European Commission

JRC TECHNICAL REPORTS

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JRC Working Papers on Taxation and Structural Reforms No 9/2018

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December 2018

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JRC Science Hub
https://ec.europa.eu/jrc

JRC114907

ISSN 1831-9408

Sevilla, Spain: European Commission, 2018

How to cite: Diego d'Andria (2018), "Effects of an ad valorem Web Tax in a Cournot-Nash market for digital advertising"; JRC Working Papers on Taxation and Structural Reforms No 9/2018, European Commission, Joint Research Centre, Seville, JRC114907

Effects of an ad valorem Web Tax in a Cournot-Nash market for digital advertising

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This version: December 2018

Abstract

We extend the theory of tax incidence under Cournot-Nash oligopolistic competition to study the effects of an *ad valorem* sales tax on Web services (so-called Web Tax) that are provided free of charge to users, and produce advertising space sold to businesses. Ads are more valuable to advertisers the more users are served by a Web service. Users have ads-neutral preferences and Web companies compete in a Cournot-Nash fashion on the advertising market but enjoy monopolistic power in the service market they serve. We demonstrate that, contrary to standard theoretical results, the equilibrium market price might be reduced by a Web Tax. The conditions for such a decrease depend upon the elasticity of ads demand.

JEL classification: D43, H2, L13.

Keywords: Web tax, digital advertising, Cournot competition, tax incidence.

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The author would like to thank Salvador Barrios, Valeska Gronert and colleagues at DG TAXUD for their valuable comments and suggestions. The findings, interpretations, and conclusions expressed in this paper are entirely those of the author. They should not be attributed to the European Commission. Any mistake and all interpretations are his and his only.

1 Introduction

The use of profit shifting strategies by multinational ICT companies is often in the headlines. Parallel to experiencing since the 1990s the positive impact of Web-based technologies on labour productivity and on product variety, governments also recognize that the very nature of some of the underlying business models, particularly their intangible nature, often entails an erosion of revenues from indirect and corporate income taxation.

The European Commission has launched on 21 March 2018 an initiative[1] with the aim to obtain a fairer allocation of tax rights in the digital market. The proposal comes after several cases in which European and national tax authorities have forced some very large Web companies to pay taxes for liabilities supposedly due from past years. Although the E.U. has been the first to take action, for instance proposing a reform of VAT rules switching destination and origin principles, the debate on Web companies and their impact on tax revenues is not exclusive to Europe, see for instance the empirical works of (among others) Goolsbee (2000), Alm et al. (2005), Ballard et al. (2007) and Einav et al. (2014) documenting base erosion at the U.S. State level of sales tax revenues due to E-commerce. The European Commission's mentioned proposal envisages a two-step approach, first a "targeted solution" introducing a tax on the sales from digital products and services (also named in the media Web Tax), then a more comprehensive approach which would be based on revised profit allocation of these multinationals across the Union and new rules reflecting digital presence according to the nexus principle. The focus of the present paper is about the Web Tax alone.

The announced aim of the European Commission's Web Tax is mainly twofold: to recover lost revenues from corporate income tax (in this respect, the Web Tax would act as a substitute for the corporate tax) and also, in the

[1] "Proposal for a COUNCIL DIRECTIVE on the common system of a digital services tax on revenues resulting from the provision of certain digital services." Brussels, 21.3.2018, COM(2018) 148 final.

European Commission's words, "to level the playing field" by reducing the tax-induced advantage of New Economy firms vis-a-vis traditional "brick-and-mortar" firms.[2] One reason why an indirect tax would be introduced as a substitute for direct taxation of profits can be traced back to the limitations that corporate tax systems face when dealing with intangible goods and assets, which facilitate transfer pricing and allow companies to sell to residents of a country or region without any physical presence there (the latter has been for a long time a prerequisite for the application of source taxation).[3] Some authors (e.g. Auerbach et al. 2008 and specifically for the digital markets, Agrawal et al. 2017) have endorsed the application of a destination-based principle to corporate taxation as a comprehensive solution to profit shifting which would apply to digital and non-digital markets. The problem with such proposal, in our view, is that both the concepts of source and destination are hard to apply as far as advertising-supported digital services are concerned. Leading companies like Alphabet/Google and Facebook generate most of their revenues from selling advertising while providing their digital services for free. They are able to provide Web services in a country while selling advertising space in another country where the advertiser is resident. In such cases it is not straightforward to determine, least to measure, where the tax-wise destination of a transaction is located: should it be where consumption of the service occurs and the ads are displayed to consumers, or should it be where the paying advertiser resides? The nexus principle as defined by OECD (2013) which advocates the use of input factors location as a proxy for the location of value generation is not fully applicable either, as production, the location of data, company servers and most of the company workforce might be in entirely different countries unrelated to the place where consumption occurs (thus, application of the nexus principle would

[2]A third rationale, discussed in the proposal, is to prevent fragmentation of markets and policies due to uncoordinated action from the EU Member States.

[3]Another reason is legal: a proposal for a direct tax would have clashed against provisions of existing treaties against double taxation.

de facto apply a source-based principle). Hence, the idea to try and capture corporate profits indirectly, by means of a Web Tax which would be levied on an imputed value of sales allocated to each European Member State where a Web multinational operates and provides services. The geographical allocation of global sales would be engineered, according to the proposal's text, also exploiting proxies such as the number of registered users and Web traffic data that a Web company produces in each European Member State.

Apart from the technical problems faced in applying such a Web Tax, its Welfare implications hinge on correctly gauging the incidence effects of the reform. We argue here that the nature of Web businesses which provide digital services for free and sell advertising to produce revenues, coupled with the very peculiar way in which these digital services match consumers with advertised content, may lead to very special conditions which make standard theory of tax incidence in oligopolistic competition regimes inapplicable. More specifically, the use of sophisticated matching algorithms based on consumers' profiling imply that the larger the base of users is for a Web company, the more efficient the matching and, consequently, the larger the value-per-view (or per-click) for advertisers. As the total number of users served affects the willingness to pay of advertisers, it stands to reason that it implies an inverse demand function that is not necessarily monotonically decreasing in quantity. This intuition serves as the starting point for the theoretical analysis that follows. In order to formally study tax incidence in this special case of a two-sided market, we assume Cournot-Nash symmetric competition. However, although Web companies compete in the model with equal market power with respect to advertising services, they can enjoy monopolistic power in their own consumer's market.

The paper is structured as follows. Section 2 provides motivation, in an informal way, for the idea that quantities produced and sold can differ for advertiser-supported Web services, and also for the claim that larger users' bases positively affect the reservation price of advertisers. It also summarizes related literature and contrasts it with our modelling choices. Section 3 illustrates a stylized model of symmetric Cournot competition and derives policy-relevant results for an *ad valorem* tax. Section 4 draws the main conclusions and points to further avenues for future research.

2 Motivation and related literature

Digital advertising has been growing steadily in the last two decades, while traditional marketing channels have not. Digital ads bring some distinct advantages to advertisers compared to broadcast ads. While the idea of targeting based on indirect proxies for consumers' types is not new to the Web and has been used extensively in printed, radio and TV media, Web services enable a much deeper matching between prospect consumers and ads. Ads can be "personalized" and sent to users with observable characteristics that predict higher chances to click on the ad, to purchase the advertised product or to be influenced in the intended way. The association between observables and the consumer's behaviour is based on a large number of data points that may include: what the user does before, during and after having being exposed to an ad; what are his or her preferences with regard to content, interests, locations and several other areas; what are the associations between observing an item (for example, a search keyword) and subsequent behaviour. The current development of machine learning algorithms promises an even deeper level of matching in the near future.

Google explains its advertising services, AdWord and AdSense, as follows[4]: "With millions of websites, news pages, blogs, and Google websites like Gmail and YouTube, the Google Display Network reaches 90% of Internet users worldwide. With specialized options for targeting, keywords, demographics, and remarketing, you can encourage customers to notice your

[4]Excerpts taken from the URLs `https://adwords.google.com/intl/en/home/how-it-works/display-ads/` and `https://support.google.com/adsense/answer/9713?hl=en&ref_topic=1628432`. Pages visited on 5/4/2018.

brand, consider your offerings, and take action." It further explains that "Google automatically delivers ads that are targeted to your content or audience" by using "contextual targeting", "placement targeting", "personalized advertising" (which is described as follows: "Personalized advertising enables advertisers to reach users based on their interests, demographics (e.g., "sports enthusiasts") and other criteria") and "language targeting". Similarly, Facebook[5] explains its advertising facilities as follows: "Two billion people use Facebook every month. With our powerful audience selection tools, you can target the people who are right for your business. Using what you know about your customers, such as demographics, interests and behaviours, you can connect with people similar to them." It then details how their platform would allow to "Find people based on what they're into, such as hobbies, favourite entertainment and more" and "based on their purchasing behaviours, device usage and other activities." Another example is provided by Reddit[6] which explains: "With over 250 million users, it can be difficult to know how best to reach your audience. Interest targeting gives you the ability to pinpoint your audience [...]. With interest targeting, you can display your ad to the right audience based off a user's browsing behavior on Reddit! [...] Targeting an interest group means you are targeting users who have expressed interest in a specific type of content. For example, a user who engages in a post relating to sports will be shown sports ads for a period of time after engaged with that type of content. As a user engages in different content their interest categorization will dynamically change, ensuring all ads are relevant to that user."

From these examples, and from many more that are easily found on the Internet, common characteristics of these technologies are made clear. First,

[5]Excerpts taken from the URLs `https://www.facebook.com/business/products/ads/ad-targeting`, Pages visited on 5/4/2018.

[6]Excerpts taken from the URLs `https://www.reddithelp.com/en/categories/advertising/targeting-your-audience/targeting-interests`, Pages visited on 5/4/2018.

users are constantly analysed with respect to their observable behaviours. Second, these behaviours are codified and stored. Third, the stored data are used by automated algorithms to match users with ads (based on keywords or criteria provided by the advertiser, or possibly through fully automated matching). The number of users on a given Web service plays a key role, as having larger numbers enhances prospects for the algorithm to find a good match for a given ad. This is particularly true for advertised products that cater to a niche demand and therefore benefit the most from having their ad seen or clicked by a good match. It is just the case to highlight that in these three notable examples (Google, Facebook and Reddit) all of them stress, as the very first information provided, the very large number of users they can potentially reach.

The consequences of these observations for economic theory are, first, that the quantity produced by a Web company can be different from the quantity sold. That is, the number of potential visualizations per user, times the number of users of a Web service, can (and will likely be) larger than the number of ad space sold to advertisers. The reason for this discrepancy is not only found in the desire to avoid congestion of the service (too much advertising could make it less appealing for users), but most importantly, because as profiling and matching algorithms improve thanks to technological advances, having more users improves the value of each ad for advertisers and therefore entails larger willingness to pay. As a Web company increases produced quantities (which is the same to say: it increases its users base) but does not increase the amount of sold ads, the price of its ads may increase without impacting aggregate prices. On the contrary as sold ads increase without a company changing the amount of service provided to users, oligopoly market prices will go down as per usual decreasing inverse demand functions. When these effects are at play, standard tax incidence theory does not transfer well because inverse demand functions are not necessarily monotonically decreasing any more. These observations ask for a specific modelling of ads-supported Web services to understand the likely effects of indirect taxation on such markets.

Related literature A relatively recent literature addresses the effects of indirect taxation on digital companies under two-sided markets, see for instance: Kind, Koethenbuerger, and Schjelderup (2008), Kind, Koethenbuerger, and Schjelderup (2010), Kind, Schjelderup, et al. (2013), Kind and Koethenbuerger (2018).

In Kind, Koethenbuerger, and Schjelderup (2010) in particular, under the assumptions that consumers pay a positive per-unit price to buy newspapers and are served by a monopolistic platform who also collects revenues by selling advertising space on its newspapers, it was found that an *ad valorem* tax on revenues from sales increases ads sales and reduces ads prices. This model is extended to a Hotelling duopoly where competition happens on prices and on the degree of product differentiation, finding similar results. In Kind and Koethenbuerger (2018), a monopolistic digital platform provides a good and advertising space, both for a price, and find that the effects of a tax depend upon whether users like or dislike advertising. In particular with ad-averse users and advertisers getting more value from ads if the users base is larger, the tax may increase output on both sides of the market (advertising and final users), while the own-tax elasticity of ads sales is always negative. The latter paper produces, like our model, the result that ads prices may be reduced by a tax, though (contrary to our model) this result requires that consumers are averse to advertising.

A paper which is also related to ours is Bourreau et al. (2018), where a monopolistic digital platform provides online services to users for a fixed access fee and zero unit price. The monopolist in this model sets prices to maximize profits and exploits personal users' data to provide them personalized services and to sell targeted advertising to online sellers. Users value the possibility to buy from well-matched sellers and receive negative utility from uploading more personal information. They find that for a free-for-use platform for users, an *ad valorem* tax generally increases prices and reduce sold ads and user-provided data. This result hinges on the assumptions that users care about the sellers' behaviour, while in our model we do not impose any assumption about consumers' evaluation of potentially useful offers.

Our model departs from these cited works in several important ways. First, we assume that Web companies compete against each other in a Cournot-Nash advertising market, but at the same time they are leaders, or monopolists, in their own Web service market (the latter assumption is supported by large evidence, see for instance Haucap et al. 2014). This assumption in our view better represents observed conditions in digital industries, where giant companies like Alphabet/Google and Facebook compete over the same advertisers but enjoy a strong market power each in its own area (e.g., respectively, search engines for Google and social networks for Facebook). This assumption should therefore improve the external validity of the model vis-a-vis models assuming a monopolistic regime. The latter is the most commonly assumed regime in the works previously referenced, with the sole exception of Kind, Koethenbuerger, and Schjelderup 2010 which discusses a Hotelling competition regime. Second, we assume consumers do not pay any price to access Web services, but they face private variable costs due to the opportunity time needed to use these services. This assumption also better serves external validity as the top global ads-supported Web services (e.g. Google, Youtube, Facebook, Reddit, Yahoo!, Twitter) all provide their services at no charge. The latter assumption, combined with the idea that the usefulness of ads improves with the number of users, bears an important consequence which is better explained in the following section: Web companies might produce more Web contacts than the number of contacts sold to advertisers.

There are other relevant differences between the present model and the previous literature. We assume consumers to be neutral w.r.t. advertising, meaning we provide an analysis that prescinds from users' tastes about ads intensity. Moreover we do not assume that Web services are platforms providing direct sales facilities (contrary for example to Bourreau et al. 2018, where the better matching for ads is valued by consumers as it leads to higher chances to make a valuable purchase), because we are interested in a situation where advertising may be related to any content, thus in principle applying to brand awareness campaigns and political advertising, too. Our results are therefore more general and apply to a broader range of ads-supported Web content. We are then able to show that the result according to which ads sales might increase and ads prices decrease after introducing an *ad valorem* tax on advertising revenues is not necessarily linked to consumers' preferences about ads or potential purchases. In our model the link between ads and users markets is due to investment behaviours of the Web companies and their ability to separately affect sold ads and the number of users, through changes in the service quality. Finally, also contrary to Bourreau et al. (2018), we do not allow users to choose how much personal data to disclose, rather we assume that the provision of personal data happens passively as a by-product of using Web services (e.g. a user employs a search engine, and in doing so reveals to the supplier of the service behavioural patterns through clicks and searched key words). We believe the latter to be closer to actual data patterns found in major services like Google, Reddit and Youtube, where active voluntary data disclosure by users is minimal.

3 The model

There are three types of agents in the model: consumers (we also use the term users, interchangeably), advertisers, and Web companies (also named firms).

Consumers are characterized by a quasilinear well-behaved utility function $u(k, z, y) = \theta(k, z) + y$ and by a budget constraint $hk + y = I$, where I

is exogenous income, y is a Hicksian-composite good with price equal to 1, k defines the units of consumption of a specific Web service of quality z (each unit k is assumed normalized in order to be equivalent to a single advertising "contact") and h is the price of each unit k. We assume that $\frac{\partial\theta}{\partial k} > 0$, $\frac{\partial\theta}{\partial z} > 0$, $\frac{\partial^2\theta}{\partial k^2} < 0$, $\frac{\partial^2\theta}{\partial z^2} < 0$, $\frac{\partial^2\theta}{\partial k\partial z} > 0$. In this setting, h is the opportunity cost of time spent online in order to consume one unit of the Web service (thus, it does not represent a price paid to Web companies), therefore the budget constraint is the potential income I that an individual might spend on the Hicksian-composite good while consuming zero units of the Web service. For simplicity and tractability, we also assume consumers are homogeneous w.r.t. h and I.

Each Web company provides its Web service free of charge and is able to improve the appeal of its Web service, thus affecting z, at a cost. An increase in z can be interpreted as improvements made to the Web service, such as better interface, larger capacity, faster responsiveness. Similarly, a reduction in z represents a deterioration of the service, for example longer queuing or downloading times due to more stringent bandwidth limitations, lower quality of media content, etc.. Web companies obtain their revenues by selling advertising space. By attracting a large number of users to their Web service they generate "contacts", which are then sold for a price to advertisers. Advertisers are a large number of businesses who choose the amount of contacts a in order to maximize their profit function $\pi_{advertisers} = g(a,q) - pa$, where $g(a,q)$ is an increasing concave function (with $\frac{\partial g}{\partial a} > 0$, $\frac{\partial g}{\partial q} > 0$, $\frac{\partial^2 g}{\partial a^2} < 0$, $\frac{\partial^2 g}{\partial q^2} < 0$, $\frac{\partial^2 g}{\partial a\partial q} > 0$) representing value added obtained by each Web contact. The value advertisers get from each contact is also increasing function of the total contacts q that are potentially reachable through the Web company they chose to buy from.

Given the aforementioned assumptions, by standard economic reasoning individual consumers' demands can be aggregated, for each Web service j, into an aggregate demand function $Z_j(z_j, h, I)$ which is increasing in z_j.

Across all Web services, advertisers can be aggregated into a single aggregate demand function $D(p, q)$, decreasing in p and increasing in q. We are assuming symmetric Web companies therefore, at the equilibrium, the values for q will be the same across firms. Each Web company faces a different Web service demand $Z_j(.)$, while they all compete for the same advertising demand $D(.)$. In order to deal with the assumption of symmetry held throughout the analysis of a Cournot-Nash oligopolistic market, we assume that each firm faces an identical demand $Z_j(.)$, which equates to assume that all firms face the same competition conditions in each of the consumption markets they serve. This assumption is meant to represent an economy where few large Web companies enjoy a large monopolistic power in each of their own served markets (e.g. Google in the search engine market, Facebook in the social networks market), and also face similar conditions with respect to the competition regime for non-leader companies. Each company is further assumed to serve a distinct market, so we rule out the possibility of Web companies competing both on ads and on the same served market.

Because for each level of service supply q the level of advertising demand D is uniquely determined by the equilibrium price, we can write $p(q, D)$ as the inverse demand for advertising given the symmetric quantity produced q, $c(q)$ as the cost function for Web companies to produce a quality level z for their Web service which is able to serve $q = Z_j(.)$ contacts, τ as the rate of an *ad valorem* tax on advertising sales, and s the number of contacts sold to advertisers. We assume that (subscripts indicate derivatives):

$$p_q \geq 0 \tag{1}$$

$$p_s < 0 \tag{2}$$

$$c_q > 0 \tag{3}$$

$$0 \leq \tau < 1 \tag{4}$$

$$s_i \leq q_i, s_i \geq 0, q_i \geq 0 \tag{5}$$

The first two conditions express that prices increase with q (because of raised willingness to pay by advertisers) and decrease with s (because of standard arguments). Costs are assumed increasing with quantity q, while the last inequality, $s_i \leq q_i$, sets the constraint that the number of contacts sold to advertisers can never be larger than the number of contacts acquired by the firm.

From previous assumptions, firms can decide to produce more than they sell, and the exceeding production still affects the demand for advertising space. This special mechanism stems from the fact that one additional produced contact has two distinct effects: it increases total costs, and it also increases the price p of each sold contact to advertisers. Sold contacts s on the other hand also bear two effects: to increase revenues, and to reduce the equilibrium price of ads. As these effects can have different intensities, contacts produced and sold coincide only under very specific parametrizations of the model such that the marginal cost (net of the marginal contribution to revenues) from one produced contact is exactly equal to the marginal revenues from one sold contact.

When $s < q$, net profit for firm i is:

$$\pi_i(q_i, s_i) = (1-\tau)p(q_i, s_i + D_{-i})s_i - c(q_i) \tag{6}$$

with D_{-i} denoting the aggregate ads demand net of the demand s_i served by firm i.

First-order conditions (FOCs) are:

$$(1-\tau)p_q s_i = c_q \tag{7}$$

$$-p_{s_i} s_i = p(q_i, s_i + D_{-i}) \tag{8}$$

Second-order conditions (SOCs) are:

$$(1-\tau)p_{qq}s_i - c_{qq} < 0 \tag{9}$$

$$2p_s + p_{ss}s_i < 0 \tag{10}$$

$$p_{sq}s_i + p_q < 0 \tag{11}$$

Note that these SOCs, together with assumptions under (1)-(5), imply $p_{sq} < 0$. Additional assumptions would further limit the range of permissible values, for example: with linear costs it is $p_{qq} < 0$; with isoelastic demand functions w.r.t. s, it is $p_{ss} < 0$.

We demonstrate the following Proposition, which is both useful for subsequent proofs and provides an effect of taxation which is, at least in principle, empirically testable:

Proposition 1. *If $s < q$, an increase in an ad valorem tax τ affects s and q with opposite signs.*

Proof. Differentiating the FOCs in (7) and (8) w.r.t. τ and rearranging we obtain (subscript i is omitted to improve readability):

$$(1-\tau)(p_{sq}s + p_q)s_\tau + [(1-\tau)p_{qq}s - c_{qq}]q_\tau = p_q s$$
$$(2p_s + p_{ss}s)s_\tau + (p_{sq}s + p_q)q_\tau = 0$$

From assumptions in (1)-(5) and the conditions needed to satisfy SOCs from (9)-(11), it descends that the quantities multiplied by s_τ and q_τ, in both equations, have the same (negative) sign. Hence to satisfy equality in the second equation, s_τ and q_τ need to have opposite sign. □

With symmetric Cournot-Nash equilibrium (see Fullerton et al. 2002 for a textbook exposition of Cournot-Nash tax incidence theory) we need the following equations where total advertising demand is $D = Ns$:

$$p = p(q_i, Ns) \tag{12}$$

$$(1-\tau)p_q s_i = c_q \tag{13}$$

$$-p_{s_i} s_i = p(q_i, Ns) \tag{14}$$

We write the producer price as $\tilde{p} = (1-\tau)p$. In order to study the incidence of the tax on equilibrium prices, we derive the following expression:

$$\frac{d\tilde{p}}{d\tau} = (1-\tau)\frac{dp(q, Ns)}{d\tau} - p = (1-\tau)(p_s s_\tau + p_q q_\tau) - p \tag{15}$$

which indicates overshifting if the increase in $\tilde{p}$ exceeds 100%.

So far we assumed that it was always verified that $s < q$. It might be the case, though, that maybe because of a pre-existing positive level of taxation, an increase in τ happens when instead $s = q$. In such case firms only optimize over a single control variable and their target function is (writing $x = s = q$ to avoid confusion with previous notation):

$$\pi_i(x_i) = (1-\tau)p(x_i, x_i + D_{-i})x_i - c(x_i) \tag{16}$$

The single FOC for this optimization problem implies $x = \left(\frac{c_x}{1-\tau} - p\right)\left(\frac{1}{p_x}\right)$, which means that the sign of x_τ is undetermined: as function $p(x_i, x_i + D_{-i})$ is not necessarily monotone, the sign of p_x can change based on the chosen parameters. From the FOC we derive x_τ. After rearranging:

$$x_\tau = \frac{c_x}{(1-\tau)^2[2p_x - \frac{c_{xx}}{1-\tau} + \left(\frac{c_x}{1-\tau} - p\right)\frac{p_{xx}}{p_x}]} \tag{17}$$

As $c_x > 0$ by assumption, sufficient conditions for having $x_\tau < 0$ are $c_{xx} \geq 0$ (thus, a linear or convex cost function), $p_x < 0$ and $p_{xx} > 0$ (which together determine a decreasing concave demand for advertising).

Constant elasticity demand function for advertising Proposition 1 holds regardless of the specific functional forms chosen for $p(.)$ and for the cost function, provided that they fulfil the requirements from the initial assumptions in (1)-(5) and the additional requirements stemming from the SOCs in (9)-(11). In order to further advance the analysis, from this point onward we will turn to a specific functional form for the inverse demand function.

We choose the following functional form:

$$p(.) = \frac{(1+q_i)^\alpha}{(s_i + D_{-i})^\beta} \tag{18}$$

with $\alpha \geq 0$ and $\beta > 0$. This function is convenient as it complies with our previous assumptions ($p_q \geq 0$ and $p_s < 0$), and it is such that $p_{sq} < 0$, $p_{ss} > 0$ and $p_{qq} < 0$ if $\alpha < 1$. Thus, it also complies with SOCs and guarantees an interior solution. If $\alpha = 0$ it reduces to the well known constant elasticity demand function used in many previous works dealing with indirect taxation under Cournot-Nash competition. It is moreover $\lim_{D\to\infty} p(.) = 0$ for any $\alpha \geq 0$.

The following Proposition determines the sign of the change in quantities sold and produced.

Proposition 2. *If $s < q$ and the inverse demand function is in the form as in (18), an increase in an ad valorem tax τ always reduces the equilibrium price p if $\beta < 1$. It always increases the the equilibrium price p if $\beta > 1$.*

Proof. Recalculating FOCs using (18) assuming $s < q$, after some algebraic manipulations and then differentiating w.r.t. τ, we obtain

$$s_\tau = \frac{1}{(1-\beta)(1-\tau)} \left[\frac{c(q)\beta^\beta (1+q)^{1-\alpha}}{(1-\tau)\alpha} \right]^{\frac{\beta}{1-\beta}} \tag{19}$$

which is larger than zero if, and only if, $\beta < 1$, and smaller than zero if and only if $\beta > 1$. It therefore implies, because of Proposition (1), $q_\tau < 0$ if $\beta < 1$

and $q_\tau > 0$ if $\beta > 1$.

To verify the sign of p_τ, again from Proposition 1 we know that s_τ and q_τ have opposite sign. As it is $\frac{dp}{d\tau} = (1-\tau)(p_s s_\tau + p_q q_\tau) < 0$, either it is true that $s_\tau < 0$ and $q_\tau > 0$ which imply, from eq. (15) and because of the assumptions imposing $p_s < 0$ and $p_q \geq 0$, that $\frac{dp}{d\tau} > 0$, or alternatively it is true that $s_\tau > 0$ and $q_\tau < 0$, which imply $\frac{dp}{d\tau} < 0$. □

A rise in the tax therefore produces an increase in the quantities sold to advertisers and a reduction in number of served consumers if the elasticity of ads demand is low, and vice versa if it is large. Note that the result in Proposition 2 is at odds with the predictions from standard Cournot-Nash oligopoly papers always predicting a reduction in quantities sold and increase in equilibrium price in response to an *ad valorem* tax.

Moving to the case with $x = s = q$, the FOC is $(1-\tau)p[1+x\left(\frac{\alpha}{x+1} - \frac{\beta}{x+D_{-i}}\right)] = c_x$. Substituting Nx in place of $x + D_{-i}$, differentiating w.r.t. τ and rearranging to obtain x_j produces the following expression:

$$x_j = \frac{c_x p}{\tilde{p}^2\left[\frac{\alpha}{x+1}(1-\frac{x}{(x+1)^2}) - \frac{c_{xx}}{\tilde{p}} - \frac{c_x \tilde{p}_x}{\tilde{p}^2}\right]} \tag{20}$$

which is negative for small enough α coupled with linear or convex costs. A small enough α also implies (from (18)) that $p_x < 0$ and therefore, that $p_j > 0$. These results are summarized in the following Proposition:

Proposition 3. *With an inverse demand function as in (18), $s = q$ and linear or convex cost function $c(.)$, there is a small enough value of α for which equilibrium prices increase with the tax τ.*

Applying the same approach as in 15, we obtain that

$$\frac{d\tilde{p}}{d\tau} = p\left[(1-\tau)\left(\frac{\alpha}{x+1} - \frac{\beta}{x}\right)\right] \tag{21}$$

which is positive (and therefore, implies overshifting) only if β is sufficiently small compared to α. Now, combining the insights from eq. (20) and (21), we see that in order to have decreasing sales and increasing price as in standard Cournot-Nash theory, and moreover to predict overshifting, it must hold at the same time that costs are linear or convex, that α sufficiently small, and also that β is small compared to α. From (21), for large x, it must therefore be true that $\beta < \alpha$. Put in other terms, it must hold true that the elasticity of advertising demand is small and, also, that the gains in terms of better targeting are small.

Number of competing firms From (17) we know that the number of firms competing in the advertising market quantitatively changes the way τ affects s through different levels in the equilibrium price p. For a given set of parameters α and $\beta < 1$, keeping q fixed while it still holds that $s < q$ the equilibrium price must always fall with the tax. As N approaches infinity the derivative p_s decreases in absolute value. An implication is that with large N, for an individual firm an increase in q impacts on prices more than an equivalent decrease in s, therefore holding the other parameters fixed, the incentive to increase s subsequent to the introduction of a tax is larger than with small N, though this does not imply as well a larger change in the equilibrium price.

Implications for Welfare analysis The model cannot provide clear conclusions with respect to the overall impact on welfare of an *ad valorem* tax, without picking concrete values for the parameters at play. Generally speaking, the result according to which the tax is expected to increase s and reduce q in presence of low β suggests a worsening of consumers' welfare, as they will now consume Web services of lower quality for the same (private and non-monetary) price. Possibly, the rise in the ratio $\frac{s}{q}$ could also be seen as a worsening element if consumers dislike receiving ads while they consume the Web services, though we purposely excluded such possibility from our analysis. Advertisers are generally better off under $s = q$ and decreasing price as both quality of the ads and quantity supplied increase.

If Welfare W is the sum of the surplus of consumers, advertisers, Web companies and the tax revenues collected, and we further assume homogeneous consumers and advertisers, expressing as mN the number of consumers in market j and as oN the number of advertisers, Welfare can be written as:

$$W = (m\theta(q) + og(\bar{s}, q) - hZ(q) - c(q))N \tag{22}$$

where s and q are the symmetric values stemming from previous analysis, $\bar{s} = \frac{s}{oN}$, and where (with a slight abuse of notation for the sake of readability) we wrote $\theta(\frac{Z}{mN}, q) = \theta(q)$ and $Z(z(q), h, I) = Z(q)$ as m and N are fixed and Z only depends on q.

In cases where $q_\tau > 0$ and thus where $s_\tau < 0$ and $p_\tau > 0$, from (22) one sees that it is more likely to obtain Welfare gains if marginal costs (both production costs and private non-monetary costs) are small, the gains from better ads targeting for advertisers are large, and users' evaluation of service improvements are large. The inverse reasoning holds true if $q_\tau < 0$, $s_\tau > 0$ and $p_\tau < 0$.

Proposition 2 indicates that a large ads price elasticity ($\beta > 1$) is the key factor determining, in the most likely case to be met in practical applications (that is, the case where $s < q$), whether the introduction of an *ad valorem* tax leads to a scenario with $q_\tau > 0$, $s_\tau < 0$ and $p_\tau > 0$. Estimates of advertising demand are available for newspaper and television advertising. Argentesi et al. (2007) study the Italian newspaper market which is a two-sided market with paying readers on one side and advertisers on the other, and find own-price elasticities ranging between -0.91 and -0.33. Wilbur (2008) focuses on U.S. television broadcasting networks and find an elasticity of -2.9, which is reported to be much larger than similar estimates produced in the 1970s, the latter always being between -1 and 0 (such change is explained by the author pointing to increased competition in that market). These estimates are both very heterogeneous and also not directly applicable to digital advertising, though they somewhat provide a proxy estimate. Our model asks for more empirical research specifically oriented at estimating Web advertising price elasticities, in order to provide dependable predictions abut the Welfare implications of the Web Tax studied here.

4 Conclusions

Base erosion of corporate taxation, both direct and indirect, is pushing governments to introduce policy reforms aimed at limiting revenue loss and tax-induced advantages benefiting Web businesses. We studied the effects of a Web Tax in a setting where Web companies compete in a Cournot-Nash fashion to sell advertising space to advertisers, while they enjoy monopolistic power in the market for their Web service. In our model, Web companies can choose to increase investments in order to improve the quality of their service thus attracting more free users, and in doing so they can enhance the value paying advertisers obtain from ads. It can be therefore beneficial for Web companies to have more potential contacts than the quantity sold to advertisers in order to keep prices high.

In such setting, we demonstrated that a Web Tax affects quantities produced and sold in opposite ways. Further assuming a specific functional form for the inverse advertising demand function, we found that the sign of these changes, and consequently whether ads price will increase or decrease after introducing a Web Tax, is function of the magnitude of the price elasticity of advertising demand. In the special case where quantities produced and sold coincide, and these react in sync to an increase in taxes, the magnitude of the impact targeting technologies have on the advertisers' evaluation of ads determines the direction of the adjustment and, in conjunction with the value for price elasticity of advertising demand, determine whether there is over- or undershifting of the tax in case prices increase with it. We also derived conditions for the tax to be Welfare improving or deteriorating.

The present paper asks for targeted empirical work to assess the price elasticity of advertising demand and to quantitatively understand how much ads targeting technologies impact on advertisers' reservation prices. Our model provides guidance for a parametric evaluation of Web Taxes based on testable quantities, which hopefully will help inform the impact assessments of future policy initiatives.

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