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Partial Stochastic Analysis with the Aglink-Cosimo Model: A Methodological Overview

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Authors:
Sergio René Araujo-Enciso, Simone Pieralli and
Ignacio Pérez-Domínguez

Editors: Simone Pieralli and Ignacio Pérez-Domínguez

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Contact information
Name: European Commission, Joint Research Centre (JRC), Directorate D — Sustainable Resources
Address: Edificio Expo. c/Inca Garcilaso, 3. E-41092 Seville (Spain)
Email: JRC-D4-SECRETARIAT@ec.europa.eu
Tel.: +34 954488300

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Executive summary

Aglink-Cosimo is a recursive-dynamic partial equilibrium model developed and maintained by the Organisation for Economic Co-operation and Development (OECD) and the Food and Agriculture Organization of the United Nations (FAO) Secretariats as a collaborative effort. The model is primarily used to prepare the OECD-FAO Agricultural Outlook, a yearly publication aiming to provide baseline projections for the main global agricultural commodities over the medium term. These deterministic projections are enhanced by a partial stochastic analysis tool, which allows the analysis of specific market uncertainties. This is done by producing counterfactual scenarios to the baseline originating from varying yields and macroeconomic variables stochastically.

The aim of this report is to propose and evaluate different methods of analysing stochastically important yields and macroeconomic uncertainty drivers. In the first stage, we identify and evaluate the best parametric method to extract unexplained variability, which we consider to be uncertainty in the macroeconomic and yield drivers. In the second stage, we test parametric and non-parametric methods side by side to simulate 10 years of potentially different macroeconomic and yield environments.

The results can be summarised as follows. For yields, we find that a parametric cubic trend method performs best in the first stage and a non-parametric hierarchical copula (Clayton) method is more appropriate in the second stage. For macroeconomic variables, a vector autoregressive model performs best in the first stage, while a non-parametric hierarchical copula (Frank) method is more appropriate in the second stage.

1 Introduction

Aglink-Cosimo is a recursive-dynamic partial equilibrium model developed and maintained by the Organisation for Economic Co-operation and Development (OECD) and the Food and Agriculture Organization of the United Nations (FAO) Secretariats as a collaborative effort. The model is primarily used to prepare the OECD-FAO Agricultural Outlook, a yearly publication aiming to provide baseline projections for the main global agricultural commodities over the medium term.

Aglink-Cosimo is a model widely used by countries that are members of both organisations. Additionally, the countries or users engage in a broad range of model development and improvement, in part to address the need for counterfactual policy analysis. An example of that type of development is the partial stochastic analysis (PSA) tool.

The PSA tool serves to assess a broad range of alternative scenarios, which diverge from the baseline by treating a number of variables stochastically. The selection of stochastic variables aims at identifying the major sources of uncertainty for EU agricultural markets. In total, 39 country-specific macroeconomic variables, the crude oil price, and 85 country- and product-specific yields are treated as uncertain within this partial stochastic framework. Apart from the international oil price, four macroeconomic variables are considered in specific countries: consumer price index (CPI), gross domestic product index (GDPI), gross domestic product deflator (GDPD) and exchange rate (XR). The countries considered are Australia, Brazil, Canada, China, Europe, India, Japan, New Zealand, Russia, and United States. The yield variables are key crop and milk yields in important world markets. Among the key crops, we include wheat, barley, maize, oats, rye, rice, soybeans, rapeseeds, sunflower, palm oil, and sugar beet and cane. Among the key markets, we include Europe, Kazakhstan, Ukraine, Russia, Argentina, Brazil, Paraguay, Uruguay, Canada, Mexico, United States, Indonesia, Malaysia, Thailand, Vietnam, Australia, China, India, and New Zealand. The original methodology was developed by the European Commission Joint Research Centre (JRC; formerly JRC-IPTS) based in Seville, with the collaboration of the European Commission Directorate General for Agriculture and Rural Development (DG-Agri). The methodology has gained attention over the years as it is useful in developing scenarios in which uncertainty plays a key role (e.g. historical large yield variability in important agricultural producers and exporters).

The PSA was first implemented in 2011. After preliminary contacts with the FAPRI network in 2010-11, a workshop was organized in March 2011 by the JRC, DG AGRI and DEFRA with the aim of implementing a stochastic methodology for uncertainty analysis with Aglink-Cosimo inspired on the FAPRI work. This was done for the first time for the EU Outlook in 2011-2021 (https://ec.europa.eu/agriculture/markets-and-prices/medium-term-outlook_en) and culminated with the publication of a JRC reference report (Burrell and Nii-Naate, 2013). Further methodological adjustments were carried out on the occasion of the EU and OECD-FAO Outlook exercises of 2013 and 2014, with a switch to TROLL in 2013 for the computation of yield deviates and the implementation of a multivariate truncation for both yields and macroeconomic variables in 2014. At the time, macroeconomic uncertainty for some variables had to be down-weighted in an ad-hoc manner to avoid explosive growth. Discussions and brainstorming exercises on possible further improvements continued during 2013 and 2014 (e.g. iMAP Reference Group meeting of July 2014).

While the PSA tool is important for assessing the potential impact of selected sources of uncertainty in the model, its results are conditional on the methods used for extracting and simulating uncertainty. Previous methods, such as those employed by

Burrell and Nii-Naate (2013), have limitations that have been partially addressed by making specific assumptions on the probabilistic distributions of the stochastic variables during the simulation stage (e.g. normalisation or truncation). In order to choose a certain methodology with scientific rigour, in April 2015 the JRC held an expert workshop in Seville to discuss existing and potential methods. Several of the recommendations made at that workshop have been considered in the work presented in this report.

The aim of this report is to propose alternative methodologies for the PSA tool, including some statistical approaches for evaluating it in a medium-term baseline context and for analysing its performance when doing scenario analysis.

This report is organised as follows. After this introductory section, Section 2 provides an overview of the methods used for extracting the yield and macroeconomic uncertainty. Section 3 focuses on the methods used in simulating the uncertainty, including the assumptions on the multivariate distributions. This third section closes with a statistical test employed for verifying the quality of the simulations. Section 4 summarises the methodologies proposed based on the theoretical framework explained in Sections 2 and 3. Section 5 explores the results of the statistical tests, aiming to select the best methods. Section 6 focuses on the impact of each method using subset scenarios. The report concludes with Section 7, in which final recommendations are given, together with some caveats.

The stochastic process is mainly divided in two parts:

- Error extraction (Part 1). 'Residual errors' are obtained from a specific method (e.g. ordinary least squares (OLS) or seemingly unrelated regression (SUR)) applied to a particular estimation model specified (e.g. Aglink-Cosimo model, cubic time trend, vector autoregressive panel regression).
- Error simulation (Part 2). Based on the error extraction, a certain distribution of errors around the baseline values is obtained.

Both sets of methodologies in Parts 1 and 2 can be paired depending on the selected choice. Some of these combinations are presented in this report.

2 Methods for error extraction

The first step in the PSA is the extraction of the uncertainty from the target model variables. When we speak about extracting or measuring uncertainty, we refer to past uncertainty, which in a sense is a deviation from some expected outcome. This section describes the methods used to extract that past uncertainty from the yield and the macroeconomic variables, as well as a method for comparing which methodology provides the best outcome.

2.1 Regression of single model equations or equation systems

The original methodology was developed by Burrell and Nii-Naate (2013). They proposed calculating the uncertainty with the one-year-ahead projection error using the deviation between the historical observations and the observations projected one year in advance, as in equation (1):

$$u_{MElt} = ME^{H}_{l,t} / ME^{B}_{l,t-1} \tag{1}$$

where $ME^{H}_{l,t}$ denotes the historical value of the variable in region *l* at time t and $ME^{B}_{l,t\,-\,1}$ denotes the projected value for the variable in region *l* at the time t – *1*. Yield projections were taken from the model, whereas projections for macroeconomic indicators originated from macroeconomic models produced by the European Commission and other international agencies. This method is equivalent to considering the 'projection error' as the uncertainty. Values were divided by the average of all the observations and, therefore, standardised to a mean of 1. While this approach provided a reasonable level of uncertainty for most of the yield and macroeconomic variables, in cases of extreme historical observations, the distribution of the simulated yield and macroeconomic variables included negative values.

In response to this concern, it was decided to calculate the yield uncertainty by means of an OLS regression using the same structure of the model equation. This approach considerably diminished the number of variables yielding negative values. However, specific cases persisted. With regard to the macroeconomic variables, no model equation allowed implementation of an OLS regression, as these variables are exogenous in the Aglink-Cosimo model. As a result of this limitation, it was decided to retain the original approach for extracting macroeconomic uncertainty and truncate the distribution to eliminate negative values in a refinement of the methodology.

Performing estimations using the structure of Aglink-Cosimo has some costs in terms of possible model revisions (e.g. splitting the coarse grains category into maize and other coarse grains in the 2016 model version) or missing historical variables. The version implemented in 2016 to draw errors consisted of fitting an OLS regression for each equation. The most common equation for modelling yield in Aglink-Cosimo has the following form:

$$Log(YLD_{clt}) = v_{cl} + \delta_{YLD,PP,cl} \cdot Log\left(\frac{PP_{c,l,t-1}}{\xi_{CPCI} CPCI_{clt-1} + (1-\xi_{CPCI}) CPCI_{clt}}\right) + \delta_{YLD,T,cl} \cdot t + u_{clt} \tag{2}$$

where *YLD* corresponds to the dependent variable (i.e. yields in this case), *PP* is the producer price deflated by the cost of production commodity index (CPCI), as defined in the documentation of the Aglink-Cosimo EU module (Araujo-Enciso et al. 2015, p. 16), *c* indicates the crop, *l* indicates the country or a region of the world where we expect correlation among yields, *u* is a random error with zero mean and uncorrelated errors among each crop/country equation, *t* represents time, and v and δ's represent coefficients to be estimated.

In this way, the extracted errors are equal to:

$$\hat{u}_{clt} = Log(YLD_{clt}) - \hat{v}_{cl} - \hat{\delta}_{YLD,PP,cl} \cdot Log\left(\frac{PP_{c,l,t-1}}{\xi_{CPCI}CPCI_{clt-1} + (1-\xi_{CPCI})CPCI_{clt}}\right) - \hat{\delta}_{YLD,T,cl} \cdot t \tag{3}$$

where the accent on the components (^) of this equation and of the following ones denote predicted values.

An additional complexity is that, in several cases, yields are modelled in Aglink-Cosimo following a different specification (e.g. Canadian wheat yields). In that case a linear time trend was estimated as follows:

$$Log(YLD_{clt}) = v_{cl} + \delta_{YLD,T,cl} \cdot t + u_{clt} \tag{4}$$

It is important to bear in mind that with this simple OLS method no correlation is considered between equations and, in principle, more variability is left unexplained.

The estimator used could be varied to be a SUR on blocks of equations where correlations between yields in a region are considered. This means that we should intuitively have lower uncertainty unexplained in the error term and in the simulation of uncertainty. However, even if this SUR estimator works for some regions, the correlation matrices are usually almost singular. Being almost singular means that the model tries to estimate a system but the information of two independent variables is in fact almost the same.

This problem could potentially be solved by including in the system of equations only variables with largest explanatory power and by using their estimated variability also for the other crops. This would, however, mean that the variability of one crop in a region would be applied to other crops in the same region, which is difficult to justify. In other words, it is cumbersome to attribute the measured variability of variables included in the regression to any of the crops excluded from the regression because collinearity problems.

We interpret these errors as the reproduction of past uncertainty in the future after excluding what the model is able to explain. In other words, we expect not all visible variability to be reproduced in future simulations but only the uncertainty that is not explained by the fluctuations in the model. We know that the behaviour of the yield variables is usually linear, thus, these results are in fact very similar to what can be estimated by including only a flexible time trend.

2.2 Methods for error extraction: cubic time trend fitting

The second method is a simple method to extract the errors by means of a cubic time trend. This method can be used for both yields and macroeconomic variables. Moreover, it requires that the errors (i.e. the regression residuals) are obtained as differences between the observed values and a fitted polynomial time trend of third order. In other words, the errors are predicted as differences between observed and predicted values.

The estimation model specified in this case is the following:

$$Y_{clt} = v_{cl} + \alpha_{cl}t + \beta_{cl}t^2 + \gamma_{cl}t^3 + u_{clt} \tag{5}$$

where *Y* represents the dependent variable, either yields or macroeconomic variables, *c* indicates the variable, *l* indicates the country in a region of the world where we expect correlation among yields or macroeconomic variables, *t* represents time, and v and α, β, γ represent parameters to be estimated.

The estimator used here is a SUR estimator. Given the inclusion of the same variables as regressors, the use of a SUR estimator results in the same coefficients as in OLS. In this way the extracted errors are equal to:

$$\hat{u}_{clt} = Y_{clt} - \hat{v}_{cl} - \hat{\alpha}_{cl}t - \hat{\beta}_{cl}t^2 - \hat{\gamma}_{cl}t^3 \quad (6)$$

We interpret these errors as a measure of past uncertainty in the future only after taking into account a cubic time trend. In other words, we can expect that most variability seen in the past would be reproduced in the simulations of future uncertainty, except for the excluded cubic time trend.

2.3 Methods for error extraction: vector autoregressive system of equations

The methods proposed in previous sections do not make any assumptions about long-run relationships between variables. However, we believe that macroeconomic variables influence each other. Therefore, neglecting such relationships could call into question the credibility of the analysis. For this reason, a third method is proposed.

This method is slightly more complex: we extract the errors while considering the dynamic nature of variables over time and the fact that there are long-run relationships among variables in a certain region. A vector autoregression (VAR) system of equations is a multi-equation model in which each endogenous variable depends on its own historical observations and the other variables in the system. The dynamic nature of the variables is considered because two lags are included among the regressor variables (on the right-hand side). The number of lags may vary across regions and could be determined by following a specific test (e.g. Akaike information criterion, AIC[1]). However, considering the length of the time series available for analysis (18 time periods from 1997 to 2015), including more lags might decrease the precision of the estimated coefficients. The correlation among variables, and thus among equations, is considered because the two-years lags of the other regional macroeconomic variables are included. Only the variables whose errors (uncertainty) are correlated are considered.

The errors are obtained as differences between the observed values and a fitted model where we include two lags in the system. These lagged variables capture much of the correlation between macroeconomic variables in a region and of the variability of macroeconomic variables over time. In other words, we expect the errors predicted to be small and, thus, the 'uncertainty' included in the simulations (Part 2) to be lower than in any of the previous methods presented. This lower amount of residual uncertainty arises because we only leave the unexplained variability year over year: it is similar to the unexplained portion of an autoregressive process.

For simplicity we include as an example the estimation model only with one-year lag. The specification would be the following:

[1] Lütkepohl (2004) provides a comprehensive overview of the most relevant selection criteria for the time lag. Among the different options we have opted for the AIC because it allows comparing models with different number of parameters.

$$\begin{aligned} \Delta Y_{1rt} &= \omega_{1rt} + \varsigma_{1rt-1}\Delta Y_{1rt-1} + \cdots + \varsigma_{1rt-1}\Delta Y_{Mrt-1} + \tau_{1r} \\ &\vdots\vdots\vdots\vdots \\ \Delta Y_{Mrt} &= \omega_{Mrt} + \varsigma_{Mrt-1}\Delta Y_{1rt-1} + \cdots + \varsigma_{Mrt-1}\Delta Y_{Mrt-1} + \tau_{Mr} \\ &\qquad \forall r = 1, \ldots, R; \end{aligned} \tag{7}$$

where ΔY_{1rt} represents the change from period t to *t-1* in a dependent variable Y, M indicates the macroeconomic variable ($m = 1, \ldots, M$), r indicates a region (i.e. typically a single country) of the world where we expect correlation among macroeconomic variables, τ is a random error with zero mean and correlated standard errors across variables, t represents time, and ς's and ω's represent parameters to be estimated.

In this way the extracted errors are equal, for each equation, to the following:

$$\hat{\tau}_{mr} = \Delta Y_{mrt} - \widehat{\omega}_{mlt} - \hat{\varsigma}_{m1t-1}\Delta Y_{m1t-1} - \cdots - \hat{\varsigma}_{mrt-1}\Delta Y_{Mrt-1} \tag{8}$$

The equation systems related to the macroeconomic variables are estimated with variables translated into growth rates and approximated by logarithmic differences. This is because typically macroeconomic variables are non-stationary (i.e. they have a unit root). For every country, one system with the following four macroeconomic variables is estimated: consumer price index (CPI), gross domestic product index (GDPI), gross domestic product deflator (GDPD) and exchange rate (XR).

The previous model includes macroeconomic indicators for only one country owing to data limitations. Historically, yearly data are available for only a limited number of years. If we were to include more countries, the number of parameters to estimate would be larger than the number of observations. For this reason, we limit the model to a country-reduced form.

As an alternative to such a reduced form, we could increase the number of observations by using data with a different time frequency, for example quarterly data from national sources. This would allow estimating the model as in equation (7), including all countries simultaneously. However, this would have implications for the uncertainty extraction as yearly deviations might differ from quarterly ones.

Note that we have selected a model with two lags. As mentioned before, it would be preferable to test for the proper number of lags considering criteria such as the AIC. However, the more lags we include, the more observations we lose from the estimation. Eventually, this would lead to an uneven panel of estimated uncertainties depending on the nature of the time series, which could cause problems in the simulation.[2]

2.4 Testing for error extraction

After proposing alternative methodologies, we looked for a measure or indicator that selects the best method. In principle, the maximum likelihood of the models as in equations (2), (5) and (7) could be compared with a likelihood-ratio test, penalising each model with the number of parameters or, alternatively, using the AIC. The problem arises when comparing regression-based methods with a deterministic calculation such as the model in equation (1). For this reason, we propose using a

[2] We tested the AIC in various VARs with different numbers of time lags, ranging from one to three. As a result, we opted to homogenise all models and include two lags for each variable.

unified simple approach for all methods, such as the mean squared error (MSE), which is defined as

$$MSE = \frac{1}{n}\sum_{i=1}^{n}\left(\hat{Y}_i - Y_i\right)^2 \tag{9}$$

where $\hat{Y}_i$ denotes either the fitted value from the models as equations (2), (5) and (7) or the one-year-ahead projected value $\widehat{ME}_{l,t-1}^{B}$, and Y_i denotes the true historical value. The smaller the value of the MSE, the better the model extracts the uncertainty.

3 Methods for error simulation

The second step of the PSA is the simulation of the extracted uncertainty for the medium-term outlook projection period. The simulation process relies on two core assumptions: (i) the choice of a distribution for the extracted uncertainty and (ii) the relationships among the exogenous uncertainties surrounding the variables of interest. In this section we discuss alternative methods for simulating these errors in the future.

3.1 Parametric approaches: multivariate normal or truncated multivariate normal distribution functions

The original methodology used in the PSA until 2016 relies on the assumption that the error is normally distributed with mean equal to zero and constant standard deviation. It is then possible to resample from the known distribution *n* times, knowing its parameters (mean and standard deviation). One of the key choices for the PSA is the relationships among the uncertainties surrounding the variables. For example, uncertainty affecting yields that originates from weather shocks can affect neighbouring areas in similar ways and with similar intensity.

For yields, the method simulating the uncertainty until 2016 assumes a parametric probability density function (PDF), such as the multivariate truncated normal distribution (MTND), denoted $\psi(\boldsymbol{\mu}, \boldsymbol{\Sigma}, \boldsymbol{a}, \boldsymbol{b}; \boldsymbol{u})$, such that:

$$\psi(\boldsymbol{\mu}, \boldsymbol{\Sigma}, \boldsymbol{a}, \boldsymbol{b}; \boldsymbol{u}) = \frac{\exp\left\{-\frac{1}{2}(\boldsymbol{u}-\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\boldsymbol{u}-\boldsymbol{\mu})\right\}}{\int_a^b \exp\left\{-\frac{1}{2}(\boldsymbol{u}-\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\boldsymbol{u}-\boldsymbol{\mu})\right\} d\boldsymbol{u}} \qquad (10)$$

where $\boldsymbol{\mu}$ denotes the mean of the extracted uncertainty vector $\mathbf{u}$, $\boldsymbol{\Sigma}$ denotes the covariance matrix and $\mathbf{a}$ and $\mathbf{b}$ are the low and high truncation points. Truncation is needed to avoid negative or extreme cases. Thus, we select the truncation interval such that is extends from half of the minimum to the maximum historical values of the extracted errors. This has been done in an ad hoc manner to allow a degree of negative skewness. In other words, we wanted to allow for a certain amount of probability mass on the lower-tail side.

For macroeconomic variables, the method simulating the uncertainty assumed until 2016 a multivariate normal distribution (MND), avoiding truncation. The only exception is the price of oil that is truncated owing to its large variability, which would otherwise lead, in some cases, to negative values. For the non-truncated macroeconomic variables, the denominator in equation (10) would be equal to 1 and the formula becomes the numerator rescaled:

$$\psi(\boldsymbol{\mu}, \boldsymbol{\Sigma}; \boldsymbol{u}) = -\frac{1}{(2\pi)^{n/2}|\boldsymbol{\Sigma}|^{1/2}} \exp\left\{-\frac{1}{2}(\boldsymbol{u}-\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\boldsymbol{u}-\boldsymbol{\mu})\right\} \qquad (11)$$

This methodology takes the residual errors from Part 1 (i.e. error extraction) and constructs a covariance matrix and a vector average, which are used as distribution parameters for a multivariate normal or truncated multivariate normal set of estimated distributions.

3.2 Semi-parametric approaches: empirical cumulative distribution functions and copulas

The imposition of a parametric distribution family can be a source of concern as it will shape the outcome of the simulations, especially if the assumed distribution function does not replicate the true distribution. To address this concern, we propose the use of semi-parametric methods. Specifically, we propose using an empirical cumulative distribution function (ECDF), where no functional form is imposed on the uncertainty. The ECDF is denoted $F_n(t) = \frac{1}{n}\sum_{i=1}^{n} 1_{x_i \leq t}$, where $1_{x_i \leq t}$ is a Bernoulli random variable. The ECDF is easily considered for univariate distributions, but application in multivariate cases of more than two variables poses some challenges. To our knowledge, there are not many examples that allow the simulation of our data directly from a multivariate ECDF as well as goodness-of-fit tests. Thus, we turn our attention to copulas as an alternative to capture the multiple relationships among the extracted uncertainties.

The original concept of copulas dates back to Sklar (1959) and has received attention in empirical applications of joint distributions (Goodwin, 2015). A copula is defined as a multivariate distribution function in the unit hypercube $[0,1]^P$ with uniform marginal distributions such that

$$C(u_1, u_2, \dots u_p) = F\left(F_1^{-1}(u_1), \dots, F_p^{-1}(u_p)\right) \tag{12}$$

where $F_1(u_1), \dots, F_p(u_p)$ are the univariate distributions. The density function of the copula can be derived from equation (12) and the marginal density functions are as in equation (13)

$$c(u_1, u_2, \dots u_p) = \frac{f\left(F_1^{-1}(u_1), \dots, F_p^{-1}(u_p)\right)}{\prod_{i=1}^{p} f\left(F_i^{-1}(u_i)\right)} \tag{13}$$

A broad range of copula types are available. The most frequently used copula types are the elliptical and Archimedean copulas. The elliptical copulas include the Gaussian and t copulas, both assuming linear relationship between the variables. However, whereas the former imposes zero tail dependence, the latter only allows for symmetrical tail dependence.

Some of the Archimedean copulas, like the Clayton copula, allow for asymmetrical tail dependence. This representation is convenient when simulating a process in which extreme events (e.g. unfavourable weather shocks) are more frequently occurring together (e.g. bad yield in two competing crops in the same region). Each copula family depicts a different type of dependency among variables. For example, the Frank copula has no tail dependence, while the Clayton copula has low tail dependence. However, Archimedean copulas represent the multivariate relationship, making use of only one correlation parameter, which makes them quite restrictive.

In order to have a flexible system allowing different correlation values for each bivariate relationship within a multiple framework, we propose using the hierarchical Archimedean copula (HAC). The HAC is a system comprising nested bivariate copulas. For example, a three-variable system in an HAC $C(u_1, u_2, u_3)$ with u_2 and u_3 nested should be written as follows:

$$C(u_1, u_2, u_3) = C_{F_0}\left(u_1, C_{F_{23}}(u_2, u_3)\right) = F_0\left(F_0^{-1}(u_1) + F_0^{-1}\left(F_{23}\left(F_{23}^{-1}(u_2) + F_{23}^{-1}(u_3)\right)\right)\right) \tag{14}$$

The advantage of this type of copula is that it maintains flexibility for choosing a marginal distribution. In this case, ECDFs can be used as marginal distributions. The method is semi-parametric in the sense that the marginal density distribution is non-parametric while the joint distribution has a functional form. While the copula can be estimated using different measures of association, such as Pearson, Spearman or Kendall, for the present methodology we selected the Kendall correlation rank.

Moreover, we selected the Clayton copula because it allows non-linear dependence in the lower tail. Such an assumption can be understood to represent a stronger correlation in the occurrence of bad weather events within a specific region. For the macroeconomic variables we chose a Frank copula, since we assumed no tail dependence. The Clayton copula has a correlation parameter range equal to $\theta \in [-1,\infty)\backslash\{0\}$, generator function $F_{\theta}(t)=\frac{1}{\theta}\left(t^{-\theta}-1\right)$ and generator inverse function $F_{\theta}^{-1}(t)= \ (1+\theta t)^{-1/\theta}$. The Frank copula has correlation parameter range $\theta \in \mathbb{R}\backslash\{0\}$, generator function $F_{\theta}(t)=-\log\left(\frac{\exp(-\theta t)-1}{\exp(-\theta)-1}\right)$ and generator inverse function $F_{\theta}^{-1}(t)= -\frac{1}{\theta}\log(1+\exp(-t)(\exp(-\theta)-1))$. The simulation with the HAC is implemented in the R package HAC developed by Okhrin and Ristig (2014).

3.3 Methods for error simulation: testing for equality of simulated and true uncertainty distributions

The test used relates to marginal distributions and analyses whether the simulated uncertainty distributions belong to the same family of the original uncertainty distributions. In this case, we employed the non-parametric method developed by Li et al. (2009) to test statistically if the densities proposed are the same. Such method is implemented in the R package *np,* developed by Hayfield and Racine (2008). The test, known as the kernel consistent density (KCD) test with mixed data types, is constructed by taking the integrated square density differences for two variables. For more details and the formulae, we refer the reader to the paper by Li et al. (2009).

4 Methodology implementation and evaluation

After providing a theoretical background of the proposed methods, we proceed to their implementation and evaluation.

For the yield uncertainty extraction and estimation, we propose two new methodologies in addition to the original.

- The first method is called cubic-parametric (CBCPAR-YIELD). It consists of extracting the error by de-trending the yield with a cubic polynomial as in equation (5) and, successively, in simulating the uncertainty via a multivariate truncated normal distribution as in equation (10).
- The second method is called cubic-nonparametric (CBCNONPAR-YIELD). In the second method we also de-trend the yield with a cubic polynomial as in equation (5), but the uncertainty is simulated assuming a marginal empirical cumulative distribution function with a HAC joint distribution as in equation (12).

These two new methodologies are compared with the original methodology where the yield is de-trended as in equation (2) and then the uncertainty simulated with a multivariate truncated normal distribution as in equation (10). This is what we term the OLSTRND-YIELD method.

Uncertainty in macroeconomic variables poses more challenges than in the case of yields. Here, we propose a total of four new methodologies.

- First we propose a cubic-parametric methodology (CBCPAR-MACRO). It consists of de-trending the macroeconomic variables with a cubic polynomial as in equation (5), and then simulating the uncertainty with a multivariate normal distribution as in equation (10).

- The second method is called cubic-nonparametric macroeconomic methodology (CBCNONPAR-MACRO). It uses a cubic polynomial to de-trend the variables in the error extraction phase as in equation (5) and then uses a marginal ECDF and a HAC joint distribution to simulate the errors in the future as in equation (12).
- The next two methods extract the uncertainty with a VAR in the error extraction phase. These methods employ yearly data. In the simulation phase, we employ parametric (assuming a MND) and non-parametric (assuming a marginal ECDF and a HAC joint distribution) simulation methods obtaining two alternatives, called, respectively, VARYEARLYPAR and VARYEARLYNONPAR.

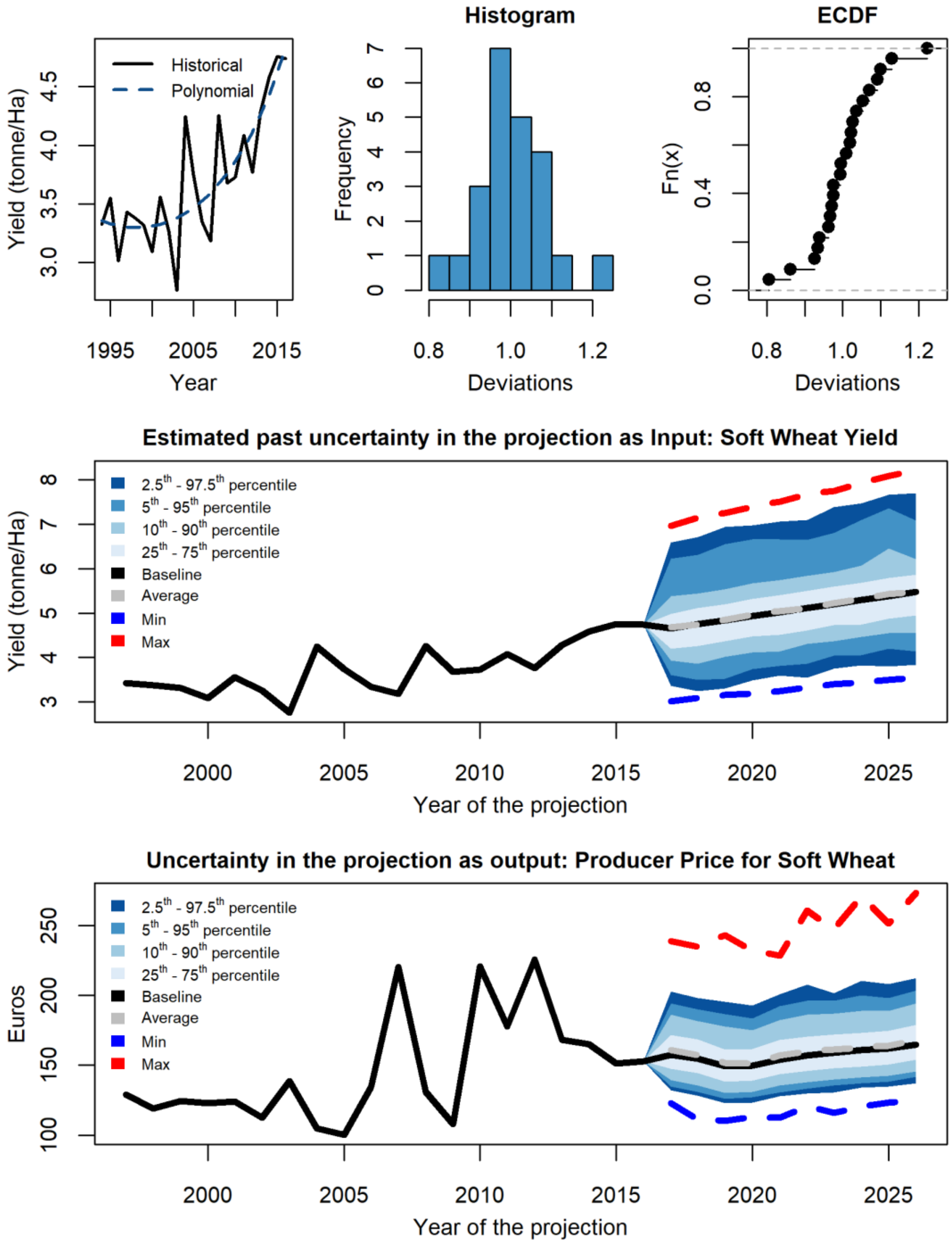
Table 1 summarises the methods proposed for both yield and macroeconomic uncertainty .

Table 1. Proposed methodologies for the PSA: macroeconomic and yield uncertainty

Uncertainty source	Name method	Uncertainty extraction method	Uncertainty simulation method
Yield	OLSTRND-YIELD	OLS (equation 2)	MTND (equation 10)
	CBCPAR-MTND-YIELD	Cubic trend (equation 5)	MTND (equation 10)
	CBCNONPAR-YIELD		ECDF and HAC (equation 12)
Macro	OLSTRND-MACRO	One-year-ahead projection error (equation 1)	MTND (equation 10) MND (equation 11)
	CBCPAR-MACRO	Cubic trend (equation 5)	MND (equation 11)
	CBCNONPAR-MACRO		ECDF and HAC (equation 12)
	VARYEARLYPAR	VAR (equation 7) yearly data	MND (equation 11)
	VARYEARLYNONPAR		ECDF and HAC (equation 12)

Figure 1 provides a graphical illustration of the methodology, implementation and evaluation using the CBCNONPAR-YIELD method as an example. The first step is to estimate the historical uncertainty or deviation from a selected variable. This example estimates the soft wheat yield in the EU-13 with the cubic trend polynomial (Figure 1, upper plot, left graph). We then estimated an empirical marginal distribution for the extracted residuals of the polynomial regression (we centred those deviations around 1). To model correlation among the variables, we included these distribution residuals in a HAC, which then provided marginal distributions accounting for correlation (Figure 1, upper plot, middle graph). The corresponding ECDF is plotted in the upper plot (right graph) of Figure 1. The selected distribution was used to generate 1 000 simulations, which were entered in the model as multiplicative factors that shift the baseline value. The outcome produced the variability around the projection baseline, having as model past uncertainty (Figure 1, middle plot). Finally, the 1 000 values for the uncertainty served to solve the Aglink-Cosimo model 1 000 times. In this example, we show the distribution of EU-13 soft wheat producer price, resulting from the 1 000 solutions (Figure 1, lower plot). The range of endogenous solutions can be obtained to observe the impact of uncertainty on the endogenous price response.

Figure 1. Implementation for the method CBCNONPAR-YIELD using soft wheat yield and producer price in the EU-13 as an example



5 Results of the implementation

This section summarises the results of the statistical methods and tests implemented for evaluating the yield and macroeconomic uncertainty estimates in the extraction and simulation steps of the PSA.

5.1 Selecting the best uncertainty extraction method

The extraction of yield uncertainty comprises two different methodologies that we aim to compare by means of the MSE: an OLS and a cubic regression. Comparing the values of the MSE for both methods suggests that the cubic de-trending provides the lowest MSE values for 76 out of 85 variables. The MSE values for the OLS methodology are lower for only nine cases. As a result, we conclude that the cubic method is the best approach to extract the uncertainty in the majority of the cases (see Table 2).

Table 2. Number of variables with lowest MSE for the extraction of the uncertainty

Method	**Number of variables with the lowest MSE**		
YIELD	OLS		Cubic
	9		76
MACRO	One-year-ahead error	Cubic	VAR yearly
	0	5	35

For the extraction of macroeconomic uncertainty, we have three methods to evaluate: (1) one-year-ahead error, (2) cubic polynomial de-trending and (3) VAR with yearly data. The comparison of the methodologies follows the same logic as for the yield uncertainty. We look at the method that provides the lowest MSE value for each of the 40 macroeconomic variables included in the PSA. The one-year-ahead error (method 1) does not provide the lowest MSE for any of the variables, the cubic polynomial (method 2) provides the lowest MSE for only five variables, and the VAR for yearly data (method 3) provides the lowest MSE for 35 variables. As the yearly VAR provides the highest number of lower MSE values, we conclude that this is the method of macroeconomic uncertainty extraction that most closely approximates the changes in the macroeconomic variables.

In summary, we can conclude that the best methods for uncertainty extraction are the cubic detrending method in the case of the yield variables and the VAR method with yearly data for the macroeconomic indicators.

5.2 Selecting the best uncertainty simulation method

The second step of the PSA consists in simulating the extracted uncertainty. The simulations are done using the methodologies described in Table 1.

The statistical test we use for assessing the performance of the methods in simulating the uncertainty is the non-parametric KCD test. The logic behind this is to test if the simulated uncertainty distribution and the extracted uncertainty distributions are statistically the same.

The results of the yield simulations are shown in Table 3. We find that the null hypothesis (i.e. the extracted and simulated uncertainty distributions are statistically the same) is rejected at the 5% level of significance for the parametric simulation methods, regardless of the extraction method selected.[3] For the semi-parametric methods, however, the null hypothesis, in almost all cases, cannot be rejected at the 5% significance threshold. The only exceptions are for sunflower seeds in Ukraine and for other coarse grains in Uruguay. These markets have outliers, which become influential points and seem to particularly distort the distribution of the simulated data.

Table 3. Absolute number and proportion of rejections of the null hypothesis by the KCD test at the 0.05 level of significance: yield uncertainty

Region	**Absolute number of null rejections with 0.05 level of significance**			**Proportion of null rejections with 0.05 level of significance**		
	OLSTRND-YIELD	**CBCPAR-YIELD**	**CBCNONPAR-YIELD**	**OLSTRND-YIELD**	**CBCPAR-YIELD**	**CBCNONPAR-YIELD**
China	5	5	0	100%	100%	0%
Europe	12	12	0	100%	100%	0%
CIS	11	11	1	100%	100%	9.1%
India	3	3	0	100%	100%	0%
NMS	11	11	0	100%	100%	0%
North America	14	14	0	100%	100%	0%
Oceania	6	6	0	100%	100%	0%
South-East Asia	5	5	0	100%	100%	0%
South America	18	18	1	100%	100%	5.5%

Note: CIS represents the Community of Independent States, while NMS represents the New Member States of the European Union, accessed after 2004.

In the literature on risk analysis, different authors have acknowledged the importance of estimating yield distributions. Some have used parametric methods, including non-normal distribution families (Ramirez et al., 2003; Sherrick et al., 2004). Other authors have explored the use of non-parametric distributions (Goodwin and Ker, 1998; Ker and Goodwin, 2000; Goodwin and Mahui, 2004; Goodwin and Hungerford, 2015). Together with the distribution, the literature emphasises the importance of testing tail

[3] Argentinian barley yields are simulated to be negative for one year for a very small portion of simulations with parametric methods. This shows the potential problems that parametric methods incur in spreading the simulated observations to unrealistic values.

dependence and the accuracy of the selected copula. In the literature, we did not find implementation of a test that allows us to discern the different types of copulas for the HAC approach. Alternatively, we could use Vine copulas, which allow direct comparison of the structures selected with a maximum likelihood test. However, such approach requires large datasets to estimate and perform the tests. In addition, the selection of the model within the vine copulas method, relies on the order of the variables and marginal distribution functional forms, which are chosen based on expert knowledge. Being aware of the sample size we have, rather than attempting to estimate a set of complex relationships involving functional forms and non-parametric kernel distributions, we opted to implement the HAC method with an empirical distribution, which is a straightforward alternative free of distributional assumptions. The outcome of the KCD test suggests that the choice of the ECDF provides simulations that resemble the true distribution of the uncertainty.

The results of the simulations of macroeconomic uncertainty are given in Table 4.

Table 4. Absolute number and proportion of rejections of the null hypothesis by the KCD test at the 0.05 level of significance: macroeconomic uncertainty

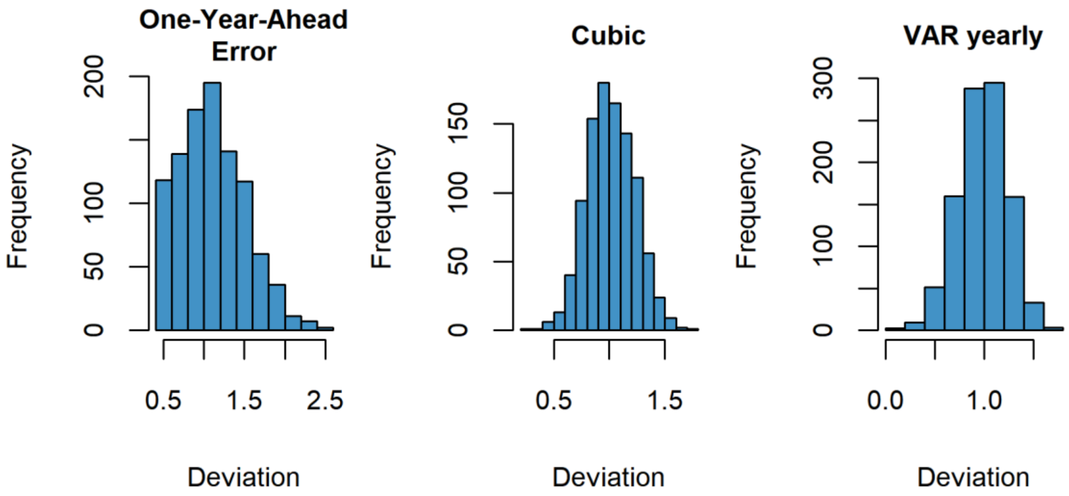
Method	Number of null rejections with 0.05 level of significance	Proportion of null rejections out of the total
OLSTRND-MACRO	40	100%
CBCPAR-MACRO	40	100%
CBCNONPAR-MACRO	2	5.0%
VARYEARLYPAR	40	100%
VARYEARLYNONPAR	1	2.5%

From the results in Table 4, we can observe the high rate of rejection of the null hypothesis for the parametric methods, which involve the assumption of an MND. This means that, by assuming an MND, the simulations will follow a distribution that differs from their actual empirical distribution: consequently, the null hypothesis is often rejected. These results are also important for the different methods used in the extraction phase. Thus, we can also say that the extraction of the uncertainty often leads to uncertainty that is non-normally distributed. This is an important finding because some methodologies rely on the assumption of a normal distribution, which is, in these cases, incorrect. The problems of the normal distribution have been acknowledged for quite some time: very often the normality translates into extreme values never observed in the past, potentially leading to unrealistic simulations (e.g. negative oil prices). In the case of macroeconomic data, the use of copulas to analyse the interaction of macroeconomic indicators has become more popular, with many applications using Archimedean copulas. Nonetheless, these applications often rely on a parametric marginal distribution rather than on non-parametric forms (as an ECDF) as this facilitates making inferences about the results and being able to forecast with those models.

As our main aim in the simulation of uncertainty is not to make a forecast but rather to replicate the previously observed uncertainty in a projection, and given the short time series available, we opted here for an empirical distribution, rather than imposing a functional form. The outcome of the KCD test confirmed our assumptions. For simulating the uncertainty using short time series, it is better to use the empirical distributions as this will avoid introducing bias in the results.

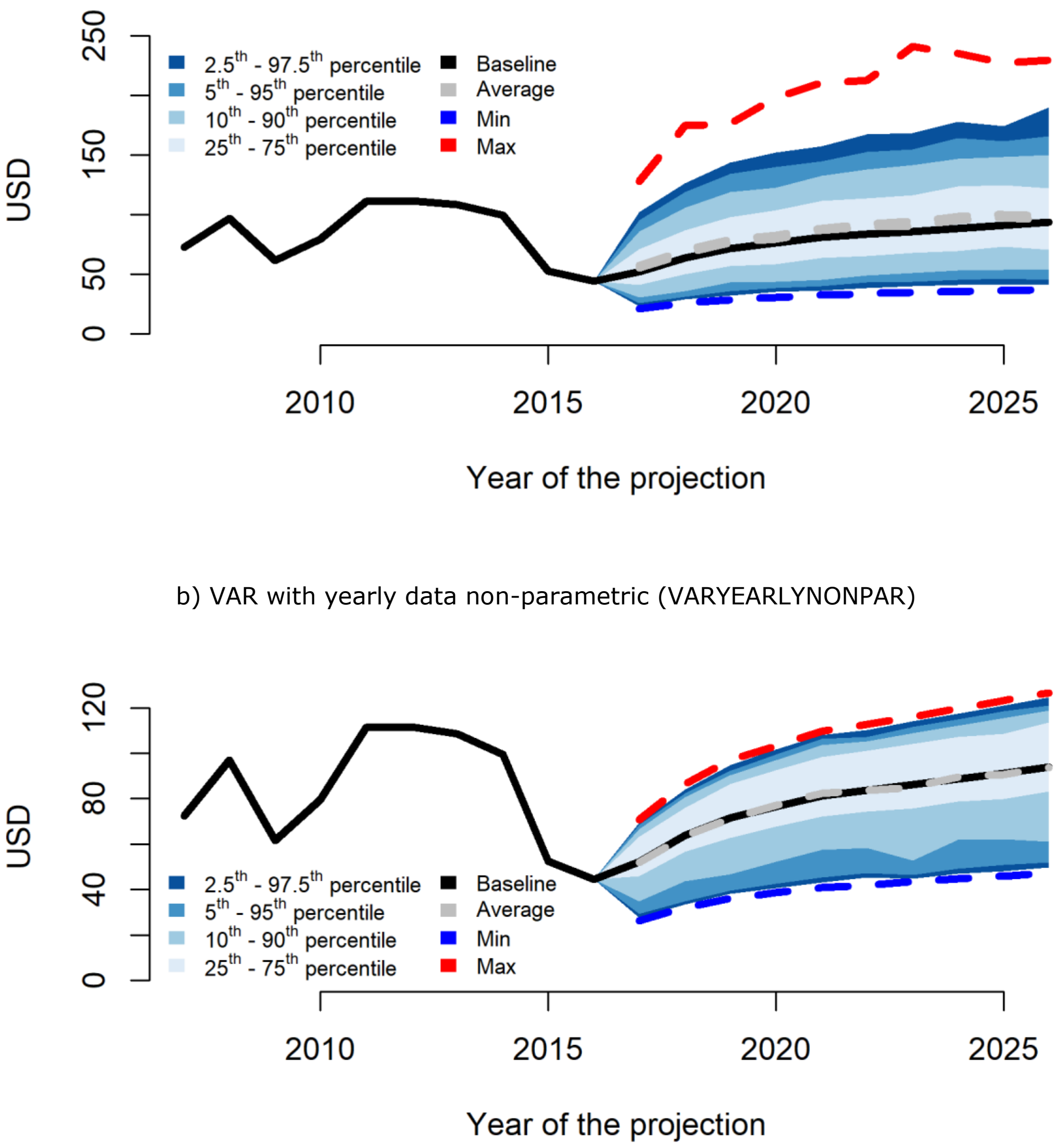
Figure 2 shows the distributions of the simulated oil price uncertainty. Note the wider range for the one-year-ahead error simulations compared with the other methods.

Figure 2. Distributions of the simulated uncertainty for oil prices



The different shapes of the marginal distributions of simulated uncertainty have implications once the simulations are performed. Figure 3 shows the simulated uncertainty surrounding the oil price for the method used until 2016 (Panel a: OLSTRND-MACRO) and the VAR with yearly data non-parametric method (Panel b: VARYEARLYNONPAR).

Figure 3. Uncertainty surrounding the oil price (per Brent barrel) for the projection period:


a) original method (OLSTRND-MACRO)

b) VAR with yearly data non-parametric (VARYEARLYNONPAR)

Both pictures offer a clear view of the effect that the different methodologies have on uncertainty. The original method allows large deviation from the baseline in the high percentiles (top 2.5%) of the simulations while the lower percentiles are more uniformly distributed than in the non-parametric method. On the other hand, the non-parametric method provides lower dispersion of extremely high prices and slightly more dispersed prices lower than the baseline.

After evaluating both stages of the PSA methodology (extraction and simulation), the conclusions are as follows. For yields, we recommend performing the extraction with a cubic time trend and the simulation with a semi-parametric copula (empirical marginal and Clayton copula). For macroeconomic variables, we recommend performing the uncertainty extraction with a VAR and the uncertainty simulation with a semi-parametric copula (empirical marginal and Frank copula).

6 Implications for scenario analysis

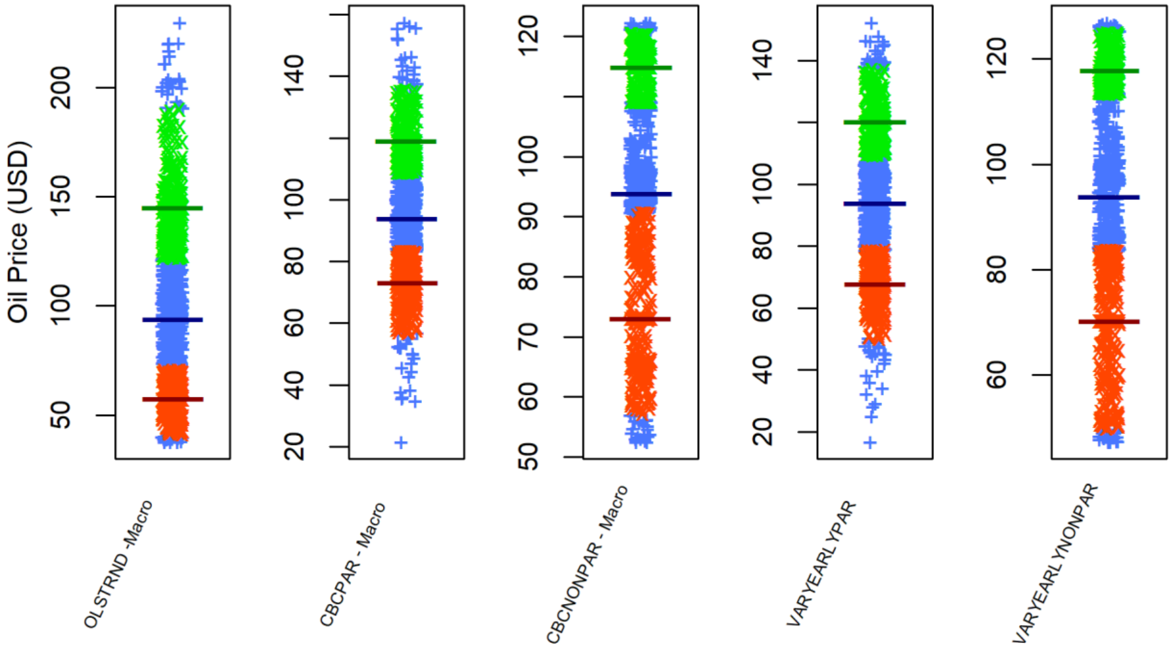
While the MSE and KCD tests allow ranking of the methods for extracting and simulating uncertainty, it is useful to examine how the different proposed methods can influence the scenario analysis. Therefore, we turn our attention to evaluating the possible impact of the different methodologies on a predefined set of scenarios.

One of the most common analyses carried out to study the implications of uncertainty is the 'subset analysis'. As its name indicates, it is based on a subsample of the stochastic simulations of the model. The simulations contained in the subset can be selected with a different number of criteria. For the purpose of this evaluation, we have developed two scenarios, one for macroeconomic uncertainty and another for yield uncertainty.

6.1 Macroeconomic uncertainty

The scenario is based on selecting two subsets of oil prices for the last year of the projection period: an upper and a lower subset. The first scenario contains the upper subset of all the simulations for the oil price within the 75th and 97.5th percentiles. The second scenario is the lower oil price subset, which contains the simulations when oil price is within the 2.5th and the 25th percentiles. The abovementioned scenarios are based on the latest baseline of Aglink-Cosimo contained in the Medium Term Agricultural Outlook published by DG-Agri in collaboration with the JRC (European Commission, 2016). Figure 4 shows the oil price spread for year 2026, identifying the simulations for each scenario by methodology.

Figure 4. Oil price spread indicating the high and low oil price scenarios/subsets for each methodology



The subset contains all simulations for which oil prices are within the percentiles. Note that each plot has a different scale, indicating that each methodology produces a different range of prices. Simulations corresponding to the low oil price subset are in red, simulations within the high oil price subset are in green and the remainder are in blue. The blue dots below and above the subsets correspond to the 0th–2.5th and 97.5th–100th percentiles. The bold lines in green and red represent the average of the high and low oil price subset, respectively, and the blue line is the baseline.

Because it would be cumbersome to analyse all the variables in the model, we restrict the analysis to those variables more likely to be affected directly and indirectly by stochastic shocks. The world prices are the ideal variables for these types of scenario because the world price clearing mechanism in the model accounts for the adjustments in domestic markets and trade.[4] Domestic markets are affected by oil prices directly on the supply side, thus causing movements of the domestic market balance and the domestic prices. The overall adjustment of all domestic markets is reflected in the world price deviations from the baseline.

Before analysing the impact of the stochastic methodologies in the subsets, it is useful to consider the main statistics of the variables of interest (world prices), obtained by solving Aglink-Cosimo using different methods. The results are summarised in Table 5.

Table 5 shows the mean, standard deviation (SD) and coefficient of variation (CV) for the five methodologies (these statistics consider the whole sample). With regard to the average world prices, the original methodology has the largest values, followed by the cubic polynomial methods (both parametric and semi-parametric) and, finally, the VAR. We notice that same uncertainty extraction methods have a similar mean for all the variables of interest. Nonetheless, the SD is different and the pattern we observe is a lower SD value for the semi-parametric methods. Such an outcome confirms our previous hypothesis: imposing normality leads to simulation of extreme cases never observed in the historical data. In turn, those outliers are responsible for a larger SD in the parametric methods. The other interesting finding regarding the CV is that its values are lower for the cubic trend methodologies, followed by the VAR and then by the year-ahead-error projection (i.e. original) method. While the year-ahead-error projection has the largest CV, and can potentially have a broader range of results, this methodology includes bias from imposing normality and does not properly detrend past variation, but rather it is influenced by how good or bad the projection of the macroeconomic indicators has been in the past.

[4] Aglink-Cosimo has a market clearance mechanism for the world markets for the following commodities: wheat, maize, coarse grains, rice, soybean, other oilseeds, sugar, vegetable oil, protein meals, pork meat, beef and veal meat, poultry, butter, cheese, skimmed milk powder and whole milk powder.

Table 5. Main statistics for the variables of interest in the five macroeconomic uncertainty methodologies

	OLSTRND			CBCPAR			CBCNONPAR			VARYEARLYPAR			VARYEARLYNONPAR		
	Mean	**SD**	**CV**	**Mean**	**SD**	**CV**	**Mean**	**SD**	**CV**	**Mean**	**SD**	**CV**	**Mean**	**SD**	**CV**
Crude oil	100	38	38%	96	20	21%	96	18	18%	93	23	25%	93	21	22%
Maize	202	16	8%	199	9	5%	198	6	3%	198	8	4%	198	6	3%
Other coarse grains	204	13	7%	200	7	3%	200	3	2%	200	6	3%	200	3	1%
Rice	450	16	3%	445	11	2%	444	6	1%	443	9	2%	444	6	1%
Wheat	245	21	9%	241	8	3%	241	4	2%	240	7	3%	241	3	1%
Other oilseeds	492	36	7%	482	21	4%	481	15	3%	479	17	4%	480	16	3%
Protein meals	407	40	10%	397	17	4%	396	9	2%	395	15	4%	396	9	2%
Soybean	486	49	10%	473	25	5%	472	14	3%	470	22	5%	472	15	3%
Beef and veal	4 072	67	2%	3 966	193	5%	3 954	125	3%	3 953	149	4%	3 959	102	3%
Pork	3 023	124	4%	2 950	131	4%	2 946	90	3%	2 936	97	3%	2 936	69	2%
Poultry	1 694	460	27%	1 662	63	4%	1 659	31	2%	1 655	51	3%	1 658	29	2%
Sheep	4 054	277	7%	4 010	100	2%	4 006	59	1%	3 999	59	1%	4 002	37	1%
Vegetable oils	939	405	43%	923	34	4%	921	24	3%	920	30	3%	921	25	3%
Butter	3 837	202	5%	3 765	181	5%	3 758	90	2%	3 740	164	4%	3 747	84	2%
Cheese	4 186	322	8%	4 121	170	4%	4 116	74	2%	4 104	146	4%	4 108	67	2%
Skimmed milk powder	3 391	338	10%	3 355	114	3%	3 351	51	2%	3 344	99	3%	3 348	45	1%
Sugar	383	45	12%	374	21	6%	373	15	4%	372	18	5%	373	12	3%
Whole milk powder	3 736	269	7%	3 684	149	4%	3 678	67	2%	3 666	132	4%	3 671	64	2%

Next we turn our attention to the analysis of the extent to which the average world price in the low oil price scenario deviates from the baseline: the results are shown in Table 6.

Table 6. Deviation of average world price with respect to the baseline for the variables of interest in the lower oil price scenario

Variable: prices	OLSTRND	CBCPAR	CBCNONPAR	VARYEARLYPAR	VARYEARLYNONPAR
Crude oil	–22.2%	–21.0%	–22.2%	–29.0%	–29.3%
Maize	–1.7%	–2.7%	–1.7%	–2.7%	–1.9%
Other coarse grains	–0.6%	–2.1%	–0.6%	–2.2%	–0.7%
Rice	–0.4%	–1.3%	–0.4%	–1.1%	–0.4%
Wheat	–0.5%	–1.8%	–0.5%	–1.8%	–0.5%
Other oilseeds	–0.7%	–2.7%	–0.7%	–2.5%	–1.2%
Protein meals	–1.0%	–3.0%	–1.0%	–3.0%	–1.2%
Soybean	–1.2%	–3.4%	–1.2%	–3.4%	–1.7%
Beef and veal	–0.8%	–4.1%	–0.8%	–2.8%	–0.8%
Pork	–1.3%	–3.9%	–1.3%	–4.2%	–1.7%
Poultry	–0.8%	–3.0%	–0.8%	–2.5%	–1.0%
Sheep	–0.2%	–1.6%	–0.2%	–0.9%	–0.3%
Vegetable oils	–0.5%	–1.9%	–0.5%	–1.5%	–0.8%
Butter	–0.4%	–2.8%	–0.4%	–2.0%	–1.4%
Cheese	–0.8%	–3.5%	–0.8%	–3.3%	–1.2%
Skimmed milk powder	–0.9%	–2.9%	–0.9%	–2.8%	–1.0%
Sugar	–1.1%	–3.8%	–1.1%	–3.3%	–1.2%
Whole milk powder	–1.2%	–3.5%	–1.2%	–3.6%	–1.6%

Note that for crude oil price the deviation is greater for the VAR methods than for the year-ahead-error projection and the cubic trend methods. If we compare the simulation step in parametric methods with semi-parametric methods, we observe that for most of the commodities the deviation with respect to the baseline is larger in the case of the parametric methods. Such a larger deviation, as we have argued before, might come from the outliers obtained by imposing normality. This finding is in line with the previous ones: the normal distribution will cause extreme observations (not observed in the past) and these will bias the standard deviation by increasing it.

The results of the subset analysis for the high oil price scenario are shown in Table 7.

Table 7. Deviation of average world price with respect to the baseline for the variables of interest in the high oil price scenario

Variable	OLSTRND	CBCPAR	CBCNONPAR	VARYEARLYPAR	VARYEARLYNONPAR
Crude oil	23.7%	27.0%	24.7%	24.7%	27.2%
Maize	2.5%	5.2%	6.5%	2.8%	3.9%
Other coarse grains	1.0%	3.6%	4.0%	1.2%	2.6%
Rice	0.8%	2.2%	2.5%	0.8%	1.3%
Wheat	1.1%	3.3%	3.8%	1.2%	2.3%
Other oilseeds	2.2%	5.1%	6.4%	2.2%	3.1%
Protein meals	1.3%	4.5%	5.7%	1.6%	3.0%
Soybean	2.0%	5.2%	7.1%	2.6%	3.7%
Beef and veal	0.4%	5.4%	4.1%	1.4%	3.1%
Pork	2.0%	5.7%	5.9%	2.1%	4.3%
Poultry	1.0%	4.5%	4.2%	1.5%	2.8%
Sheep	0.6%	2.3%	1.2%	0.4%	0.9%
Vegetable oils	2.1%	4.0%	5.3%	2.3%	2.7%
Butter	1.2%	4.5%	2.5%	1.6%	2.3%
Cheese	1.3%	4.8%	4.0%	1.5%	3.7%
Skimmed milk powder	1.0%	3.7%	3.1%	1.1%	2.9%
Sugar	2.1%	6.8%	6.3%	2.7%	4.9%
Whole milk powder	1.4%	4.6%	4.5%	1.6%	3.5%

The average subset crude oil prices show similar deviations from the baseline, with values between 23% and 27%. When we compare the parametric with the semi-parametric methods, it is not possible to discern a clear pattern of variables with a systematically lower or higher deviation. Nonetheless, it seems that in the high oil price scenario the semi-parametric methods result in a larger deviation than the parametric methods in the majority of the cases. This finding is the opposite of what occurs in the low oil price scenario, in which, we argued, outliers in the normal distribution would positively bias the SD. Indeed, outliers might be one reason for the differences in deviations between the subsets, another is the assumption of symmetry or non-symmetry of the distributions. Unlike the normal distribution, which assumes symmetry, the empirical distributions do not impose it; hence the distribution originating from the semi-parametric methods can be skewed. In the low oil price case, the average oil price for the included simulations in the parametric method is lower because of the symmetrical nature of the outliers implicit in a normal distribution. In the high oil price scenario, however, the average oil price for the included simulations in the parametric method is slightly higher than in the non-parametric methods but the spread is lower in the latter (see Figure 3). This is due to the fact that different methods replicate different ranges for the oil price uncertainty.

Overall we observe that the mean of the simulations is affected by the extraction method, while the SD and its skewness are affected by the nature of the simulation technique (parametric or semi-parametric). The SD and its skewness are more crucial for the subset analysis because they are related to the shape of the distribution and the position of the quantiles.

6.2 Yield uncertainty

So far we have argued that normality by means of symmetry and long tails of the probabilistic distribution (i.e. allowing values at either extreme of the distribution, which potentially create outliers) can bias the results, therefore, creating simulations that do not fit the extracted data. Another implication of the simulated probabilistic distribution is the correlation between yields in different regions. The correlation remains a key issue when doing subset analysis for which the subsample is conditional on observations meeting specific criteria (i.e. values below a threshold or within a range). For example, we perform a scenario analysis with a subset sample in which wheat yields in Russia, Ukraine and Kazakhstan are below the 50th percentile. Only the simulations in which the wheat yield in those three countries are below the 50th percentile are included. Moreover, we limit the analysis to the variables we consider to be most interesting for this analysis: yield, production, producer prices, and exports.

First, following the example of the scenarios for macroeconomic variables, we report, without sub-setting, descriptive statistics for the variables in each of the simulation methods tested (Table 8).

Table 8. Main statistics for the variables of interest in the three yield uncertainty methodologies

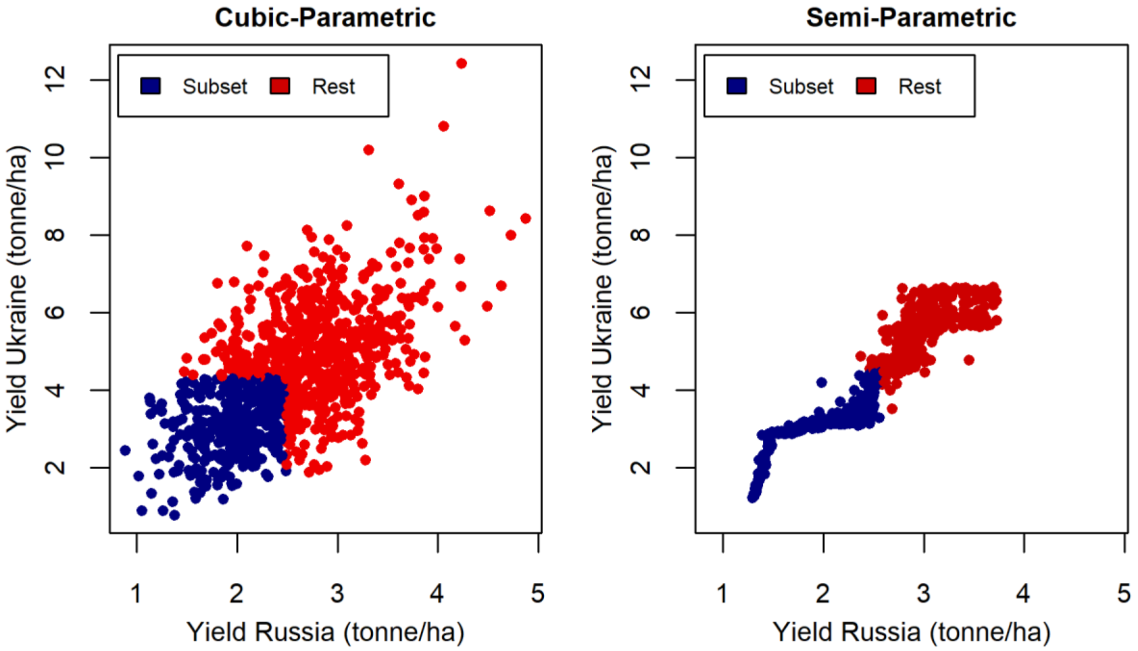
Country	Variable	CBCNONPAR			CBCPAR			OLSTRND		
		Average	SD	CV	Average	SD	CV	Average	SD	CV
Kazakhstan	Exports	6 342	2 659	42%	6 226	2 821	45%	5 954	2 772	47%
	Producer price	63 183	6 667	11%	64 125	10 072	16%	68 583	9 840	14%
	Production	15 077	2 994	20%	14 902	3 335	22%	14 558	3 260	22%
	Yield	1.35	0.54	40%	1.34	0.57	42%	1.28	0.54	42%
Russia	Exports	32 680	6 301	19%	32 853	6 878	21%	32 025	4 942	15%
	Producer price	10 556	869	8%	10 582	1 208	11%	11 241	1 008	9%
	Production	68 605	8 404	12%	68 879	9 366	14%	67 719	6 812	10%
	Yield	2.5	0.55	22%	2.52	0.66	26%	2.34	0.43	19%
Ukraine	Exports	15 418	4 033	26%	15 503	4 710	30%	14 032	4 060	29%
	Producer price	3 103	291	9%	3 121	421	14%	3 362	446	13%

Country	**Variable**	**CBCNONPAR**			**CBCPAR**			**OLSTRND**		
		Average	**SD**	**CV**	**Average**	**SD**	**CV**	**Average**	**SD**	**CV**
	Production	29 493	4 425	15%	29 605	5 287	18%	27 969	4 506	16%
	Yield	4.38	1.23	28%	4.46	1.52	34%	3.97	1.2	30%
World	**Exports**	181 800	5 828	3%	182 280	7 873	4%	178 795	5 709	3%
	Production	806 070	16 086	2%	807 597	20 602	3%	802 931	16 193	2%
	World price	242	17	7%	242	24	10%	257	21	8%

The original method (OLSTRND) has the lowest average for all variables except prices. Using the cubic trend extraction method, the parametric and semi-parametric simulation methods result in different values. The way we simulate the uncertainty distributions in the future seems more important than the uncertainty extraction methods. These simulated uncertainty distributions have a recursive effect that does not necessarily appear in the same year as the shock. Most of the yield shock is transmitted to the following year by its effect as an exogenous (shocked) variable in the returns per hectare equation. Thus, the distribution of final simulated yields has a long-lasting effect on the variables resulting from the model analysed here. With regard to the SD, the cubic polynomial with a parametric distribution (CBCPAR) has the largest absolute values for each variable. Nonetheless, in relative terms to the mean (i.e. in terms of the coefficient of variation), such a pattern is less clear.

The number of simulations fulfilling the selection criteria differs in each of the methodologies. For the OLSTRND, CBCNONPAR and CBCPAR methodologies, the number of simulations is 190, 449 and 200, respectively. Note that the two parametric methods have a similar subset size, whereas in the semi-parametric method the number of simulations fulfilling the criteria is more than doubled. This is the outcome of the Clayton copula whereby we have assumed low tail dependence and, therefore, the correlation among simulations with below-average yield is strong (see Figure 5).

Figure 5. Scatterplot for wheat yield uncertainty in Ukraine and Russia, cubic-parametric and semi-parametric methods indicating subsets



Next, we analyse the yield deviation from the baseline for each method. The values are similar for both parametric methods, OLSTRND and CBCPAR, while in the CBCNONPAR method the deviation is lower (Table 9). The results are consistent with our previous findings. The normal distribution creates extreme cases that bias the variation (see Figure 1 dispersion, where the parametric method produces extreme cases not observed in the ECDF–Clayton copula). Following the larger deviation of the yield for the parametric methods, the rest of the variables also have larger deviation than in the semi-parametric method, almost in all cases.

Table 9. Deviation of the yield scenario with respect to the baseline for the variables of interest

Country	Variable	CBCNONPAR	CBCPAR	OLSTRND
Kazakhstan	Exports	–33%	–37%	–40%
	Producer price	16%	19%	25%
	Production	–17%	–20%	–21%
	Yield	–29%	–33%	–35%
Russia	Exports	–15%	–19%	–16%
	Producer price	10%	12%	16%
	Production	–11%	–13%	–12%

Country	Variable	CBCNONPAR	CBCPAR	OLSTRND
	Yield	–20%	–24%	–23%
Ukraine	Exports	–23%	–23%	–28%
	Producer price	13%	14%	20%
	Production	–14%	–14%	–17%
	Yield	–26%	–25%	–30%
World	Exports	–3%	–3%	–4%
	Production	–2%	–2%	–2%
	World price	8%	9%	14%

Although the semi-parametric method has lower deviation with respect to the baseline, it includes a higher number of observations than the parametric methods. We believe that such a method is more accurate. First, the normal distribution biases the variation, and hence it ‘exaggerates’ the past uncertainty in the tails symmetrically. This is avoided with an ECDF. Second, the normal distribution imposes a linear relationship, while the Clayton copula is more flexible and assumes a non-linear relationship that simulates the behaviour of observed uncertainty.

7 Implications of the proposed methodologies

The implementation of new methodologies for the PSA with Aglink-Cosimo poses some challenges that have already been acknowledged. The first and most important is the limited number of observations in the past from which to extract the uncertainty. There are fewer observations available for macroeconomic variables than for yield variables. In the methodologies proposed for macroeconomic uncertainty we deal with this issue by performing VARs including variables for each country considered. For the extraction of the past yield uncertainty, the cubic polynomial offers reasonable results and the current number of observations allows the estimation. Nonetheless, as the number of historical observations remains low, new data incorporated in the analysis can potentially shift the results, especially if they come from a harvest failure or a bumper crop year.

Among the proposed methods, we tested the use of parametric and semi-parametric methods. The latter do not impose a distribution on the marginal distributions, which we think is less restrictive and, thus, preferable. The parametric methods have been shown to bias the variance of the true distributions, hence yielding outliers in the simulations, especially in the distribution tails. By distributing the uncertainty equally, such methods do not accurately replicate the past uncertainty. Indeed, the KCD tests confirmed our hypothesis, as the results for the parametric simulation method reject the null hypothesis of this test. This finding therefore confirms that the extracted uncertainty does not follow a normal distribution for any of the extraction methods. Thus, for simulations, it is best to make use of the empirical distributions rather than imposing a functional form that can bias the simulations.

Additionally, looking at the bias of the variance, the normal distribution assumes a linear relationship among the variables. On the other hand, the copula method corrects this issue by assuming non-linearity with tail dependence. We think that this more flexible system is better. In this sense, it would be interesting to carry out further research to determine the true nature of the correlations among actual variables, potentially with bootstrapping procedures as described in Genest et al. (2009).

Overall, despite having lower variation, the semi-parametric methods should be preferred over their parametric counterparts. The simulated uncertainty represents, in this case, more closely the true distribution of the measured uncertainty. For a stochastic method considering the yield variables, our recommendation is to use the cubic trend for extracting the uncertainty and the Clayton copula with ECDF marginal distributions for simulating the uncertainty. For a stochastic method considering the macroeconomic indicators, we recommend the use of a VAR for extracting the uncertainty and a Frank copula with ECDF marginal distributions for simulating the uncertainty.

The methods presented are just a few of the many options that could be implemented. More flexible methods, such as Vine copulas, could provide further insights into the yield uncertainty but are more complex and data demanding. The methodologies we propose in this report are well documented in the literature, easy to implement, and require fewer data than other methods.

The semi-parametric methods are more flexible than the parametric ones, which impose a certain functional form. For this reason, they are also more prone to changes if observations are added to the historical data, especially if the data come from extreme cases. This behaviour should be reflected in our simulations in as much as it reflects the fact that uncertainty has a changing nature.

Finally, the uncertainty considered in the PSA is based on past observations. We acknowledge that in the future we might have new sources of variability (e.g. climate change). Nonetheless, our aim is not to speculate on unobserved future sources of uncertainty, but rather to provide a benchmark for policy, technical and scientific analysis, based on plausible assumptions on past observed variability.

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