y > 0$ .

#### Proposition 4

Let  $aa'$ ,  $bb'$  and  $cc'$  be three yield-equivalent reforms that reduce all tax liabilities in group 99 and increases all tax liabilities in group 100 at the same rate  $\ell$ , and share this rate proportionally to, respectively, their original tax liability, original net income and gross income, their ranking in terms of redistribution is ambiguous.

*Proof*

Consider that gross income in group 99 is distributed equally among all individuals. In this case  $G_Y^{99} = G_T^{99} = G_{Y-T}^{99} = 0$ , so Equations (12), (14) and (18) will be:

$$\Pi^{RS(2aa')} = \Pi^{RS(1)} + \frac{t}{1-t} \ell \{ 1 + p G_T^{100} \} \quad (19)$$

$$\Pi^{RS(2bb')} = \Pi^{RS(1)} + \frac{t}{1-t} \ell \{ 1 + p G_{Y-T}^{100} \} \quad (20)$$

$$\Pi^{RS(2cc')} = \Pi^{RS(1)} + \frac{t}{1-t} \ell \{ 1 + p G_Y^{100} \} \quad (21)$$

Since  $G_T^{100} > G_Y^{100} > G_{Y-T}^{100}$ , then  $\Pi^{RS(2aa')} > \Pi^{RS(2cc')} > \Pi^{RS(2bb')}$ .

Consider now that gross income in group 100 is distributed equally among all individuals. In this case  $G_Y^{100} = G_T^{100} = G_{Y-T}^{100} = 0$ , so Equations (12), (14) and (18) will be:

$$\Pi^{RS(2aa')} = \Pi^{RS(1)} + \frac{t}{1-t} \ell\{1 - (1-p)G_T^{99}\} \quad (22)$$

$$\Pi^{RS(2bb')} = \Pi^{RS(1)} + \frac{t}{1-t} \ell\{1 - (1-p)G_{Y-T}^{99}\} \quad (23)$$

$$\Pi^{RS(2cc')} = \Pi^{RS(1)} + \frac{t}{1-t} \ell\{1 - (1-p)G_Y^{99}\} \quad (24)$$

Since  $G_T^{99} > G_Y^{99} > G_{Y-T}^{99}$ , then  $\Pi^{RS(2aa')} < \Pi^{RS(2cc')} < \Pi^{RS(2bb')}$ .

Given that these two cases give opposite results, the ranking of the three reforms in redistribution terms is generally ambiguous. Therefore, the relative order among the three tax reform alternatives remain an empirical issue.

### Proposition 5

For asymmetric partitions of the population where  $p \rightarrow 0$  the following order is fulfilled:  $\Pi^{RS(2bb')} > \Pi^{RS(2cc')} > \Pi^{RS(2aa')}$ .

*Proof*

Applying  $p \rightarrow 0$  to Equations (12), (14) and (18) we obtain:

$$\lim_{p \rightarrow 0} \Pi^{RS(2aa')} = \Pi^{RS(1)} + \frac{t}{1-t} \ell\left(1 - G_T^{99(1)}\right) \quad (25)$$

$$\lim_{p \rightarrow 0} \Pi^{RS(2bb')} = \Pi^{RS(1)} + \frac{t}{1-t} \ell\left(1 - G_{Y-T}^{99(1)}\right) \quad (26)$$

$$\lim_{p \rightarrow 0} \Pi^{RS(2cc')} = \Pi^{RS(1)} + \frac{t}{1-t} \ell\left(1 - G_Y^{99(1)}\right) \quad (27)$$

Since  $G_T^{99} > G_Y^{99} > G_{Y-T}^{99}$ , then  $\Pi^{RS(2bb')} > \Pi^{RS(2cc')} > \Pi^{RS(2aa')}$ .

## 3 An illustration for Spain

To illustrate the results of the previous sections we use Spanish PIT microdata from 2011 to simulate the three tax reforms of Proposition 4:  $aa'$  (changes proportional to tax liability),  $bb'$  (changes proportional to net income) and  $cc'$  (changes proportional to gross income). In particular we use the 2011 Spanish PIT Return Sample disseminated by the Spanish Institute for Fiscal Studies (Instituto de Estudios Fiscales, IEF) and the Spanish Tax Agency (Agencia Estatal de Administración Tributaria, AEAT) which contains more than 2 million observations representative of more than 19 million tax returns<sup>5</sup>. To ensure microdata consistent with the methodology, we have made zero all negative incomes, and then have removed the observations that showed inconsistent results in the original tax (gross income < tax base, tax base < gross tax liability, gross tax liability < net tax liability, average legal rate > maximum marginal legal rate). Some of those cases were errors, while others are a consequence of the

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<sup>5</sup> Detailed information on the database can be found in Pérez *et al.* (2014) [in Spanish].

dual specifications of the Spanish PIT. As a result we use 2,031,577 observations (99.79% of the original observations) that represent 19,430,040 tax returns (99.82%). Table 1 shows the descriptive statistics of the microdata regarding gross income.

**Table 1: Descriptive statistics of the 2011 Spanish PIT microdata**

Concept	Group 99	Group 100	Total
Number of observations	1,885,082	146,495	2,031,577
Population represented	19,235,740	194,300	19,430,040
Total (EUR)	409,096,697,360	39,226,087,286	448,322,784,646
Average (EUR)	21,268	201,885	23,074
Gross income Standard dv. (€)	15,461	557,243	60,537
Minimum (€)	0	102,063	0
Maximum (€)	102,063	96,182,743	96,182,743
Gini index	0.37659546	0.35587053	0.41801533

Source: Own elaboration from 2011 Spanish PIT Return Sample provided by AEAT and IEF.

In order to assess the reforms we cannot use the 2011 Spanish PIT as a reference, since like any other real income tax it does not fit the  $T = t(y)$  model, because tax liabilities depend not only on income but also on other variables (income type, age, personal and family characteristics, region, tax incentives, etc.). In order to stay as close as possible to the real tax we simulate a stylized tax  $T = t(y)$  with the same revenue and redistribution effect as the real tax applied in 2011. To ensure that average rates are also distributed in a similar way we keep the basic structure of the real tax, i.e.  $T = t(y) = f(y - d(y)) - c(y)$ , where  $f(\cdot)$  represents the tax schedule,  $d(y)$  are tax deductions and  $c(y)$  tax credits. All these parameters depend only on total income or are constant. In particular, in our microsimulation exercises  $f(\cdot)$  is the real tax schedule applied in 2011 to “general income” (Spanish PIT also incorporates a different schedule for “savings income”), while  $d(y)$  has a fixed part and a part that is proportional to income (but with a fixed limit), and  $c(y)$  is constant (but limited to ensure that  $T \geq 0$ ). All these values try to reproduce the real variability originated by tax treatments based on non-income attributes. Table 2 shows the final parameters chosen.

**Table 2: Design parameters of the stylized tax equivalent to the 2011 Spanish PIT**

Parameter		Comments	
$f(\cdot)$	Taxable Income (EUR) – Marginal Rate	Progressive tax schedule applied to “general income” in 2011	
	0 - 17707.20		24%
	17707.20 - 33007.20		28%
	33007.20 - 53407.20		37%
	53407.20 - 120000.20		43%
	120000.20 - 175000.20		44%
$d(\cdot)$	$\min(2500 + \min(.1372186y, 50000), y)$	EUR 2,500, plus 13.72186% of gross income with a limit of EUR 50,000. The deduction cannot be higher than gross income.	
$c(\cdot)$	$\min(1591, f(y - d(y)))$	EUR 1,591 with the limit of the gross tax liability previously calculated	

Source: Own elaboration.

**Table 3: Results of the simulations**

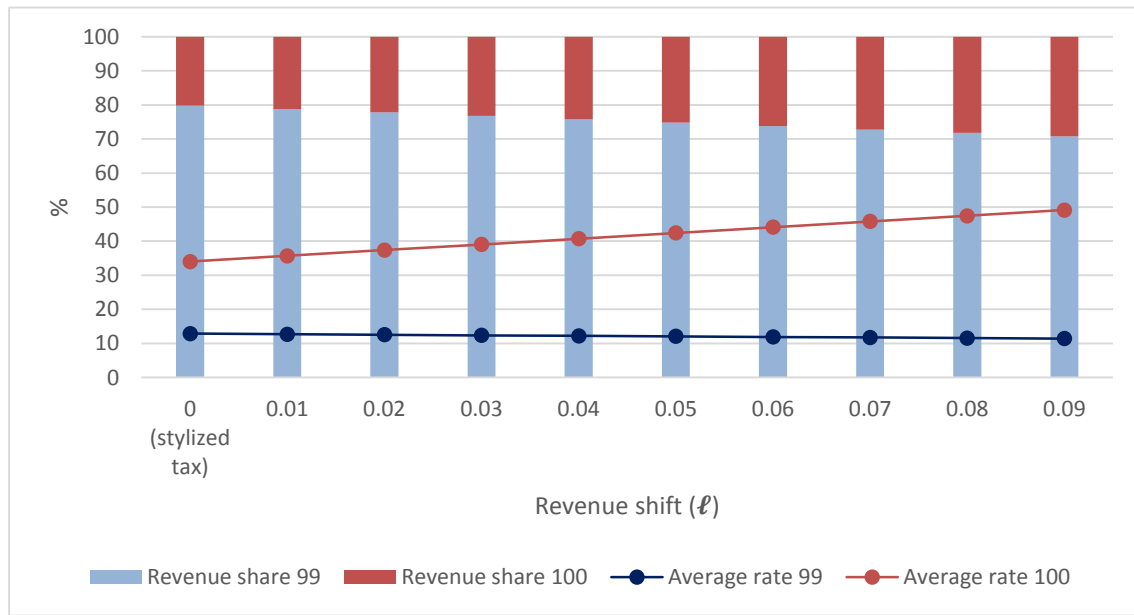
		Stylized tax	Tax burden shift ( $\ell$ )										
			0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09		
Shares	$s_Y$	0.087495	0.087495	0.087495	0.087495	0.087495	0.087495	0.087495	0.087495	0.087495	0.087495		
	$s_T$	0.202389	0.212389	0.222389	0.232389	0.242389	0.252389	0.262389	0.272389	0.282389	0.292389		
	$s_{Y-T}$	0.067691	0.065968	0.064244	0.062520	0.060797	0.059073	0.057349	0.055626	0.053902	0.052178		
Tax rates	$t^{99}$	0.128512	0.126901	0.12529	0.123679	0.122068	0.120456	0.118845	0.117234	0.115623	0.114011		
	$t^{100}$	0.340089	0.356893	0.373697	0.390501	0.407304	0.424108	0.440912	0.457715	0.474519	0.491323		
	$t$	0.147024	0.147024	0.147024	0.147024	0.147024	0.147024	0.147024	0.147024	0.147024	0.147024		
Gross income	$G_Y^{99}$	0.376595	0.376595	0.376595	0.376595	0.376595	0.376595	0.376595	0.376595	0.376595	0.376595		
	$G_Y^{100}$	0.355871	0.355871	0.355871	0.355871	0.355871	0.355871	0.355871	0.355871	0.355871	0.355871		
	$G_Y$	0.418015	0.418015	0.418015	0.418015	0.418015	0.418015	0.418015	0.418015	0.418015	0.418015		
<i>aa'</i>	Tax liabilities	$G_T^{99}$	0.636066	0.636066	0.636066	0.636066	0.636066	0.636066	0.636066	0.636066	0.636066		
		$G_T^{100}$	0.434294	0.434294	0.434294	0.434294	0.434294	0.434294	0.434294	0.434294	0.434294		
		$G_T$	0.695528	0.699275	0.703021	0.706767	0.710514	0.714260	0.718007	0.721753	0.725499	0.729246	
	Net income	$G_{Y-T}^{99}$	0.338333	0.338882	0.339430	0.339975	0.340519	0.341060	0.341600	0.342137	0.342672	0.343206	
		$G_{Y-T}^{100}$	0.315454	0.312349	0.309077	0.305625	0.301977	0.298116	0.294024	0.289677	0.285052	0.280122	
		$G_{Y-T}$	0.370181	0.369536	0.368890	0.368245	0.367599	0.366955	0.366310	0.365667	0.365024	0.364383	
Linear PIT Reforms	<i>bb'</i>	Tax liabilities	$G_T^{99}$	0.636066	0.638426	0.640810	0.643220	0.645654	0.648113	0.650596	0.653103	0.655636	0.658196
			$G_T^{100}$	0.434294	0.428699	0.423607	0.418953	0.414683	0.410751	0.407119	0.403754	0.400627	0.397714
			$G_T$	0.695528	0.701036	0.706508	0.711946	0.717347	0.722711	0.728037	0.733323	0.738571	0.743780
	Net income	$G_{Y-T}^{99}$	0.338333	0.338333	0.338333	0.338333	0.338333	0.338333	0.338333	0.338333	0.338333	0.338333	
		$G_{Y-T}^{100}$	0.315454	0.315454	0.315454	0.315454	0.315454	0.315454	0.315454	0.315454	0.315454	0.315454	
		$G_{Y-T}$	0.370181	0.369030	0.367878	0.366727	0.365576	0.364426	0.363276	0.362128	0.360981	0.359836	
<i>cc'</i>	Tax liabilities	$G_T^{99}$	0.636066	0.638125	0.640208	0.642319	0.644455	0.646617	0.648805	0.651020	0.653262	0.655531	
		$G_T^{100}$	0.434294	0.430602	0.427241	0.424170	0.421352	0.418758	0.416361	0.414140	0.412077	0.410155	
		$G_T$	0.695528	0.700814	0.706071	0.711302	0.716504	0.721677	0.726820	0.731933	0.737014	0.742063	
Net income	$G_{Y-T}^{99}$	0.338333	0.338404	0.338474	0.338544	0.338614	0.338683	0.338753	0.338822	0.338891	0.338959		
	$G_{Y-T}^{100}$	0.315454	0.314398	0.313286	0.312112	0.310871	0.309558	0.308166	0.306688	0.305115	0.303438		
	$G_{Y-T}$	0.370181	0.369094	0.368007	0.366920	0.365834	0.364748	0.363663	0.362579	0.361495	0.360414		

Source: Own elaboration from 2011 Spanish PIT Return Sample provided by AEAT and IEF.

Taking this stylized tax as a starting point we simulate the three types of reform ( $aa'$ ,  $bb'$  and  $cc'$ ) for several values of  $\ell$ . We start by  $\ell = 0.01$  (i.e. we shift 1% of the overall revenue from group 99 to group 100) and keep increasing this value (in steps of 0.01) while the effective average tax rate of group 100 is lower than 0.50. Although these simulations are only an illustration of the previous theoretical developments, we understand that this is a reasonable limit for the average tax rate of that group. Table 3 shows all the shares, rates and Gini and concentration indices calculated in the simulations.

Figure 1 shows graphically the revenue shares of group 99 ( $1 - s_T$ ) and 100 ( $s_T$ ) and their corresponding average tax rates ( $t^{99}$  and  $t^{100}$ ) for all the simulated taxes.

**Figure 1: PIT reform simulations: Revenue shares and average effective tax rates**



The results are the same for the three types of reforms, because the share of taxes between groups and their average tax rates only depend on the value of  $\ell$ . The differences arise when we analyse the share of taxes within each group and the redistributive impact they have. To show these effects we use the decomposition in Equation (5).

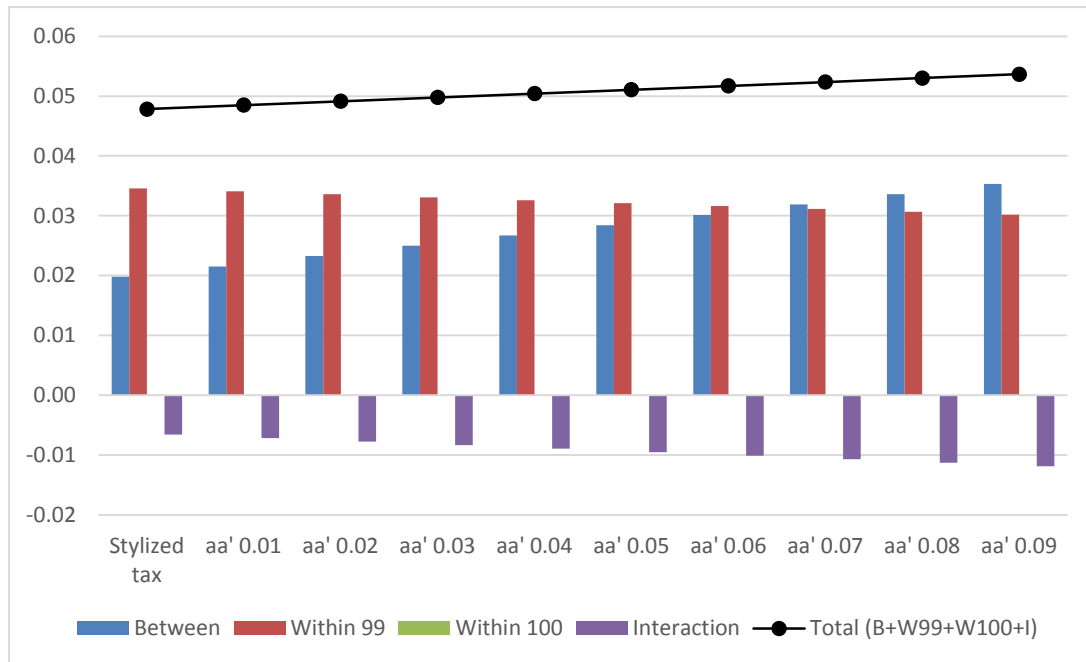
Figures 2 to 4 show the results for each type of reform. These three figures confirm empirically the results of Propositions 1 to 3: all the reforms are more redistributive than the original tax. Furthermore, Figure 5 shows that the ranking from more to less redistributive is  $bb'$ ,  $cc'$ ,  $aa'$ , what is consistent with Proposition 5 (which is obtained under the assumption  $p \rightarrow 0$ ). However, the distance between  $aa'$  and  $cc'$  is much higher than between  $cc'$  and  $bb'$ , which is related to the higher distance between  $G_T$  and  $G_Y$  than between  $G_Y$  and  $G_{Y-T}$  (see Proposition 5). We also see that within each reform type the redistributive effect rises as  $\ell$  increases, which is a direct consequence of Propositions 1, 2 and 3.

Regarding the changes in the different partial effects considered we also see that in all the reforms the 'between effect' increases while the 'interaction' effect decreases. Both the



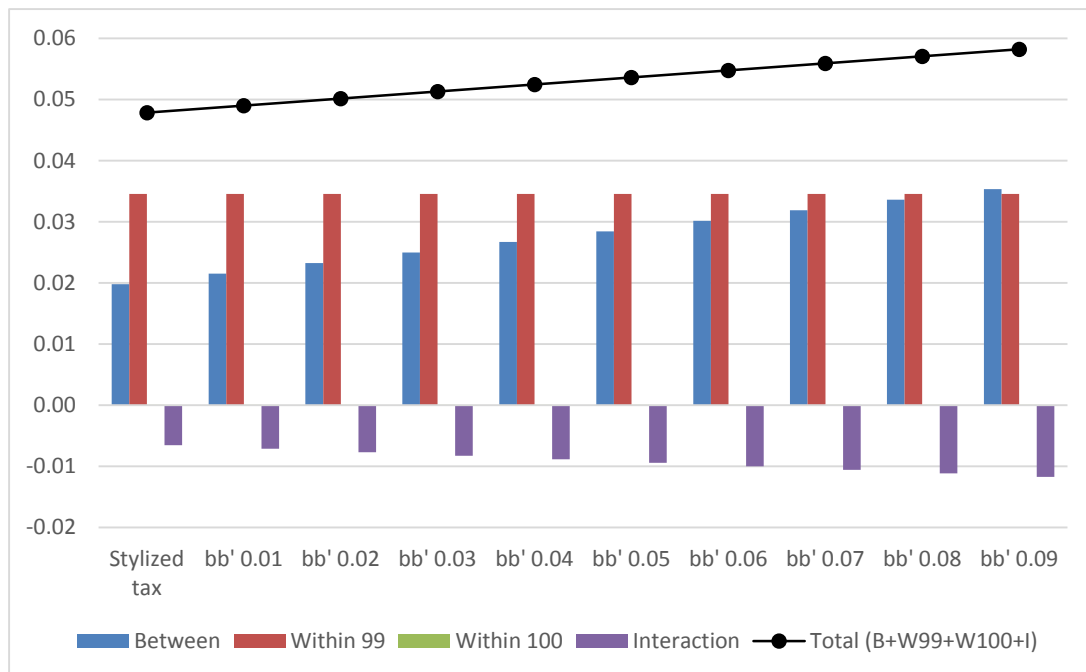
'within 99' and 'within 100' effects are constant for  $bb'$ , which can be derived directly from Equation (3); for  $aa'$  and  $cc'$  the 'within 99 effect' decreases and the 'within 100 effect' increases, but the latter effect is almost negligible due to the small population share of the last centile. In general the total redistributive effect is driven mostly by the 'within 99 effect', although the 'between effect' exceeds it for high values of  $\ell$ . Finally the 'interaction' effect grows in the opposite direction as  $\ell$  increases, so it smooths the increase of total redistribution.

**Figure 2: Redistributive effect of reforms type  $aa'$**



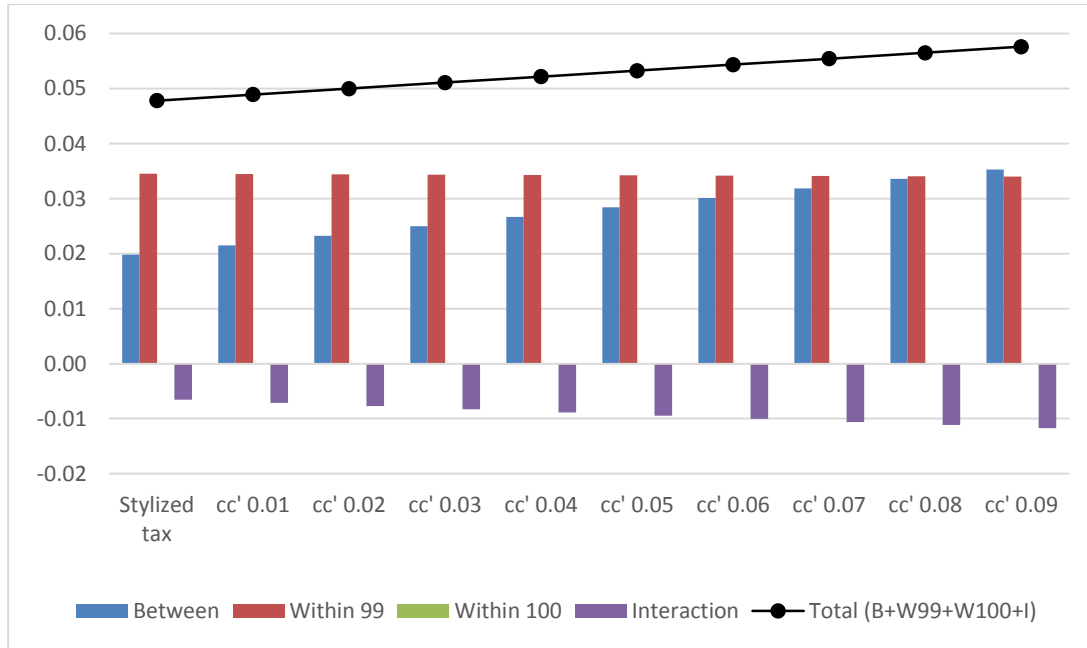
Source: own elaboration

**Figure 3: Redistributive effect of reforms type  $bb'$**



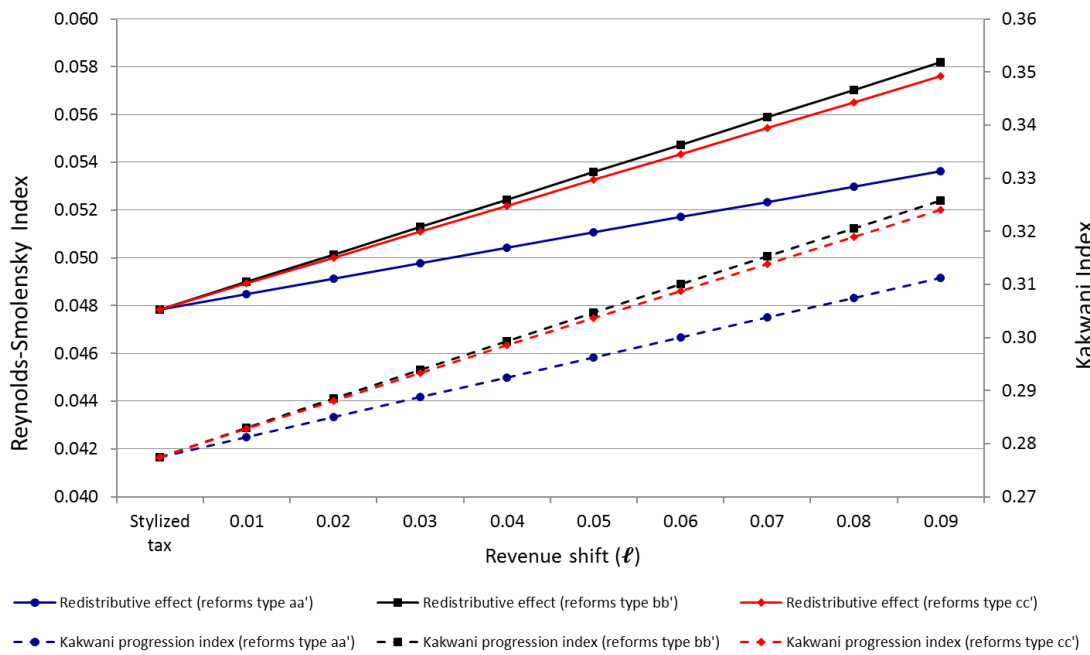
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**Figure 4: Redistributive effect of reforms type cc'**



Source: own elaboration

**Figure 5: Comparison of redistributive effect and progressivity**



Source: own elaboration

## 4 Conclusions

Throughout this paper we have developed a methodology to assess PIT reforms that shift part of the tax burden towards the top 1% income earners, keeping overall revenue constant. Based on the Kakwani (1977) decomposition of the Reynolds-Smolensky index, and using the decompositions by income groups by Lambert and Aronson (1993) and Alvaredo (2011), we have developed a theoretical framework that allow us to obtain conclusions in terms of redistribution on a set of reforms based on Pfähler (1984). We also illustrate the results with an empirical exercise for Spain using a PIT microdata sample.

The main conclusions of this paper are the following:

- The overall redistribution of this type of reforms can be decomposed in a 'between' effect (that measures the pure effect of the shift) and a 'within effect' for each group (that measures how the distribution of tax changes within the group affects total redistribution). Depending on the way we make the decomposition there may be an additional 'interaction term'.
- In principle, the redistributive result of this type of reforms is ambiguous in redistributive terms, since there are positive effects (between and within for the "rich") and negative effects (within for the "poor" and interaction).
- For three types of reforms based on Pfähler (1984) (that consist of allocating the tax changes proportionally to tax liabilities, net income or gross income) we show that the redistributive effect is always higher than before the reform.
- The ranking among those three types of reform is ambiguous except when the population size of the rich group is sufficiently small (empirically verified for Spain when  $p = 1\%$ ). In this case the best option is allocating tax changes proportionally to net income, and the worst doing it proportionally to tax liabilities.

As we exposed in the introduction, the motivation of this paper was to shed light on the potential capacity to reduce income inequality through a tax increase on the highest income individuals. The main objective of our study has been to develop a theoretical framework for accurately analysing the underlying drivers of the redistributive effects of this kind of reforms. Nevertheless, we think that there is still space for further research. In particular, as a second step we consider it crucial to extend our theoretical framework to incorporate behavioural responses to the tax changes proposed.

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## Appendix

Following Pfähler (1984), options  $a$ ,  $b$  and  $c$  can be sorted according to the different local progressivity measures, as shown in the following propositions. We show the results and proofs only for tax cuts in group 99, being the sorting of alternatives the opposite for tax increases in group 100.

### Proposition A.1 (Pfähler, 1984)

For a tax cut, the three alternatives  $a$ ,  $b$  y  $c$  can be ordered according to their residual progression in terms of its inverse  $RP^* = 1/RP$ :

$$RP^{*(1)}(y) = RP_b^{*(2)}(y) > RP_c^{*(2)}(y) > RP_a^{*(2)}(y), \forall y = (0, \infty) \quad (\text{A.1})$$

In our case, reducing the tax for group 99 proportionally to net income (option  $b$ ) keeps the residual progression of the original tax unchanged. But additionally we know that this option is better in terms of  $RP$  to, in this order, a reduction proportional to gross income ( $c$ ) and a reduction proportional to the tax liability ( $a$ ).

*Proof*

For the proof, we use Ruiz-Huerta *et al.* (1995):

Consider  $RP^*(y) = 1/RP(y)$  so that the residual progression can be expressed as:

$$RP^*(y) = \frac{1-t(y)}{1-m(y)} > 1 \quad (\text{A.2})$$

In the case of a reduction proportional to net income ( $b$ ), the average ( $t$ ) and marginal ( $m$ ) rates of taxpayers included in group 99 after the tax cut will be:

$$t_b^{(2)} = \frac{T^{(1)}(y)}{y} - \beta \frac{y-T^{(1)}(y)}{y} = t^{99(1)} - \beta(1 - t^{99(1)}) \quad (\text{A.3})$$

$$m_b^{(2)} = T^{(1)'}(y) - \beta(1 - T^{(1)'}(y)) = m^{99(1)} - \beta(1 - m^{99(1)}) \quad (\text{A.4})$$

From these definitions it is easy to prove that  $RP^{*99(1)}(y) = RP_b^{*99(2)}(y)$ :

$$RP_b^{*99(2)}(y) = \frac{1-t^{99(1)}+\beta(1-t^{99(1)})}{1-m^{99(1)}+\beta(1-m^{99(1)})} = \frac{(1+\beta)(1-t^{99(1)})}{(1+\beta)(1-m^{99(1)})} = RP^{*99(1)}(y) \quad (\text{A.5})$$

If the tax cut is applied proportionally to gross income ( $c$ ), the two tax rates will be:

$$t_c^{(2)} = \frac{T^{(1)}(y)-\zeta y}{y} = t^{99(1)} - \zeta \quad (\text{A.6})$$

$$m_c^{99(2)} = \frac{d(T^{(1)}(y)-\zeta y)}{dy} = T^{(1)'}(y) - \zeta = m^{99(1)} - \zeta \quad (\text{A.7})$$

Now we prove that  $RP_b^{*99(2)}(y) > RP_c^{*99(2)}(y)$ :

$$RP_c^{*99(2)}(y) = \frac{1-t^{99(1)}+\zeta}{1-m^{99(1)}+\zeta} < \frac{(1-t^{99(1)})}{(1-m^{99(1)})} = RP^{*99(1)}(y) = RP_b^{*99(2)}(y) \quad (\text{A.8})$$

Finally, if the tax cut is applied proportionally to tax liability ( $a$ ), the two tax rates will be:

$$t_a^{(2)} = \frac{T^{(1)}(y)(1-\alpha)}{y} = t^{99(1)}(1-\alpha) \quad (\text{A.9})$$

$$m_a^{99(2)} = \frac{d(T^{(1)}(y)(1-\alpha))}{dy} = T^{(1)'}(y)(1-\alpha) = m^{99(1)}(1-\alpha) \quad (\text{A.10})$$

Since  $\zeta = \alpha \bar{t}$ , and for  $m^{99(1)} > t^{99(1)}$ ,  $\zeta < \alpha \bar{t}$ , then  $RP_a^{*99(2)}(y) < RP_c^{*99(2)}(y)$ :

$$RP_a^{*99(2)} = \frac{1-t^{99(1)}+\alpha t^{99(1)}}{1-m^{99(1)}+\alpha m^{99(1)}} < \frac{1-t^{99(1)}+\zeta}{1-m^{99(1)}+\zeta} = RP_c^{*99(2)}(y) \quad (\text{A.11})$$

Consequently we prove that the following ranking applies to the three options considered:

$$RP^{*99(1)}(y) = RP_b^{*99(2)}(y) > RP_c^{*99(2)} > RP_a^{*99(2)} \quad (\text{A.12})$$

#### Proposition A.2 (Pfähler, 1984)

For a tax-cut reform, the three alternatives a, b y c can be ordered according to their average rate progression in terms of its transformation  $ARP^* = yARP$ :

$$ARP_a^{*(2)}(y) < ARP_c^{*(2)}(y) = ARP^{*(1)}(y) < ARP_b^{*(2)}(y), \forall y = (0, \infty) \quad (\text{A.13})$$

Now, reducing the tax for group 99 proportionally to gross income (option c) keeps the  $ARP$  of the original tax unchanged. But additionally we know that a reduction proportional to tax liability ( $a$ ) shows the lowest  $ARP$ , while a reduction proportional to net income ( $b$ ) shows the highest  $ARP$ .

*Proof*

For taxpayers in group 99 we have:

$$ARP_a^{*99(2)}(y) = m^{99(1)}(1-\alpha) - t^{99(1)}(1-\alpha) = (1-\alpha)(m^{99(1)} - t^{99(1)}) \quad (\text{A.14})$$

$$ARP_b^{*99(2)}(y) = [m^{99(1)} - \beta(1 - m^{99(1)})] - [t^{99(1)} - \beta(1 - t^{99(1)})] = (1 + \beta)(m^{99(1)} - t^{99(1)}) \quad (\text{A.15})$$

$$ARP_c^{*99(2)}(y) = (m^{99(1)} - \zeta) - (t^{99(1)} - \zeta) = m^{99(1)} - t^{99(1)} = ARP^{*99(1)}(y) \quad (\text{A.16})$$

It is trivial that for  $\alpha, \beta > 0$  and for  $m > t$  the following ranking applies:

$$ARP_a^{*99(2)}(y) < ARP_c^{*99(2)}(y) = ARP^{*99(1)}(y) < ARP_b^{*99(2)}(y) \quad (\text{A.17})$$

**Proposition A.3 (Pfähler, 1984)**

For a tax-cut reform, the three alternatives a, b y c can be ordered according to their liability progression (revenue elasticity):

$$LP_a^{(2)}(y) = LP^{(1)}(y) < LP_c^{(2)}(y) < LP_b^{(2)}(y), \forall y = (0, \infty) \quad (\text{A.18})$$

Thus, we see that reducing the tax for group 99 proportionally to tax liability (option a) keeps the LP of the original tax unchanged. And finally, we know that this option is better in lower in terms of LP to, in this order, a reduction proportional to gross income (c) and a reduction proportional to net income (b).

*Proof*

Liability progression is defined at any income level y as the elasticity of tax liability to pre-tax income, so that:

$$LP(y) = \eta_{T(y),y} = \frac{dT(y)}{dy} \frac{y}{T(y)} = \frac{yT'(y)}{T(y)} = \frac{m(y)}{t(y)} > 1 \quad (\text{A.19})$$

For taxpayers in group 99 we get the values for liability progression:

$$LP_a^{*99(2)}(y) = \frac{m^{99(1)}(1-\alpha)}{t^{99(1)}(1-\alpha)} = \frac{m^{99(1)}}{t^{99(1)}} = LP^{*99(1)}(y) \quad (\text{A.20})$$

$$LP_b^{*99(2)}(y) = \frac{m^{99(1)} - \beta(1 - m^{99(1)})}{t^{99(1)} - \beta(1 - t^{99(1)})} \quad (\text{A.21})$$

$$LP_c^{*99(2)}(y) = \frac{m^{99(1)} - \varsigma}{t^{99(1)} - \varsigma} \quad (\text{A.22})$$

Since in a progressive tax  $m^{99(1)} > t^{99(1)}$ , for  $\varsigma > 0$  we verify that for any positive value of gross income  $LP_a^{*99(2)}(y) < LP_c^{*99(2)}(y)$ . Additionally, since  $\beta > \varsigma$ , the progressivity condition  $m^{99(1)} > t^{99(1)}$  ensures that  $LP_c^{*99(2)}(y) < LP_b^{*99(2)}(y)$ , therefore:

$$LP_a^{*99(2)}(y) = LP^{*99(1)}(y) < ARP_c^{*99(2)}(y) < ARP_b^{*99(2)}(y) \quad (\text{A.23})$$

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