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Bounded-rationality and heterogeneous agents: Long or short forecasters?

Beqiraj Elton
Di Bartolomeo Giovanni
Di Pietro Marco
Serpieri Carolina

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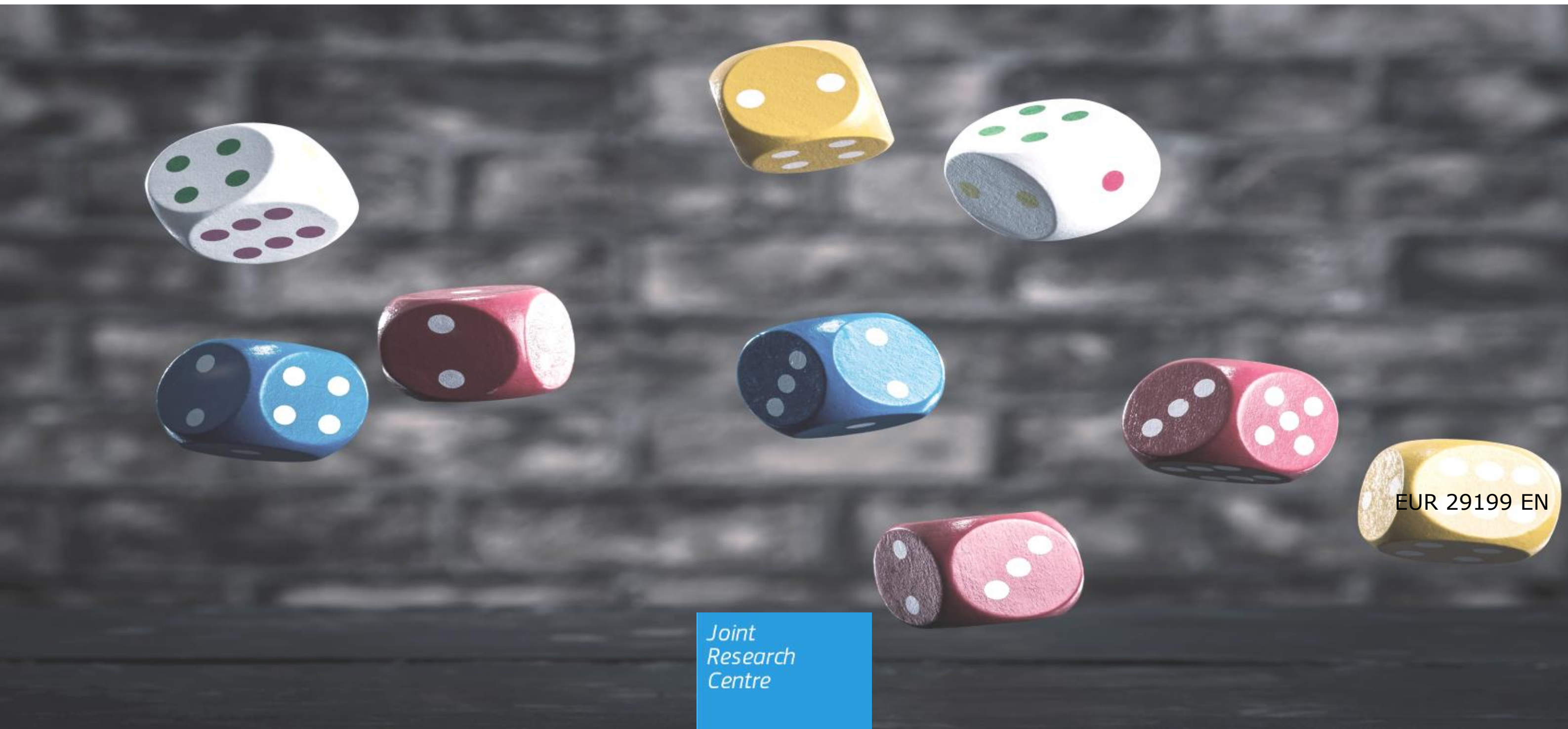

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Contact information
Name: Carolina Serpieri
Address: Via E. Fermi 2749
I-21027 Ispra (VA) – Italy – TP 26A
E-mail: carolina.serpieri@ec.europa.eu
Tel.: +39/0332786292

Name: Elton Beqiraj
Address: Dipartimento di Economa e Diritto,
Facoltà di Economia, Sapienza Università di Roma
Via del Castro Laurenziano,9 00161, Roma, Italia
E-mail: elton.beqiraj@uniroma1.it

Name: Giovanni Di Bartolomeo
Address: Dipartimento di Economa e Diritto,
Facoltà di Economia, Sapienza Università di Roma
Via del Castro Laurenziano,9 00161, Roma, Italia
E-mail: giovanni.dibartolomeo@uniroma1.it
Tel.: +39 06 49766400

Name: Marco Di Pietro
Address: Dipartimento di Economa e Diritto,
Facoltà di Economia, Sapienza Università di Roma
Via del Castro Laurenziano,9 00161, Roma, Italia
E-mail: marco.dipietro@uniroma1.it

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Authors

Elton Beqiraj, Sapienza University of Rome
elton.beqiraj@uniroma1.it

Giovanni Di Bartolomeo, Sapienza University of Rome
giovanni.dibartolomeo@uniroma1.it

Marco Di Pietro, Sapienza University of Rome
marco.dipietro@uniroma1.it

Carolina Serpieri, JRC European Commission
carolina.serpieri@ec.europa.eu

Abstract

Our paper estimates and compares behavioral New-Keynesian DSGE models derived under two alternative ways to introduce heterogeneous expectations. We assume that agents may be either short-sighted or long-horizon forecasters. The difference does not matter when agents have rational expectations, but it does when a fraction of them form beliefs about the future according to some heuristics. Bayesian estimations show that a behavioral model based on short forecasters fits the data better than one based on long forecasters. Long-horizon predictors exhibit very poor predictive ability, whereas the short forecasters' model also outperforms the rational expectation framework. We show that the superiority is due to its ability to capture heterogeneous consumers' expectations. Finally, by Monte-Carlo-filtering mapping, we investigate the indeterminacy regions to complement existing literature.

1. Introduction

The empirical evidence finds a substantial heterogeneity in the formation of beliefs and usually rejects the rationality of inflation forecasts.[1] Ample evidence of various kinds suggests that heterogeneous expectations are very relevant. Evidence for heterogeneous beliefs is also found in laboratory experiments.[2]

Two popular approaches to incorporate heterogeneous expectations in the New Keynesian model were proposed by Branch and McGough (2009) and Massaro (2013). Both assume that at least part of the population form expectations by some heuristics, but they differ in the assumed time–horizon of forecasts. Focusing on short-horizon forecasters, Branch and McGough (2009) derive aggregate dynamics depend on one–period–ahead subjective heterogeneous forecasts.[3] Massaro (2013) assumes, instead, that agents choose optimal plans while considering forecasts of macroeconomic variables over an infinite horizon, as a result, the predicted aggregate dynamics hinge on long-horizon forecasts.[4]

The popularization of models based on short-horizon forecasts and long-horizon expectations raises important and relevant research questions: (a) do short- and long-horizon forecasts fit the data well? (b) Is the empirical fit of short- and long-horizon forecasts better than the one of rational expectation hypothesis (REH)?

Our paper's approach to these questions is based on Bayesian estimations. We investigate the performance in fitting data of the two alternatives in terms of marginal likelihood and estimated moments. We also explore the indeterminacy regions implied by the two approaches by Monte-Carlo-Filtering mapping.

Our main findings can be summarized as follows.

1. A behavioral New Keynesian model derived with short forecasters fits the data better than one based on long forecasters. Short forecasters also outperform rational expectations. Comparisons are based on log–marginal likelihoods and moments. The moments implied by the model with short-horizon predictors are the closest to the observed data. The model with long-horizon predictors exhibits instead a poor predictive ability.
2. We provide evidence that shows how the superiority of the short forecasters' model over the others considered is due to the behavioral specification of the dynamic aggregate demand equation. Short–horizon predictors better fit heterogeneity in consumers' expectations than long-horizon ones. By contrast, the heterogeneity in producers' expectations is less relevant to discriminate between the models.
3. The analysis based on the Monte-Carlo Filtering mapping complements the results on monetary policies obtained by Branch and McGough (2009) and Massaro (2013). Exploring a reasonable range of parameters, the estimates of the behavioral New Keynesian model with long–horizon predictors are strongly affected by its determinacy conditions. Precisely, it tends to overestimate the degree of price stickiness and the central bank's response to the output gap.
4. The main policy implications that one may draw from our paper are that DSGE models for policy analysis should incorporate heterogeneous expectations.

[1] Mankiw *et al*. (2004) is the main reference. Early studies are Roberts (1997) and Campbell and Mankiw (1989). Similar results are also obtained by Carroll (2003), Branch (2004), Andolfatto *et al*. (2008), Pfajfar and Santoro (2010), Andrade and Le Bihan (2013), and Coibion and Gorodnichenko (2015), Dovern (2015), Dräger *et al*. (2016).

[2] See Hommes *et al*. (2005), Adam (2007), and Hommes (2011).

[3] Agents are boundedly rational and behave similar to Euler equation learners in the sense that optimal decisions are based on the perceived Euler equation. See Honkapohja *et al*. (2013).

[4] See also Preston (2006).

Moreover, all agents should be modeled according to the Branch and McGough's (2009) approach in such a model setup.

Our research is related to several branches of the literature. In particular, they concern monetary policy analysis in the New Keynesian model with heterogeneous expectations and the empirical fit of REH models vs. models of bounded rationality and the empirical fit of DSGE models in general.

In the REH, the forecasting horizon does not matter. By contrast, it does when agents use heuristics to form their expectations. To grasp the intuition, in the realm of DSGE models, consumers use short-sighted forecasts when they behave according to the Euler equation. In such a case, they have to forecast the next period inflation and consumption to decide their current action. Instead, they have to look at long–horizon forecasts, if their life cycle–permanent income matters. However, REH imply that short and long forecasts are consistent and make no difference for the consumer's choices. The life cycle-permanent income is obtained from the Euler equation and budget constraint, iterating forward and applying the law of iterated expectations. Once heuristics are considered, the one–by–one equivalence is broken and heuristics for short–term expectations and long–term forecasts are no more equivalent.

Considering some forms of imperfection in the expectations formation process, recent theoretical studies show that several puzzles of traditional macro models can be solved. For instance, heterogeneous expectations are a parsimonious way to obtain models consistent with inflation inertia, output persistence, and "discounted forward guidance."[5] These studies remain rooted in classical economics and use its powerful tools. By parsimonious approaches, agents are modeled as maximizing utility subject to constraints, but agents may use heuristics to form expectations. In this framework, these agents can be defined individually (instead of fully) rational (Deak *et al*., 2017). It is worth mentioning that other similar approaches have been proposed within the classic economics. It has been, e.g., assumed that agents may form expectations by using some learning algorithms (Evans and Honkapohja, 2001) or that they are subjected to infrequent information updating, i.e., inattentiveness.[6]

The existing two popular approaches to incorporate heterogeneous expectations in the New Keynesian model have different normative and positive implications. The technical details and the differences from the REH are discussed in Honkapohja *et al*. (2013), Massaro (2013), Branch and McGough (2016). Branch and McGough (2009) and Massaro (2013) also study the normative implications of the two approaches for monetary policy, when the central bank sets the nominal interest rate according to a Taylor-type rule. Investigating the determinacy regions associated to their different behavioral New-Keynesian DSGE monetary models, they find that heterogeneous expectations can undermine some standard results. Optimal policies, distributive effects, and welfare analysis are also affected by the assumption about the prediction horizon of boundedly rational agents (cf. Gasteiger, 2014, 2017; Di Bartolomeo *et al*., 2016; Beqiraj *et al*., 2016).

In the empirical field, Milani (2007) shows that when the conventional assumption of rational expectations is relaxed to assume learning by economic agents, "mechanical" sources of persistence may be omitted. Learning can induce realistic levels of persistence in the models, so the estimated degrees of habit formation and indexation fall from values close to one to values close to zero. Model with learning fits the data better than the model with rational expectations augmented with indexation and habits. Milani (2011) considers the role of psychological factors, market sentiments, and less-than-

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[5]The latter is related to the so-called forward guidance puzzle. See McKay *et al*. (2016) and Di Bartolomeo *et al*. (2017).

[6]See, among others, Gabaix and Laibson (2002), Mankiw and Reis (2002, 2003), Sims, (2003), Moscarini (2004).

fully-rational shifts in beliefs on economic fluctuations. The empirical results show that expectations shocks are a major driving force of the U.S. business cycle; they account for roughly half of business cycle fluctuations. Relaxing the rational expectations assumption, he allows for learning by economic agents and by expectation shocks he captures waves of optimism and pessimism. These waves in turn affect the formation of expectations, leading agents to form forecasts that deviate from those implied by their learning model.

Closely related to our work, Deak *et al*. (2017) estimate a New Keynesian behavioral model with heterogeneous agents and bounded-rationality. In their model, a fraction of individually rational agents use simple adaptive expectations rules to forecast aggregate variables exogenous to their micro-environment. The remaining agents are instead fully rational. By Bayesian estimations, they compare their benchmark to some alternatives.[7] Their model fits the data better than in the standard rational expectations New Keynesian framework or in a model based on Euler learning. Compared to Deak *et al*. (2017), we consider that agents may be either long or short forecasters. Therefore, we extend their work since they do not consider time horizon alternatives for the boundedly rational agents, which is instead our primary goal. To the best of our knowledge, we are the first to compare estimations of behavioral New Keynesian models embedding expectations heterogeneity under the form of long–horizon and short–sighted forecasts.

The rest of the paper is organized as follows. Section 2 illustrates the theoretical background of our research and introduces different expectations formation processes. Section 3 presents the results of our estimates. Section 4 discusses the underlying properties of the expectation model that best fit the data. Section 5 concludes.

2. Theoretical background

We consider a simple generalization of the small–scale New Keynesian DSGE model to account for heterogeneity in expectations. The economy is populated by a continuum of households represented by an interval $[0,1]$. Each household consists of a continuum of agents which are employed across firms and share dividends within the household. Households maximize the same utility function, but may form their expectations according to different processes, expectation operators ($\mathcal{E}$) are thus indexed by i.

Agents of kind i choose their consumption, $\{C_i\}_t^\infty$, and labor supply, $\{N_i\}_t^\infty$, path to maximize the expected present discounted value of their utility, i.e.,

$$\mathcal{E}_{i,0}\sum_{t=0}^{\infty}\beta^t\left(\frac{exp\,(z_t^d)C_{i,t}^{1-\sigma}}{1-\sigma}-\upsilon\frac{N_{i,t}^{1+\omega}}{1+\omega}\right) \tag{1}$$

where $\beta \in (0,1)$ is the discount factor; υ denotes the labor disutility scaling parameter; $\mathcal{E}_{i,t}$ refers to type i expectation operator at t; $C_t^i \equiv \left(\int_0^1 C_t^i\,(j)^{\frac{\epsilon-1}{\epsilon}}dj\right)^{\frac{\epsilon}{\epsilon-1}}$ is the composite consumption good, where $C_t^i(j)$ is the quantity of good $j\ \in\ [0,1]$ consumed by the household i in period t; z_t^d is a AR(1) preference shock. The preference shock has the following form:

$$z_t^d = \rho_d z_{t-1}^d + e_t^d, \tag{2}$$

where e_t^d is a white noise stochastic disturbance. Independently to their ability to forecast economic variables, we assume that all the agents have unbiased (rational) expectations about the evolution of their preferences.[8]

[7]The benchmark is based on Eusepi and Preston (2011).

[8]The assumption does not affect the empirical estimations but it seems the most reasonable.

The consistent aggregate price index for consumption is defined by the following expression $P_t \equiv \left[\int_0^1 P_t(j)^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}}$ with ϵ denoting the elasticity of substitution among goods.

Similarly, we assume that the supply side of the economy is characterized by a continuum of firms of each production type j, operating under monopolistic competition. Firms maximize profits, but may use different processes to form their expectations. As long as prices are sticky, heterogeneity in expectations matters. We assume a Calvo (1983) price setting framework: firms can reset prices with probability $(1-\xi_p) \in (0,1)$ each time t. Moreover, labor is the only input needed to produce good j according to $Y_t = \exp(a_t) N_t$ with $\exp(a_t)$ denoting the total factor productivity shock which follows an AR(1) structure:

$$a_t = \rho_a a_{t-1} + e_t^a \tag{3}$$

where e_t^a is a *i.i.d.* process.

2.1 Rational agents

In the above described simple setup, assuming homogeneous rational agents, the traditional log-linearized version of the New Keynesian–DSGE model is easily represented by the following set of equations (lower-case letters indicate log-deviations):

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(r_t - E_t \pi_{t+1} + E_t \Delta z_{t+1}^d) \tag{4}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa m c_t + z_t^m \tag{5}$$

$$mc_t = w_t + n_t - y_t \tag{6}$$

$$w_t = \sigma y_t + \omega n_t - z_t^d \tag{7}$$

$$y_t = a_t + n_t \tag{8}$$

where $\kappa = (1-\xi_p)(1-\beta\xi_p)/\xi_p > 0$ and E_t indicates the mathematical expectation operator conditional on the model and the information available at the end of period $t-1$ ($\mathcal{E}_{i,t} = E_t$, $\forall\, i$). Regarding the variables, y_t is the output gap; π_t is the inflation; r_t indicates the nominal expected interest rate; mc_t is the real marginal cost; w_t is the real wage; $\Delta z_{t+1}^d = z_{t+1}^d - z_t^d$ and z_t^m is a AR(1) shock to the markup evolving as:

$$z_t^m = \rho_m z_{t-1}^m + e_t^m \tag{9}$$

where e_t^m is a white noise.

The economic interpretation of the model (4)–(5) follows. Equation (4) represents the dynamic IS (Euler equation);[9] (5) describes the New Keynesian Phillips curve; (6) defines the real marginal cost; (7) is the labor supply; (8) is the production function.

In order to close the model, we specify the behavior of the central bank. We assume that the central bank uses the following simple Taylor rule to set interest rates:

$$r_t = \delta_\pi \pi_t + \delta_y y_t + e_t^r \tag{10}$$

[9]Note that we have considered the market clearing condition: $y_t = c_t$.

It thus responds to both inflation and real activity according to the coefficients δ_π and δ_y, respectively, and e_t^r is a (white noise) monetary policy shock.

2.2 Heterogeneity in expectations

We now assume that the economy is populated by heterogeneous agents, who may differ in the way they form their expectations. In such a case, the predicted aggregate dynamics depend on the assumption about horizon forecasts. Specifically, first-order conditions and budget constraints can be represented by linking current and one-period ahead (expected) variables or, using forward iterations, by relating current and any–period ahead (expected) variables. As long as expectations are rational, these representations are equivalent. But, when the micro-foundations underpinning the New Keynesian model are solved relaxing the rational expectations assumption, the expectations horizon matters since the representations are no longer equivalent (Preston, 2006).

Along the above insights, to derive parsimonious micro-founded representations of DSGE sticky–price models that are consistent with boundedly rational individuals, we consider two polar cases. We assume that boundedly rational agents may be either short-sighted (SSFs) or long-horizon forecasters (LHFs). SSFs form their expectations focusing on one-period ahead forecasts based on heuristics. LHFs, instead, use heuristics to forecast macroeconomic variables over an infinite horizon.[10]

In general, assuming SSFs, the dynamic IS (4) and Phillips curve can be represented as

$$y_t = \int_0^1 \mathcal{E}_{i,t}\left[y_{i,t+1} - \frac{1}{\sigma}(r_t - \pi_{i,t+1})\right] - \frac{E_t \Delta z_{t+1}^d}{\sigma} \tag{11}$$

$$\pi_t = \int_0^1 \mathcal{E}_{i,t}\left(\beta \pi_{i,t+1} + \kappa mc_{i,t}\right) + z_t^m \tag{12}$$

which are simply derived from the aggregation of the consumers' Euler equations and the firms' pricing rules.

By contrast, assuming LHFs, first-order conditions are combined with budget constraints and iterated forward, yielding[11]

$$y_t = \int_0^1 \mathcal{E}_{i,t} \sum_{s=t}^{\infty} \beta^{s-t}\left((1-\beta) y_s - \frac{\beta}{\sigma}(r_s - \pi_{s+1})\right) - \frac{E_t \Delta z_{t+1}^d}{\sigma} \tag{13}$$

$$\pi_t = \int_0^1 \mathcal{E}_{i,t} \sum_{s=t}^{\infty} (\xi_p \beta)^{s-t}\left((1-\xi_p)\beta \pi_{s+1} + \kappa mc_s\right) + z_t^m \tag{14}$$

It is easy to verify that assuming homogeneous rational agents (i.e., replacing E_t to $\mathcal{E}_{i,t}$), both (11) and (13) collapse to (4); similarly, (12) and (14) converge to (5). In this scenario, the forecasting horizon dilemma does not apply.

Henceforth, we assume that the economy is populated by two types of agents, who differ in their expectations formation process. A fraction α have rational expectations (rational agents), while the remaining $1-\alpha$ (boundedly rational agents) form expectations according to a mechanism of bounded rationality.[12]

[10]The former approach is formally derived by Branch and McGough (2009), while the latter is obtained by Massaro (2013). We refer to these papers for a full and detailed derivation of the equations.

[11]A full derivation is provided by Massaro (2013) or Beqiraj *et al*. (2016).

[12]For the sake of brevity, α is fixed. See Di Bartolomeo *et al*. (2016) for a discussion.

Each rational agent i forecasts macroeconomic variables according to the following rule

$$\mathcal{E}_{i,t}^{\mathcal{R}} x_{t+1} = x_{t+1} + \zeta_{i,t} \tag{15}$$

where $\mathcal{E}_{i,t}^{\mathcal{R}}$ is the expectation operator used by the rational agent i to forecast x_{t+1} at t and $\zeta_{i,t}$ is an i.i.d. expectation error with zero mean defined in the support identified by the agents considered (i.e., in the case of rational agents: $(0, \alpha)$). Aggregation among rational agents then yields

$$\int_0^{\alpha} \mathcal{E}_{i,t}^{\mathcal{R}} x_{t+1} di = \alpha E_t x_{t+1} \tag{16}$$

The remaining $1-\alpha$ agents have cognitive limitations and use heuristics to forecast macro variables. As previously illustrated, the predicted aggregate dynamics depend on the assumption about horizon forecasts.

2.3 Short-term forecasters

We begin by assuming SSFs. They set their behavior at time t based on their expectations on consumption (output) or price at price $t+1$. Households satisfy an Euler equation, while firms set their price according to a Calvo's pricing rule. However, differently from the traditional case, SSFs' expectations are based on some heuristics and affected by systematic errors.

We assume that all SSFs form their beliefs based on a simple perceived linear law of motion, i.e., $x_t = \theta x_{t-1}$. Therefore,

$$\mathcal{E}_{i,t}^{\mathcal{B}} x_t = \theta x_{t-1} + \zeta_{i,t} \tag{17}$$

where $\mathcal{E}_{i,t}^{\mathcal{B}}$ is the expectation operator used by SSFs and θ is the *adaptation operator*, which implies that SSFs form adaptive ($\theta < 1$) or extrapolative ($\theta > 1$) expectations (we refer to $\theta = 1$ as the case of naive expectations). Applying the law of iterated expectations, we obtain $\mathcal{E}_t^{\mathcal{B}} x_{t+1} = \theta^2 x_{t-1} + \theta\zeta_{i,t} + \zeta_{i,t+1}$, then aggregating

$$\int_{\alpha}^{1} \mathcal{E}_{i,t}^{\mathcal{B}} x_{t+1} di = (1-\alpha)\theta^2 x_{t-1} \tag{18}$$

By imposing some minimal restrictions to the heuristics used by the SSFs, Branch and McGough (2009) show that (17) implies a micro-founded representation of the sticky price New Keynesian model very similar to the traditional case (4)-(10).

The generalization of the New Keynesian sticky price model to heterogeneous expectations, in fact, implies that the conditional expected values in (4) and (5) are replaced by a convex combination of expectations operators (rational (16) and heuristics (18)). The aggregate IS demand curve and the Phillips curve become:

$$y_t = \alpha E_t y_{t+1} + (1-\alpha)\theta^2 y_{t-1} - \frac{r_t - \alpha E_t \pi_{t+1} - (1-\alpha)\theta^2 \pi_{t-1} + E_t \Delta z_{t+1}^d}{\sigma} \tag{19}$$

$$\pi_t = \beta[\alpha E_t \pi_{t+1} + (1-\alpha)\theta^2 \pi_{t-1}] + \kappa m c_t + z_t^m \tag{20}$$

The rest of the model is instead still described by (6)-(10).

2.4 Long-term forecasters[13]

LHFs use heuristics to forecast macroeconomic variables over an infinite horizon. The selection of heuristics takes place at the beginning of period t, when they observe and compare past performances. Each predictor $\theta_i \in \Theta, \forall i \in [0,1]$, is evaluated according to the past squared forecast error (performance measure). The distribution of beliefs then evolves over time as a function of past performances according to the continuous choice model (Diks and van der Weide, 2005). The distribution of beliefs is normal and its evolution is characterized by a mean equal to x_{t-1} and a finite variance that is decreasing in the agents' sensitivity to differences in performances.

Aggregating among boundedly rational agents, we obtain

$$\int_{\alpha}^{1} \mathcal{E}_{i,t}\, x_{t+1} di = (1-\alpha) \int_{\Theta} \theta_i\, di = (1-\alpha)\theta x_{t-1} \tag{21}$$

By using (21) into (13) and (14), we get

$$y_t = \alpha E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[(1-\beta) y_s - \frac{\beta}{\sigma}(r_s - \pi_{s+1})\right] +$$
$$+(1-\alpha)\left[y_{t-1} - \frac{\beta}{\sigma}\frac{(1-\beta)r_t + \theta\beta r_{t-1} - \theta\pi_{t-1}}{1-\beta}\right] - \frac{E_t \Delta z_{t+1}^d}{\sigma} \tag{22}$$

and

$$\pi_t = \alpha E_t \sum_{s=t}^{\infty} \left(\xi_p \beta\right)^{s-t} \left[(1-\xi_p)\beta\pi_{s+1} + \kappa mc_s\right] +$$
$$+(1-\alpha)\left[\frac{\theta k}{1-\xi_p\beta} mc_{t-1} + \frac{\theta(1-\xi_p)\beta}{1-\xi_p\beta}\pi_{t-1}\right] + z_t^m \tag{23}$$

where we assumed that the current interest rate is observed by boundedly rational agents, while current output and inflation are not.

3. Empirical analysis

This section briefly outlines how to draw from the posterior distribution of structural parameters and to compute the log–marginal likelihood associated with each model. First, we present a description of the data used in our estimation and the prior densities of the assumed estimated parameters. Second, we test the stability of the different models via Monte Carlo Filtering. Third, we report the estimation of the structural parameters and compare the log–marginal likelihood of the models. Finally, we provide the validation of models incorporating bounded rationality by comparing their second moments and correlations against the current data.

3.1 Data and methodology

We estimate the models described in the previous section by Bayesian techniques.[14] Specifically, we estimate the two behavioral New Keynesian models: the SSFs' model,

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[13]LHFs are modeled as in Preston (2006) and Massaro (2013) to whom we refer for details.

[14]We used Dynare MatLab routines to simulate and estimate the models (see Adjemian *et al.*, 2011).

where agents behave according to their short–run expectations ((6)–(10) and (19)–(20)); and the LHFs' model ((6)-(10) and (22)–(23)), where agents take their decisions considering their long–horizon expectations. For the sake of comparison, we also estimate the baseline New Keynesian model (4)–(10), where agents are homogeneous and rational such that here their horizon forecast does not matter.

After writing each model in state-space form, the likelihood function is evaluated according to the Kalman filter, whereas prior distributions are used to introduce additional non-sample information into the parameter estimation. Once a prior distribution is elicited, the posterior density for the structural parameters can be obtained by re-weighting the likelihood by a prior. The posterior is computed using numerical integration by applying the Metropolis-Hastings (MH) algorithm for Monte Carlo integration. For the sake of simplicity, all structural parameters are supposed to be independent of one another.[15]

We consider a sample of United States quarterly data ranging from 1984:Q1 to 2008:Q2.[16] Namely, these observables are the log of the real GDP as a measure for the output gap, the Federal Funds rate as a proxy for the nominal interest rate, the log-difference of the GDP deflator as a proxy for inflation, and the log of the expected inflation series. Expected inflation is taken from the Survey of Professional Forecasters (available from the Federal Reserve Bank of Philadelphia).[17] All the other data are drawn from the FRED database maintained by the Federal Reserve Bank of St. Louis. Data on nominal interest rate and inflation have been demeaned, whereas data on real GDP has been detrended by the Hodrick–Prescott filter.[18]

Beyond the structural shocks described in the previous section, we add a measurement error (e_t^{π}) to the inflation observable and one shock to the expected inflation evolution, $e_t^{\pi^E}$.[19] To recap, the model is affected by six shocks and four observables are employed for the estimation; as a result, our econometric procedure does not imply problems deriving from stochastic singularity.[20]

3.2 Prior distributions and calibrated parameters

The prior distributions of the estimated parameters are weakly informative and centered on values similar to the calibrations used by Branch and McGough (2009) and Massaro (2013). The pivotal parameters of our estimation are α, denoting the share of rational agents, and θ, representing the adaptation parameter. We assign to α a Beta distribution with mean 0.8 and standard deviation 0.1: this decision comes from the fact that being α a share, it can take value between 0 and 1. The adaptation parameter θ has a Gamma distribution centered on the naive case, i.e., 1, and with standard deviation 0.2.

The priors of the other parameters are elicited as follows: the relative risk aversion coefficient (σ) has mean of 0.157 with standard deviation 0.1 and follows a Gamma

[15]For an exhaustive analysis of Bayesian estimation methods, see Geweke (1999), An and Schorfheide (2007), or Fernández-Villaverde (2010).

[16]The end of our sample is chosen in order to avoid to deal with the zero–lower bound on nominal interest rate.

[17]See Milani (2007) and Del Negro and Eusepi (2011).

[18]Following Lubik and Schorfheide (2004) and Ascari *et al.* (2011), the computation of the Hodrick–Prescott detrended measure of output is made employing a sample (1960:Q1–2008:Q2) longer than the one used for the estimation. In this way, we increase the number of observations exploited to compute the cyclical component of output. Consequently, our detrended output measure has a nonzero mean for the sample used for the estimation; thus, we need to demean it.

[19]The need of adding a measurement error to the inflation arises mainly for two reasons. First, its introduction helps the inversion of the Hessian matrix computed at the posterior mode. Second, it improves the model fit, in particular by looking at the sign of the cross–correlation between inflation and output (see Section 3.5).

[20]Technical issues deriving from misspecification are widely discussed in Lubik and Schorfheide (2006) and Fernández–Villaverde (2010).

distribution. The inverse of the Frisch elasticity (ω) has a Gamma distribution with mean 2 and standard deviation 1. The Calvo parameter (ξ_p) is centered on a 0.75 mean with standard deviation 0.15 and has a Beta distribution. Taylor rule parameters are both normally distributed: the feedback parameter of inflation (δ_π) is centered on a mean equal to 1.5 and with standard deviation 0.2, whereas the policy rate response to the output gap (δ_y) has a mean of 0.2 with standard deviation 0.1.

The six exogenous shocks follow an Inverse Gamma distribution centered on a mean equal to 0.01 and with two degrees of freedom. The autoregressive component of the shocks follows a Beta distribution with mean 0.5 and standard deviation 0.2.

As common practice in estimated DSGE models, some parameters need to be calibrated either because they are difficult to estimate or because can lead to identification problems.[21] Specifically, the discount factor (β) is calibrated to 0.99 and the elasticity of substitution between goods (ϵ) is calibrated to 7.84, implying a net mark-up of around 15%.

3.3 Mapping the stability: Monte Carlo Filtering

In a Bayesian approach, identifying the determinacy region of the model is a fundamental step. It allows to initialize the estimation within the portion of the acceptable domain of model coefficients, excluding indeterminacy or instability. We know that introducing heterogeneous expectations in behavioral New Keynesian DSGE models strongly shrinks the stability domain of the model (Branch and McGough, 2009; Massaro, 2013). Therefore, we explore the parameter space to detect those that mostly drive the model towards indeterminate or unstable regions using a Monte Carlo Filtering technique (described below).

Massaro (2013) and Branch and McGough (2009) also provide some numerical analysis of determinacy. By using some common calibrations,[22] they focus on how the determinacy properties of the models are affected by changes in the parameters of a simple Taylor rule. Both conclude that a–more–than–one response of the interest rate to inflation does not necessarily guarantee a unique equilibrium in a world with heterogeneous agents.

In the set of parameters considered by Massaro (2013), LHFs imply that a–more–than–one response of the interest rate is no longer a sufficient condition, but it becomes a necessary condition for determinacy. Moreover, the higher is the interest rate response to inflation, the more the central bank should also respond to the output gap to ensure determinacy. In the model with SSFs provided by Branch and McGough (2009), the more–than–one response suffices to drive the model towards the determinate region only when it is combined with a small–feedback reaction to the output gap. But, as agents become more extrapolative, i.e., $\theta > 1$, the monetary authority must respond more aggressively to inflationary pressure to guarantee the equilibrium determinacy.

We complement the above results by a Monte Carlo Filtering (MCF) mapping, exploring a larger space of parameters. MCF mapping is a procedure implemented to detect the parameters that mostly drive the model towards indeterminate and unstable regions. As explained by Ratto (2008), a multi-parameter Monte Carlo simulation is performed, sampling model parameters $(X_1, ..., X_k)$ from prior ranges, and propagating parameter values through the model. Two regions are distinguished: a target stable behavior region B, where the Blanchard–Kahn (1980) rank conditions are satisfied, and an unacceptable

[21]We check the correct identification of the subset of estimated parameters using the Identification toolbox for Dynare, which implements the identification condition developed by Iskrev (2010a, 2010b). For a review of identification issues arising in DSGE models, see Canova and Sala (2009).

[22]Branch and McGough (2009) focus on SSFs and consider the calibrations used by Woodford (1999), Clarida *et al*. (2000) and McCallum and Nelson (1999). Massaro (2013) calibrates a model with LHFs as Galí and Gertler (1999).

behavior region $\overline{B}$, characterized by model instability or indeterminacy. Running N Monte Carlo draws we obtain, for each parameter $i = 1,2,\dots k$, two subsets: $(X_i|B)$ of size n and $(X_i|\overline{B})$ of size $\overline{n}$, where $n + \overline{n} = N$. These subsets represent draws from unknown probability density functions, $f_n(X_i|B)$ and $f_{\overline{n}}(X_i|\overline{B})$.

A certain parameter X_i is a key factor in driving model behavior whether the two distributions $f_n(X_i|B)$ and $f_{\overline{n}}(X_i|\overline{B})$ are significantly different; therefore, there will be subsets of values in the predefined range of X_i that are more likely to fall under B than under $\overline{B}$. Instead, when the two distributions are not significantly different, X_i plays no role in driving the behavior of the model and any value in its predefined range is likely to fall either in B or in $\overline{B}$.

The identification of the parameters that mostly drive the DSGE model into the target behavior, is obtained by comparing the distributions f_n and $f_{\overline{n}}$, using a two-sided Smirnov-Kolmogorov test for each parameter. The test is defined in relation to the cumulative distribution functions of X_i by:

$$d_{n,\overline{n}}(X_i) = \sup \ \| F_n(X_i|B) - F_{\overline{n}}(X_i|\overline{B}) \| \tag{24}$$

The null hypothesis of the test is $f_n(X_i|B) = f_{\overline{n}}(X_i|\overline{B})$. Finally, we need to define the significance level τ at which the statistic $d_{n,\overline{n}}(X_i)$ rejects the null. In general, the greater $d_{n,\overline{n}}(X_i)$ (or, alternatively, the smaller is τ) the more important is parameter X_i for driving model behavior.

We run the MCF mapping[23] in the models we estimate to grab information about which parameters mostly drive the acceptable behavior and which region of the parameter space drives the models towards a unique equilibrium. In Table 1 we summarize the results of the Smirnov-Kolmogorov test in leading to the acceptable behavior. We label by "SSFs" the model with short-sighted forecasts, while "LHFs" indicates the model with long-horizon predictors. An asterisk denotes that the null hypothesis is rejected and the parameter in question affects the behavior of the model.

Table 1 - Smirnov-Kolmogorov statistics in driving acceptable behavior

Param.	Acceptable behavior		Indeterminacy		Instability	
	SSFs	LHFs	SSFs	LHFs	SSFs	LHFs
α	0.149*	0.406*	0.454*	0.796*	0.228*	0.039
θ	0.504*	–	0.235*	–	0.337*	–
σ	0.022	0.079	0.020	0.263*	0.015	0.009
ω	0.025	0.180*	0.027	0.089	0.010	0.010
ξ_p	0.084	0.187*	0.057	0.118	0.037	0.013
δ_π	0.442*	0.305*	0.513*	0.522*	0.123*	0.028
δ_y	0.027	0.136	0.044	0.105	0.016	0.009
ρ_a	0.013	0.075	0.023	0.121	0.009	0.003
ρ_d	0.017	0.045	0.025	0.099	0.013	0.000
ρ_m	0.017	0.073	0.041	0.063	0.016	0.000

Differently from the standard framework, a failure to reach a unique equilibrium can lead to both instability or indeterminacy. The presence of boundedly rational agents introduces backward-looking components in the model dynamics, as a result unstable eigenvalues that ensure determinate equilibrium in a completely forward-looking model may now induce unstable dynamics.

[23]This analysis is performed using the Global Sensitivity Analysis (GSA) toolbox for Dynare.

In both models α and δ_π are pivotal parameters in driving the acceptable behavior. In particular, in both cases the cumulative probability distribution for unacceptable behavior is shifted on the left,[24] indicating that the probability of model stability increases for high values of α and δ_π. Looking at the indeterminacy, the cumulative probability distribution of δ_π is still shifted on the left, i.e., high values of δ_π avoid indeterminacy, while the cumulative probability distribution of α is shifted on the right, signaling that high values of α increase the probability of falling in the indeterminacy region. These results hold for both models.

The short-sighted framework is also affected by the adaptation parameter θ. On the one hand, a low value for θ drives the model towards the acceptable behavior and avoids explosiveness. On the other hand, it increases the likelihood of indeterminacy.

Regarding the remaining parameters, the determinacy of the model with LHFs compared to the SSFs's framework is more sensible to parameter changes. In particular, a higher degree of price rigidity raises the probability to hit the determinacy region.

Summarizing, the stability properties of the two models are not very different, we can choose a comparable starting calibration for the initialization of our estimation. However, in a LHFs context, more parameters may lead to deviate from the determinacy region and the model is more likely to fail to reach the unique equilibrium.

3.4 Estimation results and model comparison

The estimation of the structural parameters of our two behavioral New Keynesian models is reported in Table 2. We also report estimates from the baseline New Keynesian model (labeled "NK"), where all the agents are rational. The table reports priors (mean and density) and posteriors (with their $[5th,\ 95th]$ probability intervals), and the log–marginal likelihood for each model. The posterior distributions are obtained using the MH algorithm. The mean and posterior percentiles come from two chains of $200{,}000$ draws each from the MH algorithm, for which we discarded the initial 30% of draws. The scale for the jumping distribution in MH algorithm has been calibrated to achieve an acceptance rate around 25%.

Table 2 – Prior and posterior distributions for structural parameters

Par.	Prior density	Prior mean (Std.dev.)[25]	Posterior mean [5th pct, 95th pct]		
			SSFs	LHFs	NK
σ	Gamma	0.157	0.193	0.042	0.162
		(0.10)	[0.057,0.318]	[0.028,0.056]	[0.026,0.290]
ω	Gamma	2.00	5.037	1.332	5.023
		(1.00)	[3.304,6.671]	[0.780,1.869]	[3.335,6.677]
ξ_p	Beta	0.75	0.521	0.668	0.567
		(0.15)	[0.347,0.695]	[0.646,0.689]	[0.416,0.727]
α	Beta	0.80	0.540	0.874	1
		(0.10)	[0.367,0.703]	[0.838,0.910]	[-]
θ	Gamma	1.00	0.757	1	1
		(0.20)	[0.606,0.903]	[-]	[-]
δ_π	Normal	1.50	2.485	1.219	2.586
		(0.20)	[2.278,2.723]	[1.126,1.314]	[2.441,2.772]
δ_y	Normal	0.20	0.098	0.330	0.116
		(0.10)	[0.024,0.173]	[0.216,0.441]	[0.048,0.181]
ρ_a	Beta	0.50	0.919	0.305	0.884

[24]The figures produced by Dynare are available upon request.

[25] For the Inverse Gamma distributions, the degrees of freedom are indicated.

		(0.20)	[0.863,0.978]	[0.097,0.505]	[0.816,0.960]
ρ_d	Beta	0.50	0.910	0.426	0.913
		(0.20)	[0.881,0.939]	[0.191,0.663]	[0.884,0.940]
ρ_m	Beta	0.50	0.579	0.230	0.567
		(0.20)	[0.252,0.907]	[0.052,0.393]	[0.247,0.921]
e_t^a	Inv. Gamma	0.01	0.0030	0.0023	0.0032
		(2.00)	[0.002,0.004]	[0.002,0.003]	[0.003,0.004]
e_t^r	Inv. Gamma	0.01	0.0035	0.0044	0.0036
		(2.00)	[0.002,0.005]	[0.003,0.006]	[0.002,0.005]
e_t^d	Inv. Gamma	0.01	0.0147	0.0041	0.0123
		(2.00)	[0.010,0.020]	[0.002,0.006]	[0.009,0.016]
e_t^m	Inv. Gamma	0.01	0.0052	0.0027	0.0056
		(2.00)	[0.002,0.008]	[0.002,0.004]	[0.003,0.004]
e_t^π	Inv. Gamma		0.0022	0.0055	0.0022
			[0.002,0.003]	[0.005,0.006]	[0.002,0.003]
$e_t^{\pi^E}$	Inv. Gamma	0.01	0.0018	0.0053	0.0020
		(2.00)	[0.002,0.002]	[0.005,0.006]	[0.002,0.002]
Log marginal likelihood			1784.474	1517.106	1774.219

In the behavioral New Keynesian model with SSFs*,* we estimate a share of rational agents close to 55%, implying that about 45% of individuals are boundedly rational. The adaptation parameter is estimated to 0.75, non–rational agents are adaptive. The other parameters are akin to the deep parameters estimated in the standard New Keynesian model.

The response of the central bank to the inflation is strong, while its reaction to the output gap is moderated; a similar result has been found by Clarida *et al.* (2000) for the Great Moderation period. The Calvo parameter is estimated to 0.52 entailing that prices are adjusted about every two quarters. Interestingly, this value is in line with the microeconometric evidence on price setting that calls for price updates every $2-3$ quarters (see Bils and Klenow, 2004; Klenow and Malin, 2011). Finally, the technology shock shows a large degree of autocorrelation, a common result in estimated DSGE models (see, e.g., Smets and Wouters, 2007).

In the behavioral New Keynesian model with LHFs, the estimated share of rational agents is quite large. The estimated value is 87%, implying a low fraction of boundedly rational agents (i.e., 13%).[26] The other parameter estimates also differ from those obtained when bounded rationality is formed according to a short-sighted mechanism (and from those obtained from the standard New Keynesian model). The estimated coefficients of the Taylor rule imply that the monetary authority reacts to inflation implementing the Taylor principle, but as δ_π is relatively small, the estimated response to the output gap is quite high. The estimated degree of price rigidity is higher compared to the model with SSFs, involving an average duration of a price spell of about three quarters. All the shocks exhibit a small degree of autocorrelation.

We interpret the different estimations of the Taylor rule in the SSFs' and LHFs' framework as the result of the fact that models have different determinacy regions which are differently affected by Taylor rule coefficients. Specifically, the behavioral New Keynesian model with LHFs is less likely to match determinacy. The estimation seems to drive estimated Taylor coefficients towards the values that fulfill the model stability. The model with LHFs requires in fact to raise the feedback coefficient associated to the output gap to ensure determinacy. The estimation of the Calvo parameter also goes in that direction, from the stability mapping, illustrated in the previous subsection, we

[26]The adaptation parameter is 1 as in Massaro (2013).

know that a higher degree of price rigidity helps the LHFs' model to meet the determinacy region.

We now investigate how the manner in which bounded rationality is formed (SSFs or LHFs) affects the fit of a model. To understand which model fits better the data we compare them via Bayes factor. The model with the highest log marginal likelihood better explains the data (see Kass and Raftery, 1995).[27]

The log–marginal likelihoods associated with the two model specifications considered herein are reported in the last row of Table 2. The difference, in terms of log–marginal likelihood, between the two models is 267.36. According to Jeffreys' scale of evidence,[28] this difference must be considered as "decisive" evidence in favor of SSFs.

For the sake of completeness, we compare the SSFs' behavioral New Keynesian model to the standard rational expectation framework. The parameter estimates are quite similar. Looking at the Bayes factor, the difference in terms of log–marginal likelihood between these two models is about 10, entailing a "substantial" evidence in favor of the model with SSF.

Our result is not surprising as the SSFs model incorporates lagged terms both in the IS and the Phillips curves: the presence of these backward components introduces persistence in the model and allows to capture the degree of inertia present in the data. The basic New Keynesian model is instead purely forward looking and fails to capture output persistence and inflation inertia.

The standard New Keynesian model however strongly performs better than the model with LHFs—despite the latter specification is also able to account for output persistence and inflation inertia. The result further confirms the poor predictive ability associated with LHFs.

Some additional investigation about the model's ability to fit the data and their validation is provided in the next subsection.

3.5 Models validation

This section further attempts to understand how well behavioral–macro models presented here fit the data and capture their dynamics. We compare a set of business cycle statistics implied by the estimated models to those measured in the data. We study the second moments, autocorrelation, and cross-correlation with respect to the output of the observable variables included in the estimation. These summary statistics have been standard means of validating models in the literature on DSGE models. Comparing models' moments is useful to assess the absolute fit of a model to macroeconomic data, inspecting whether the model correctly predicts population moments, such as the variables' volatility or their autocorrelation (see, e.g., Justiniano *et al.*, 2011; Cantore *et al.,* 2015).

In Table 3 we present our results. We report the second moments and correlations arising from our estimations and compare them with those found in the current data for the sample range (i.e., 1984:Q1–2008:Q2).

Table 3 - Second moments and correlations for observable variables and actual data

Series	Standard deviation	Autocorrelation (order=1)

[27] For a similar approach see, e.g., Rabanal and Rubio-Ramirez (2005), Riggi and Tancioni (2010), Di Bartolomeo and Di Pietro (2017).

[28] Jeffreys (1961) developed a scale to evaluate the Bayes factor indication. Odds ranging from 1:1 to 3:1 give "very slight evidence", odds from 3:1 to 10:1 are "substantial", odds from 10:1 to 100:1 give "strong to very strong evidence", and odds greater than 100:1 are "decisive evidence."

	Data	SSFs	LHFs	NK	Data	SSFs	LHFs	NK
Output	0.0103	0.0121	0.0143	0.0106	0.89	0.92	0.13	0.89
Inflation	0.0022	0.0019	0.0097	0.0017	0.58	0.56	0.96	0.45
Exp. Inflation	0.0019	0.0021	0.0110	0.0022	0.71	0.24	0.74	0.21
Interest rate	0.0058	0.0037	0.0096	0.0034	0.94	0.92	0.96	0.90

Series	Standard deviation relative to output				Cross-correlation with output			
	Data	SSFs	LHFs	NK	Data	SSFs	LHFs	NK
Inflation	0.213	0.157	0.678	0.160	0.27	0.04	-0.46	0.08
Exp. Inflation	0.184	0.173	0.769	0.207	0.39	0.01	-0.28	0.01
Interest rate	0.563	0.305	0.671	0.320	0.47	0.33	-0.47	0.38

Moments implied by the model with SSFs are the closest to the data. The only pitfall of the SSFs' specification is that it captures a too low first–order autocorrelation for the expected inflation. Model with LHFs shows a bad fit: both observed inflation and expected inflation are too volatile (about five times higher than the data), output exhibits a slight degree of autocorrelation, and the cross-correlation with output for both interest rate and inflation has the wrong sign.

In general, the fit of the standard New Keynesian model is as good as the fit arising under a SSFs' mechanism. However, the latter model estimates moments slightly closer to the data, looking at the standard deviations and the autocorrelations. These factors favor it in the Bayesian race above described.

Summarizing, the model with long-horizon predictors exhibits very poor predictive ability, while the overall model fit derived under a short-sighted mechanism is the best in predictions and the closest to the data. The next section will shed some light on the sources of the empirical superiority of SSFs' over LHFs' model.

4. Investigating the best predictive ability of the SSFs' model

We have learned that there is a huge difference between behavioral–macro models based on SSFs or LHFs in terms of parameter estimates and model–predictive ability. As discussed, compared to the basic New Keynesian model, empirical superiority of the short-sighted framework is somehow expected, as the standard setup cannot properly capture output persistence and (intrinsic) inflation inertia (see, e.g., Fuhrer, 2011). By contrast, the poor predictive ability of the LHFs' model is surprising. Possibly, it derives from the restriction of the parameter space imposed by the determinacy condition. To meet this region, posteriors are forced to overestimate the output coefficient of the Taylor rule and the price stickiness.

The sources of the different performances of the two behavioral macro–model can be further investigated by considering mixed frameworks. In line of principle, firms and households could use different horizons when forming their expectations: firms could be LHFs and households SSFs or vice versa. We refer to these cases as mixed models. They are characterized by IS and Phillips curves derived under alternative assumptions for the forecasting mechanism, which are differently affected by bounded rationality. By mixed models, we can empirically test (i) whether different kinds of agents, specifically firms or households, use disparate schemes for their forecasts; (ii) whether a mixed model fits the aggregate data better than other specifications or not.

We focus on the mixed case where firms are LHFs and households are SSFs, i.e., a specification characterized by a Phillips curve *a la* Massaro (2013) and an IS curve *a la* Branch and McGough (2009).[29] The model in question does not figure out particular

[29] Mixed model needs further clarifications regarding the underlying assumptions on heterogeneous expectations and places a particular structure on higher order beliefs. In the Yeoman Farmer model *a la* Branch and McGough (2009), heterogeneous expectations among households straight forward translate into the

determinacy problems. We do not consider (and estimate) the case when firms are SSFs and households have long-horizon beliefs since it never fulfills the determinacy region.[30]

Our mixed model (labeled "mixed" in the table below) is formally composed by the demand curve (19) and the Phillips curve (23). The remaining equations are (6)–(10). The estimates are presented in Table 4. As usual, we report the posterior means and $[5th \text{ pct}, \ 95th \text{ pct}]$ percentiles. For sake of comparison, we also report again the estimated parameters for the short-sighted model from Table 3. Remember that the model with SSFs won the likelihood race among the other models considered in our investigation.

Table 4 –Prior and posterior distributions for structural parameters

Par.	Prior density	Prior mean (Std.dev.)	Posterior mean $[5\text{th pct}, \ 95\text{th pct}]$ SSFs	Posterior mean $[5\text{th pct}, \ 95\text{th pct}]$ Mixed
σ	Gamma	0.157	0.193	0.213
		(0.10)	[0.057,0.318]	[0.092,0.332]
ω	Gamma	2.00	5.037	4.425
		(1.00)	[3.304,6.671]	[2.859,5.882]
ξ_p	Beta	0.75	0.521	0.788
		(0.15)	[0.347,0.695]	[0.581,0.955]
α	Beta	0.80	0.540	0.667
		(0.10)	[0.367,0.703]	[0.521,0.805]
θ	Gamma	1.00	0.757	1.142
		(0.20)	[0.606,0.903]	[0.770,1.487]
δ_π	Normal	1.50	2.485	2.621
		(0.20)	[2.278,2.723]	[2.487,2.772]
δ_y	Normal	0.20	0.098	0.086
		(0.10)	[0.024,0.173]	[0.017,0.151]
ρ_a	Beta	0.50	0.919	0.912
		(0.20)	[0.863,0.978]	[0.853,0.977]
ρ_d	Beta	0.50	0.910	0.910
		(0.20)	[0.881,0.939]	[0.882,0.940]
ρ_m	Beta	0.50	0.579	0.625
		(0.20)	[0.252,0.907]	[0.327,0.932]
e_t^a	Inv. Gamma	0.01	0.0030	0.0029
		(2.00)	[0.002,0.004]	[0.002,0.004]
e_t^r	Inv. Gamma	0.01	0.0035	0.0030
		(2.00)	[0.002,0.005]	[0.002,0.004]
e_t^d	Inv. Gamma	0.01	0.0147	0.0125
		(2.00)	[0.010,0.020]	[0.008,0.017]
e_t^m	Inv. Gamma	0.01	0.0052	0.0056
		(2.00)	[0.002,0.008]	[0.002,0.009]
e_t^π	Inv. Gamma		0.0022	0.0023
			[0.002,0.003]	[0.002,0.003]

expectations of firms. However, the model with decentralized markets as Massaro (2013) requires to explain why the owner of a firm could form LHFs that are fundamentally different from the expectations of the shareholders that are SSFs. Even in this framework a risk sharing mechanism operates in the sense that each household consists of a continuum of agents which are employed across firms and share dividends across the household to ensure against the Calvo risk. Furthermore, in both modeling setup, rational agents are not sophisticated enough to back out the expectations of boundedly agents since they have wrong second order beliefs, i.e., they are not fully rational. Then the problem does not arise in the sense that even the rational entrepreneur would not be able to recognize that agents different from him behave in the market and eventually work in his firm and the risk sharing mechanism would eventually correct from the distortions created by the heterogeneity.

[30]Sensitivity analysis were performed with the GSA toolbox. Results are available upon request.

$e_t^{\pi^E}$	Inv. Gamma	0.01	0.0018	0.0019
		(2.00)	[0.002,0.002]	[0.002,0.002]
	Log marginal likelihood		1784.474	1783.531

The estimated parameters of the mixed model are similar to those estimated in the model where all the agents are SSFs. Some remarkable differences involve the fraction of rational agents that now is slightly higher and the adaptation parameter, estimated to be greater than one and entailing extrapolative expectations. Therefore, the degree of price rigidity is higher and translates into an average price duration of about four quarters. The mixed specification inherits some stability characteristics from the LHFs' model. In such a case, MCF shows that a high degree of price rigidity and larger share of rational agents is required to fulfill model determinacy.

The log–marginal likelihood associated with the mixed model is almost identical to that obtained in a context of SSFs. According to the Jeffreys' scale of evidence, we can state that this difference is negligible. If we inspect the second moments, they are quite identical among these two specifications.[31]

We ran two further empirical tests to check the robustness of the results discussed in this section.

1. We estimated both models with an interest rate smoothing in the Taylor rule. In the previous section, we had not considered the smoothing parameters because (i) we strictly followed the specification by Branch and McGough (2009) and Massaro (2013) who inspect the stability properties of their models without considering smoothing in the Taylor rule (ii) introducing a smoothing coefficient in the set of estimated parameters negatively affects the stability region of the model with LHFs, further tightening the parameter space where model has acceptable behavior and making its estimation hard.
2. An additional robustness check we did was to estimate the models reported in Table 4 constraining the adaptation parameter to be equal to one, i.e., imposing naive expectations.

Both robustness checks above–described led to results very similar to those presented in Table 4, in terms of estimated parameters and in terms of model comparison.[32]

To summarize, modeling mechanism of bounded rationality for price–setters by SSFs or LHFs leads to very similar dynamics. Instead, the way in which forecasts are done has relevant implications for the households. If households use long-horizon predictors dramatically deteriorates the model fit and, as well, its forecast ability. We thus recommend to model bounded rationality following a short-sighted forecast scheme for the household sector, whereas assuming short-sighted or long-horizon predictors for the firms does not remarkably worsen the model fit.

5. Conclusions

Behavioral New-Keynesian DSGE models are becoming quite popular in macroeconomics (Branch and McGough, 2009, 2016; Massaro 2013). Assuming that a fraction of agents form beliefs about the future according to some heuristics, we have compared two alternative mechanisms used to introduce heterogeneous expectations. The alternatives

[31]Standard deviation, autocorrelation and cross-correlation with output associated with the mixed model are not reported here to save space. However, they are available upon request.

[32]For the sake of brevity, we do not report the tables with parameter estimates. They are available upon request.

differ in the forecast horizon of the agents (short vs. long), which does not matter when all the agents have rational expectations.

Based on the different expectation–formation mechanisms, we have built different DSGE models. The models have then been estimated by Bayesian techniques and alternatives have been compared by marginal likelihoods. We have also explored the indeterminacy regions implied by the two alternatives using Monte-Carlo-Filtering mapping.

Our main results can be summarized as follows.

1. Marginal likelihoods show that a behavioral New-Keynesian model including short forecasters fits the data better than one based on long forecasters. Short forecasters also outperform rational expectations.
2. Moments implied by the model with short-horizon predictors are the closest to the observed data. By contrast, the model with long-horizon predictors exhibits a poor predictive ability.
3. The superiority of the short forecasters' model over the others considered is due to the behavioral specification of the dynamic aggregate demand equation, which better fits heterogeneity in consumers' expectations. Conversely, heterogeneity in the supply side of the economy is less relevant to discriminate between the models.
4. The Monte Carlo filtering mapping generalizes the results of Branch and McGough (2009) and Massaro (2013) obtained for a more restrictive set of parameters. A model based on long-horizon predictors is less stable. As a result, for our estimations, it tends to overestimate the degree of price stickiness and the central bank's response to the output gap to match the stability region.

Our results suggest that the enhancement of more complex medium-sized New Keynesian models with a fraction of boundedly rational agents who are short–run forecasters could be fruitful. We let this to future researches. Our findings also give a suggestion to the economists who study the expectations formation processes using microdata. They suggest to focus on expectations of households rather than on those of price-setters. Aggregate–macro outcomes in fact seem to be more sensible to how the former are modeled.

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