European Commission

JRC TECHNICAL REPORT

Redistributive policies in general equilibrium

JRC Working Papers on
Territorial Modelling and Analysis
No 08/2021

Authors:
Economides, G.
Philippopoulos, A.
Sakkas, S.

2021

Joint Research Centre

Contact information
Name: Simone Salotti
Address: Edificio Expo, C/Inca Garcilaso 3, 41092 Sevilla (Spain)
Email: simone.salotti@ec.europa.eu
Tel.: +34 954488406

EU Science Hub
https://ec.europa.eu/jrc

JRC126365

Seville: European Commission, 2021

How to cite this report: Economides, G., Philippopoulos, A., and Sakkas, S. (2021). Redistributive policies in general equilibrium. JRC Working Papers on Territorial Modelling and Analysis No. 08/2021, European Commission, Seville, JRC126365.

The **JRC Working Papers on Territorial Modelling and Analysis** are published under the supervision of Simone Salotti and Andrea Conte of JRC Seville, European Commission. This series mainly addresses the economic analysis related to the regional and territorial policies carried out in the European Union. The Working Papers of the series are mainly targeted to policy analysts and to the academic community and are to be considered as early-stage scientific papers containing relevant policy implications. They are meant to communicate to a broad audience preliminary research findings and to generate a debate and attract feedback for further improvements.

Redistributive policies in general equilibrium*

George Economides[†], Apostolis Philippopoulos[‡§], and Stylianos Sakkas[¶]

September 2, 2021

Abstract

We develop a general equilibrium OLG model to evaluate a wide menu of popular redistributive policies in a unified context. We work in two steps: First, we study how initial conditions in human and financial capital, as inherited from family background, shape individuals' human capital, and hence their work opportunities, income and wealth, and eventually macroeconomic outcomes. Second, we study which policies can reduce this type of inequality without, hopefully, damaging macroeconomic efficiency.

Keywords: Inequality, efficiency, government expenditures, public policy.

JEL: D63, H21, H50

*We thank Manos Matsaganis and Panos Tsakloglou for discussions and data. We gratefully acknowledge support from EC-JRC (EMI Pool 039182) for this project. Any views and errors are ours and not of the European Commission.

†Athens University of Economics and Business, and CESifo

‡Athens University of Economics and Business, and CESifo

§Corresponding author: Apostolis Philippopoulos, Department of Economics, Athens University of Economics and Business, 76 Patission street, Athens 10434, Greece. Tel: +30-210-8203357. Email: aphil@aueb.gr

¶European Commission, Joint Research Centre

1 Introduction

Concerns over growing inequality of income and wealth have been at the center of public and policy debate in recent years and especially since the outbreak of the global financial crisis of 2008 (for reviews, see e.g. Bourguignon (2015, 2018)). The rise in inequality in several countries, or the simple perception that the public opinion may have of its rise, have led to news calls for redistributive policies.

In this paper, we develop a general equilibrium model in order to to evaluate a wide menu of redistributive policies in a unified context. Although there is a rich and still growing literature on the topic, to the best of our knowledge, there has not been a quantitative comparison of the most commonly used or debated redistributive policies within a single unified context.

The aim of the paper is twofold. First, it studies the role of individuals' initial conditions and how the latter shape the accumulation of human capital and hence work opportunities, income and wealth at personal level and how all this is reflected in macroeconomic outcomes. The second aim is to study what policy can do to reduce this type of inequality, whose roots are differences in initial conditions, without, hopefully, damaging macroeconomic efficiency. We focus on initial conditions in inherited human and financial capital. In other words, in our work, the drivers of inequality are differences in these initial conditions and the subsequent creation of human capital that distinguishes people between skilled and unskilled or equivalently between rich and poor.

It is well recognized that initial conditions (father-son relationship or inter-generational correlation) play an important role in shaping opportunities over life thereby affecting social (im)mobility and (in)equality persistence. This applies to parental human capital, financial wealth and usually a mix of the two (see e.g. Glomm and Ravikumar (1992), Galor and Zeira (1993), Benabou (1996), Cunka and Heckman (2007), Ehrlich and Kim (2007), Corak (2013), Huggett et al (2011), Heckman and Mosso (2014), Autor (2014), Francesconi and Heckman (2016), Dettmer et al (2020) and also see the data in section 2 below). It is also well recognized that the distinction between skilled and unskilled people can explain a big part of income inequality at least among "the other 99%" (see e.g. Acemoglu (1998), Autor et al (1998), Goldin and Katz (2008), Acemoglu and Autor (2011), Acemoglu (2009, chapter 15), Aghion and Howitt (2009, chapter 8), Autor (2014) and also see the data in section 2 below).

The vehicle used for our analysis is a dynamic OLG model. Regarding households, we have two distinct types that differ in initial levels of financial and human capital which both depend on their family background; we call them rich-born and poor-born households. Each type can live for three periods as young, adult and old in an overlapping generations setup. Choices, in all three periods, can differ depending on initial conditions. However, we also allow for social mobility depending on the level of human capital accumulated during the young age. Regarding firms, given the evidence that skill-biased technological progress is behind the wage premium to skilled workers (the so-called skill premium), we use the technology introduced by Stokey (1996) and Krusell et al (2000) according to which capital accumulation is more complementary to skilled labor than the unskilled one; hence the skill premium. Regarding the government, redistributive policies are captured by a rich menu of taxes and public spending, as well as by how individual categories of public spending are allocated between the skilled/rich and the unskilled/poor.

Our main results are as follows. A further increase in public spending on education, health and work-supplement services, when these public services are made available to all people, is good not only for aggregate output but also for social mobility and income equality. A number of targeted redistributive policies can deliver similar beneficial results, although of a smaller magnitude; this happens, for example, in the case of earmarking more public resources to health and pensions for the unskilled. At the other end, there are policies that hurt the income of everybody and also worsen income inequality (at least in the medium term); this applies to an increase in non-targeted spending on pensions and transfers to working adults, as well as an increase in the progressivity of labor income taxes. Finally, there are cases in which there are tradeoffs so that social judgements need to be made. For example, the exclusion of skilled people from public spending on education and work-supplement services, a further increase in personal capital income taxes, a further increase in corporate taxes and a furthher increase in non-targeted pensions provided to everybody, may reduce income inequality but have detrimental effects on aggregate performance.

There are at least three general messages from these results. First, incentives to work, save and invest are crucial to both aggregate and individual outcomes. Incentives produce second-round or general equilibrium effects that may move outcomes in the opposite direction from first-round effects. Second, in general equilibrium, the public perception, further fuelled by several political leaders and commentators, that there is always an unpleasant tradeoff between efficiency and equity is not correct. There are policies that can improve both. Third, in a general equilibrium setup, there are social complementarities. For instance, an increase in public spending on the education of the poor, to the extent that this is financed by a decrease in public spending on the education of the skilled, is bad for the aggregate economy so everybody becomes worse off even if inequality is reduced. And vice versa: a policy that improves the effective return to unskilled labor also benefits the skilled even if the skill premium falls.

The rest of the paper is as follows. Section 2 presents data. Section 3 presents the model. Section 4 is on parameterization and model solution. Section 5 simulates the effects of policy reforms. Conclusions are in section 6. Details are placed in an Appendix.

2 A look at the data

In this section, we provide two types of data. We start with data on inequality and then present fiscal instruments typically used to combat inequality.

2.1 Inequality and some of its characteristics

Graph 1 shows the evolution of income inequality over 2005-2019 in the eurozone also distinguishing between core and periphery countries. Income inequality is measured by the Gini coefficient which is one of the most commonly used measures of income and wealth inequality (see e.g. Bourguignon (2015, 2018)). The higher this index, the worse the income inequality. As can be seen, income inequality has been systematically higher in the periphery countries, although the gap between core and periphery has become smaller in the aftermath of the 2008 global financial crisis.

Graph 1: Income inequality in the Eurozone

Graph 2 shows the relationship between the skill premium and the Gini coefficient in the year 2014 in a number of European countries. The correlation is positive meaning that in countries where the return to tertiary education is high, relative to that to high-school education, inequality is also higher. This supports our assumption that education and human capital are key drivers of inequality.

Graph 2: Skill premium and income inequality in European countries

Graph 3 shows the relationship between income inequality and "earnings elasticity" in the late 2000s in a group of European countries. Income inequality is again measured by the Gini coefficient, while, earnings elasticity is a measure of economic immobility showing how closely related an offspring's economic status is to that of his/her parents. The higher this measure, the more a child's status is determined by parental status or equivalently the lower economic mobility is. The scatterplot shows a positive relationship meaning that higher immobility is associated with higher inequality. It is worth noticing that most periphery countries exhibit low mobility and high inequality which is the opposite from Nordic-Scandinavian countries. A message is that initial conditions, and in particular the parents' level of education and/or wealth, matter to income inequality. The more these conditions matter, the more the existing inequalities are determined by unequal opportunities in early life. This supports our modelling assumptions for the importance of inherited, initial conditions. It can also rationalize redistributive policies.

Graph 3: Economic immobility and income inequality in European countries

2.2 Fiscal policy

Tables 1a and 1b illustrate the key characteristics of fiscal policy in the Eurozone. Specifically, Table 1a presents public spending categories according to their function (averages over 2001-2019), while Table 1b presents the main types of tax revenues (averages over 2001-2018).

On the spending side, we have public spending on social protection (which includes spending on pensions, family support, unemployment benefits, housing, sickness and disability, etc), public spending on health (which includes public health, medical products and equipment, hospital services, outpatient services, etc), public spending on general public services (which includes public debt payments, administrative costs, executive and legislative organs, fiscal affairs and other transfers of a general character between different levels of government), public spending on education (which includes spending on pre-primary and primary education, secondary and post secondary non-higher and higher education), public spending on economic affairs (which includes public infrastructure spending such as public transport, fuel and energy, mining, manufacturing and construction, communications, licenses and other related support programs) and public spending on public order-safety and defense (which includes military defense, civil defense, foreign military aid, police and fire protection services, law courts and prisons, etc). Other minor (quantitatively) types of spending are on environmental protection, housing and community amenities and recreation and culture.

On the revenue side, there are direct taxes on personal income from labor and capital, as well as corporate income taxes on firms' profits, while the main indirect taxes are taxes on sales, value-added or imported goods (indirect taxes are labeled as consumption taxes in Table 1b and throughout the paper). Also, although there are differences across countries, for the average of the Eurozone, an important share of tax revenues comes from social security contributions.[1]

As can be seen, public spending on social protection is the biggest spending item in the data, being followed by spending on health and general public services and, in turn, by spending on education and economic affairs. Regarding taxes, the main revenues come from consumption taxes and personal (labor plus capital) income taxes. Social security contributions paid by employers contribute more than those paid by employees.

In what follows, we will develop a model that gives a natural role to most of these spending and tax categories.

Table 1a: Structure of public spending in the Eurozone

Table 1b: Structure of tax revenues in the Eurozone

3 Model

In this section, we develop the model used. We start with an informal description and then formalize things.

3.1 Informal description of the model

Economic agents and their roles We consider a closed economy populated by households, firms and the government. Regarding households, we distiguish two types. Both types can live for three periods as young, adult and old in an overlapping generations setup.[2] Also, for both types, the first period is devoted to education, the second to active economic life and the third to retirement. On the other hand, the two types

[1] See European Commission (2014b, 2018b) and OECD (2017).

[2] See e.g. de la Croix and Michel (2002) for a review of overlapping generations models. Acemoglu (2009, chapter 9) also reviews the OLG model.

differ in initial levels of financial and human capital, which both depend on their family background, and these differences in initial conditions shape different lifetime opportunities and choices. However, we also allow for the possibility of social mobility depending on the level of human capital accumulated during the young age. Specifically, the poor-born can manage to become skilled, and so climb up the income ladder for the rest of their lifes, if their human capital in the beginning of adult life is high enough; and symmetrically opposite for the rich-born, namely, although they start with an advantage, they can become unskilled, and so go down the income ladder for the rest of their lifes, if their human capital in the beginning of their adult life gets low enough. This is discussed in detail below. Households are modeled in subsection 3.2.

Regarding firms, given the evidence that skill-biased technological progress is behind the wage premium to skilled workers (the so-called skill premium), and this has been happening despite the concurrent rise in the numbers of college graduates, we use the technology introduced by Stokey (1996) and Krusell et al (2000) and used by many others since then. Namely, the firm uses physical capital and two types of labor services, skilled and unskilled, where the former is more complementary to capital than unskilled labor; hence the skill premium. Firms are modeled in subsection 3.3.

Regarding the government, we assume a rich menu of taxes and public spending as they are recorded in the data. The governemt is presented in subsection 3.4.

Human capital formation and social mobility The human capital generated by a young individual (which shapes his/her economic status when he/she becomes adult and old) depends on the human capital of his/her predecessors, his/her own effort at school, public policies like government education spending, and, if he/she is rich-born, on private tuition fees. That is, leaving aside the role played by public policies, the rich-born start with an advantage: they inherit a better human capital as well as financial wealth that both allow them to build more human capital ceteris paribus. In turn, if the human capital of a household in the beginning of his/her adult life is high enough (see right below what this means), this household becomes a skilled worker and this allows him/her to enjoy a higher wage rate and to participate in the asset markets. Income from these assets, as well as a public pension, make his/her income when old and this allows him/her to leave a financial bequest which in inherited by the newly rich-born of the next generation. If, on the other hand, the human capital of a household in the beginning of his/her adult life proves to be low enough (see right below what this means), this household works as an unskilled worker, which means a lower wage rate and inability to participate in the asset markets. The old member of this poor family lives only of his/her public pension and is not able to leave a financial bequest to the newly poor-born of the next generation.

Nevertheless, although opportunities, choices, occupations and incomes can differ between the rich-born and the poor-born and these differences can persist over one's lifetime and across generations, we also allow for social mobility depending on the level of human capital that individuals possess. In particular, when young in period t, the poor-born face a probability $0 \leq q_{t+1}^{s} \leq 1$ of becoming skilled when adult at $t+1$ and hence improve their economic status when adult and old, and a probability $0 \leq 1 - q_{t+1}^{s} \leq 1$ of becoming unskilled and hence remain poor. They manage to become skilled and so climb up the income ladder when their effective human capital exceeds a threshold level, where the latter is a combination of their endogenously determined human capital at the start of $t+1$ (as explained above) and an exogenous idiosyncratic shock drawn from a uniform distribution. Symmetrically, those who are born rich at t face a probability $0 \leq q^u_{t+1} \leq 1$ of becoming unskilled and hence move down when adult and old, and a probability $0 \leq 1-q^u_{t+1} \leq 1$ of becoming skilled and so remain rich. This probability is also endogenous and modeled in a symmetrically opposite way from they way we model q^s_{t+1}. In other words, the born-rich become unskilled and so climb down the income ladder when their effective human capital falls below a threshold level.

3.2 Households

For expositional convenience, we first present the households' problems (constraints and utility functions) assuming away social mobility. After presenting these relatively simple problems, we will add the possibility of social mobility and solve the households' optimization problems accordingly.

Households can live for three periods (as young, adult and old), consume in each period, invest in education when young being supported by their parents, work when adult and retire when old; as pointed out by e.g. De la Croix and Michel (2002, chapter 5), assuming three-period lived households is the simplest way to capture the three main stages of life (education, active economic life and retirement).

3.2.1 The problem of rich-born households without social mobility

Each rich-born individual is indexed by superscript r. A rich young individual starts with his/her grandparents' bequest and spends effort time and private tuition fees in education. When he/she becomes adult, he/she works as a skilled person and saves in the form of capital and government bonds; in other words, in the absence of social mobility, he/she remains rich. When he/she reaches the old age, he/she uses his/her own savings as well as a public pension, and dies leaving an optimally chosen bequest which will be inherited by the newly rich-born people. We now present the rich household's constaints and utility function.

Budget constraints When a person is born rich at time t and remains so in the rest of his/her life, the budget constraints when young, adult and old at t, $t+1$ and $t+2$ respectively are:

$$\left(1+\tau^c_t\right) c^{r,y}_t + z^{r,y}_t = \left(1-\tau^b_t\right) b^{r,y}_{t-1} + \Psi^{r,y}_t \tag{1a}$$

$$\left(1+\tau^c_{t+1}\right) c^{r,m}_{t+1} + k^{r,m}_{t+1} + d^{r,m}_{t+1} + \Psi^{r,m}_{t+1} = \left(1-\tau^{n,r}_{t+1}-\tau^s_{t+1}\right) (g^{w,r,m}_{t+1})^{\chi} w^r_{t+1} h^{r,m}_{t+1} l^{r,m}_{t+1} + g^{t,r,m}_{t+1} \tag{1b}$$

$$\left(1+\tau^c_{t+2}\right) c^{r,o}_{t+2} + b^{r,o}_{t+2} = \left[1-\delta^k + \left(1-\tau^k_{t+2}\right) r^k_{t+2}\right] k^{r,m}_{t+1} + \left(1-\tau^k_{t+2}\right) \pi^{r,o}_{t+2} + \left(1+r^d_{t+2}\right) d^{r,m}_{t+1} + s^{r,o}_{t+2} \tag{1c}$$

where $c^{r,y}_t$, $c^{r,m}_{t+1,}$ and $c^{r,o}_{t+2}$ are r's consumption when young, adult and old respectively, $b^{r,y}_{t-1}$ is an endowment inherited from his/her grandparents/old of the previous generation in case a new person is born in a rich family, $\Psi_t^{r,y}$ is a gift received by a young rich person from his/her parents/adults, $z_t^{r,y}$ is private spending on education when young and rich,[3] $h_{t+1}^{r,m}$ is the stock of human capital of a skilled adult (see below for the motion of human capital), $l_{t+1}^{r,m}$ is work hours of a skilled adult, w_{t+1}^{r} is the wage rate earned by skilled people, $g_{t+1}^{w,r,m}$ is government spending per skilled adult earmarked for work supplement public services where the pararameter $\chi \geq 0$ is a measure of their efficiency, $g_{t+1}^{t,r,m}$ is a transfer payment to each rich adult from the government, $k_{t+1}^{r,m}$ is savings in the form of physical capital, $d_{t+1}^{r,m}$ is savings in the form of government bonds, $\Psi_{t+1}^{r,m}$ is a gift from parents/adults to their children,[4] r_{t+2}^{k} and r_{t+2}^{d} denote respectively the returns to physical capital and government bonds, $\pi_{t+2}^{r,o}$ is dividends received from the ownership of firms, $s_{t+2}^{r,o}$ is the pension provided to each rich old person by the government, and $b_{t+2}^{r,o}$ is a financial bequest to the next cohort of rich people.[5] Finally, $0 \leq \tau_t^c$, τ_t^k, $\tau_t^b < 1$ are proportional tax rates on consumption, personal capital income and bequests respectively, $0 \leq \tau_t^s < 1$ is a proportional social security contribution paid by employees,[6] while $0 \leq \tau_t^{n,r} < 1$ denotes the progressive average tax rate on the labor income paid by the rich.

We assume that equation (1c) holds with probability $0 \leq q_{t+2}^{r,o} \leq 1$ only, while, with probability $0 \leq 1 - q_{t+2}^{r,o} \leq 1$, the rich adult dies before reaching the old age (this probability is modelled below). When the agent dies before reaching the old age, he/she leaves an enforced or unintended bequest, denoted as $\Omega_{t+2}^{r,o}$. The latter is his/her whole wealth, namely, from (1c) above:

$$\Omega_{t+2}^{r,o} \equiv \left[1 - \delta^k + \left(1 - \tau_{t+2}^k\right) r_{t+2}\right] k_{t+1}^{r,m} + \left(1 - \tau_{t+2}^k\right) \pi_{t+2}^{r,o} + \left(1 + \rho_{t+2}\right) d_{t+1}^{r,m} \tag{2}$$

where modelling details are provided in Appendix A. Notice that the initial endowment inherited by the rich-born young of the next cohort, $b_{t+2}^{r,y}$, will be a weighted average of the bequest voluntarily chosen by the old, $b_{t+2}^{r,o}$, and the enforced bequest, $\Omega_{t+2}^{r,o}$, where the weights are respectively the probability of reaching the old age, $q_{t+2}^{r,o}$, and the probability of suddenly passing away, $1 - q_{t+2}^{r,o}$ (see Appendix D for details).

Motion of human capital The human capital of r household at the beginning of $t+1$ when adult/work life starts is:

$$h_{t+1}^{r,m} = \left(1 - \delta^{r,h}\right) h_t^{r,y} + B^r \left(e_t^{r,y}\right)^{\theta} \left[\gamma \left(z_t^{r,y}\right)^{\nu} + (1 - \gamma) \left(g_t^{r,e} + \kappa g_t^{r,h}\right)^{\nu}\right]^{\frac{1-\theta}{\nu}} \tag{3}$$

where $h_t^{r,y}$ is r's human capital inherited by his/her predecessors, $e_t^{r,y}$ is effort time spent in education when young, $z_t^{r,y}$ is private tuition fees spent by a young person born in a rich family, $g_t^{r,e}$ and $g_t^{r,h}$ are

[3] We have also allowed young households to borrow from, say, adults or old households to finance their spending on consumption and education. We report that the main results do not change.

[4] $\Psi_{t+1}^{r,m}$ and $\Psi_t^{r,y}$ are obviously linked to each other (see the market-clearing conditions in Appendix D).

[5] Bequests are modelled as in e.g. Acemoglu (2009, chapter 9.6) and Coeurdacier et al. (2015). $b_{t+2}^{r,o}$ and $b_{t-1}^{r,y}$ are obviously linked to each other (see the market-clearing conditions in Appendix D).

[6] This modelling is as in e.g. Bruce and Turnovsky (2013). Alternatively, we could assume that the labor income tax, τ_t^n, is imposed after we deduct social security contributions, namely, to have $(1 - \tau_{t+1}^n)(1 - \phi_{t+1}) w_{t+1}^r l_{t+1}^{r,m} h_{t+1}^{r,m}$ in the adult's budget constraint (see e.g. Conesa and Garriga (2008)). We report that our main results do not depend on the particular way we model the social security tax (results are available upon request).

respectively government spending per rich young person allocated to education and health services, the parameter $0 \leq \kappa \leq 1$ measures how much public spending on health contributes to the quality of human capital (unhealthy people cannot be efficient, irrespectively of their education level), and $B^r > 0$, $0 \leq \theta \leq 1$, $0 \leq \gamma \leq 1$, $0 \leq \nu \leq 1$ are parameters.[7] In other words, private tuition fees and public policies are combined into a composite via a CES technology, with an elasticity of substitution $\frac{1}{1-\nu}$ and relative importance γ, and then this composite combines with effort, $e_t^{r,y}$, via a Cobb-Douglas technology, with a relative share θ for $e_t^{r,y}$. The resulting end-of-period stock, $h_{t+1}^{r,m}$, is used by the adult during his/her work life at $t+1$.[8]

Utility function The discounted lifetime utility of a person who is born rich at time t and remaines rich in the rest of his/her life is:

$$
\begin{aligned}
u_t^i = & \frac{(c_t^{r,y})^{1-\sigma}}{1-\sigma} - \chi_n \frac{(e_t^{r,y})^{1+\eta}}{1+\eta} + \chi_g \frac{(g_t^u)^{1-\eta_g}}{1-\eta_g} + \\
& +\beta \left\{ \frac{(c_{t+1}^{r,m})^{1-\sigma}}{1-\sigma} - \chi_n \frac{(l_{t+1}^{r,m})^{1+\eta}}{1+\eta} + \chi_g \frac{(g_{t+1}^u)^{1-\eta_g}}{1-\eta_g} \right\} \\
& +\beta^2 q_{t+2}^{r,o} \left\{ \frac{(c_{t+2}^{r,o})^{1-\sigma}}{1-\sigma} + \chi_g \frac{(g_{t+2}^u)^{1-\eta_g}}{1-\eta_g} + \beta \chi_b \frac{(b_{t+2}^{r,o})^{1-\eta_b}}{1-\eta_b} \right\}
\end{aligned} \tag{4}
$$

where, as said above, $c_t^{r,y}$, $c_{t+1,}^{r,m}$ and $c_{t+2}^{r,o}$ are r's consumption when young, adult and old respectively, $e_t^{r,y}$ is effort time spent in education when young, $l_{t+1}^{r,m}$ is effort time spent in work when adult and $b_{t+2}^{r,o}$ is a bequest chosen by the old.[9] Also, g_t^u denotes per capita public spending on "utility-enhancing" public goods and services (see below for their definition), while, the parameter $0 < \beta < 1$ is the subjective time preference rate and σ, χ_n, η, χ_g, η_g, χ_b, η_b are preference parameters. Notice that there only a probablity $q_{t+2}^{r,o}$ of reaching the old age.

3.2.2 The problem of poor-born households without social mobility

Each poor-born individual is indexed by superscript p. Differently from a rich-born person, a poor young individual starts without inherited financial capital and with relatively little (i.e. unskilled-type) human capital. He/she can accumulate human capital but, when he/she becomes adult, he/she works as as unskilled person and cannot save; in other words, in the absence of social mobility, he/she remains poor. So, when he/she reaches the old age, he/she lives on social security (namely, on a public pension) and dies without leaving a financial bequest to the next generation.[10] We now present the poor household's constraints and

[7]This specification (i.e. that individual human capital accumulation is an increasing function of both private and public spending on education) reflects the idea that public spending applies more to primary and secondary education, while private spending applies more to college education and on-the-job training.

[8]Following the related literature, individual human capital can be augmented by both private resources (time and expenditures) and public policy; see also e.g. Glomm and Ravikumar (1992), Kaganovich and Zilcha (1999), Blankenau and Simpson (2004), Blankenau (2005) and Arcalean and Schiopu (2010). Regarding the functional form used, see also e.g. Arcalean and Schiopu (2010). Note that our functional form can nest several cases in this literature.

[9]The way we model the bequest motive follows e.g. Acemoglu (2009, chapter 9.6) and Coeurdacier et al. (2015)

[10]As pointed out by De la Croix and Michel (2002, chapter 5), in general, there are three ways of accumulating human capital: First, the individual decision on the length/effort of education. Second, public spending on education. Third, individual spending on education, where the latter can be financed in various ways like parental funding and/or borrowing

utility function.

Budget constraints When a person is born poor at time t and remains so in the rest of his/her life, the budget constraints when young, adult and old at t, $t+1$ and $t+2$ respectively are:

$$\left(1+\tau_t^c\right) c_t^{p,y} = \Psi_t^{p,y} \tag{5a}$$

$$\left(1+\tau_t^c\right) c_{t+1}^{p,m} + \Psi_{t+1}^{p,m} = \left(1-\tau_{t+1}^{n,p}-\tau_{t+1}^{s}\right)\left(g_{t+1}^{w,p,m}\right)^{\chi} w_{t+1}^{p} h_{t+1}^{p,m} l_{t+1}^{p,m} + g_{t+1}^{t,p,m} \tag{5b}$$

$$\left(1+\tau_{t+2}^c\right) c_{t+2}^{p,o} = s_{t+2}^{p,o} \tag{5c}$$

where variables are as defined above in the rich person's problem if we replace the superscript r with the superscript p. As above, equation (4c) holds with probability $0 \leq q_{t+2}^{p,o} \leq 1$ only, while, with probability $0 \leq 1-q_{t+2}^{p,o} \leq 1$, the rich adult dies before reaching the old age (this probability is modelled below).[11] [12]

Motion of human capital The human capital of p household at the beginning of $t+1$ when adult/work life starts is:

$$h_{t+1}^{p,m} = \left(1-\delta^{p,h}\right) h_t^{p,y} + B^p \left(e_t^{p,y}\right)^{\theta} \left[(1-\gamma)\left(g_t^{p,e}+\kappa g_t^{p,h}\right)^{\nu}\right]^{\frac{1-\theta}{\nu}} \tag{6}$$

where variables are as defined above if we replace the superscript r with the superscript p. Notice that the poor people do not afford private tuition fees.

Utility function The discounted lifetime utility of a person who is born poor at time t and remains poor in the rest of his/her life is:

$$\begin{aligned} u_t^p = & \frac{\left(c_t^{p,y}\right)^{1-\sigma}}{1-\sigma} - \chi_n \frac{\left(e_t^{p,y}\right)^{1+\eta}}{1+\eta} + \chi_g \frac{\left(g_t^u\right)^{1-\eta_g}}{1-\eta_g} + \\ & +\beta \left\{ \frac{\left(c_{t+1}^{p,m}\right)^{1-\sigma}}{1-\sigma} - \chi_n \frac{\left(l_{t+1}^{p,m}\right)^{1+\eta}}{1+\eta} + \chi_g \frac{\left(g_{t+1}^u\right)^{1-\eta_g}}{1-\eta_g} \right\} \\ & +\beta^2 q_{t+2}^{p,o} \left\{ \frac{\left(c_{t+2}^{p,o}\right)^{1-\sigma}}{1-\sigma} + \chi_g \frac{\left(g_{t+2}^u\right)^{1-\eta_g}}{1-\eta_g} \right\} \end{aligned} \tag{7}$$

where the only difference from the rich person's utility function is that now there are no financial bequests.

from the market. Here, in the case of poor-born households, we we do not include the third way so as to focus on the role of unequal opportunities. Our main results do not change if we allow for parental funding of tuition fees to the extent that poor parents can spend less than rich parents on their children education. For credit markets and human capital, see e.g. the review in Acemoglu (2009, chapter 21.6) where credit market imperfections prohibit borrowing and lending to finance human capital accumulation.

[11] We do not allow for a separate transfer to the young ($g_t^{t,p,y}$) simply because in practice it is the parents who receive a child benefit.

[12] We assume that credit market imperfections do not allow for borrowing or lending. See also e.g. Galor and Zeira (1993). Acemoglu (2009, chapter 21.6) analyses the credit market in a model with human capital.

3.2.3 Probabilities of changing economic status

We assume that depending on the amount of human capital accumulated when young, a person born in a poor family with unskilled parents and grandparents can become skilled and so climb up the economic ladder if his/her human capital in the beginning of adult life is relatively high, and, vice versa, namely, a person born in a rich family with skilled parents and grandparents can become unskilled if his/her human capital is relatively little in the beginning of adult life.

To formalize these probabilities, we assume that an unskilled worker becomes skilled when his/her effective human capital is higher than an exogenous threshold denoted as $S^p \geq 0$. Following e.g. Angelopoulos et al (2017), we assume that the effective human capital of an unskilled person, p, is the outcome of his/her endogenously determined human capital, $h_t^{p,m}$, and an exogenous idiosyncratic shock, ϕ_t^p, that can account for luck, aspirations, school quality, etc. In other words, at time t, an unskilled person becomes skilled when his/her beginning-of-period effective stock of human capital exceeds this threshold, $\phi_t^p (h_t^{p,m})^{\xi^p} > S^p$, where the parameter $0 \leq \xi^p \leq 1$ is the elasticity of skill acquisition with respect to human capital when one moves up. For simplicity, let us assume that ϕ_t^p follows the uniform diistribution with a pdf $\frac{1}{\phi^h - \phi^l}$, where $\phi^h > \phi^l \geq 0$ are the maximum and minimum levels of this distribution (see e.g. DeGroot (1989, chapter 3)). Then, calculating the cumulative distribution function, the probability of becoming skilled from unskilled - and hence the fraction of unskilled/poor young agents who turn to skilled/rich when they become adults - is:

$$q_t^s = 1 - \frac{\left(\frac{S^p}{(h_t^{p,m})^{\xi^p}} \right) - \phi^l}{\phi^h - \phi^l} \tag{8}$$

Working similarly, the probability of becoming unskilled from skilled - and hence the fraction of skilled/rich young agents who turn to unskilled/poor when they become adult - is:

$$q_t^u = 1 - \frac{\left(\frac{(h_t^{r,m})^{\xi^r}}{S^r} \right) - \phi^l}{\phi^h - \phi^l} \tag{9}$$

where $S^r \geq 0$ and $0 \leq \xi^r \leq 1$ are the new parameters associated with moving down.

Appendix B presents the evolution of population shares under the above possibilities.

3.2.4 Probability of reaching the old age

For simplicity, we assume that the probability of an adult reaching the old age, $q_t^{r,o}$ and $q_t^{p,o}$, depends only on current public spending on health as a fraction of GDP. This is denoted as $q_t^{r,o} = q_t^{p,o} \equiv q_t^o = q\left(\frac{G_t^h}{N_t^f y}\right) \equiv$

$q\left(s_t^{g^h}\right)$, where $q\left(.\right)$ is increasing and concave.[13] For convenience, we will use the functional form:

$$q_t^o \equiv \Xi\left(1 + \frac{s_t^{g^h}}{1 + s_t^{g^h}}\right) \tag{10}$$

where the parameter $0 < \Xi < 1$ will be calibrated so as the probability to be within usual ranges (see also e.g. Chakraborty, 2004, and Dioikitopoulos, 2014).

3.2.5 Formula for progressive taxation

Following e.g. Guo and Lansing (1998) and Chen and Guo (2013), the progressive average labor income tax rate on household type $i = r, p$ takes the form:

$$\tau_t^{n,i} \equiv 1 - \Phi\left(\frac{w_t(g_t^{w,m})^{\chi} h_t^m l_t^m}{w_t^i(g_t^{w,i,m})^{\chi} h_t^{i,m} l_t^{i,m}}\right)^{\varphi} \tag{11}$$

where $0 < \Phi < 1$ and $0 \leq \varphi < 1$ are policy parameters and the numerator

$$w_t(g_t^{w,m})^{\chi} h_t^m l_t^m \equiv \frac{w_t^r(g_t^{w,r,m})^{\chi} h_t^{r,m} l_t^{r,m} + w_t^p(g_t^{w,p,m})^{\chi} h_t^{p,m} l_t^{p,m}}{2}$$

denotes the average labor income (we use the arithmetic mean as a measure of average income but we could also use the weighted average with the weights being the population shares).

3.2.6 Decisions and optimality conditions with social mobility

The newly-born households at t make their decisions (namely, their education effort and, in the case of the born-rich, their tuition fees) without knowing whether they will end up being skilled/rich or unskilled/poor when they become adult and old. When they reach the adult stage at $t+1$, and before any further decisions are made, this uncertainty is resolved so that they now know their status with certainty for the rest of their life. More formally, if one is born in a rich family at t, there is a probability $0 \leq 1 - q_{t+1}^u \leq 1$ of him/her remaining skilled/rich and a probability $0 \leq q_{t+1}^u \leq 1$ of becoming unskilled/poor in adult life, and, similarly, if one is born in a poor family at t, there is a probability $0 \leq 1 - q_{t+1}^s \leq 1$ of him/her remaining unskilled/poor, and a probability $0 \leq q_{t+1}^s \leq 1$ of becoming skilled/rich, in adult life. Once one happens to be skilled or unskilled in adult life, he/she remains so in old life.

Therefore, the rich-born choose $e_t^{r,y}$ and $z_t^{r,y}$, and the poor-born choose $e_t^{p,y}$, under status uncertainty. In turn, those who happen to be skilled/rich when adult, choose $l_{t+1}^{r,m}$, $k_{t+1}^{r,m}$, $d_{t+1}^{r,m}$, $\Psi_{t+1}^{r,m}$ and $b_{t+2}^{r,o}$ under status certainty, while, those who happen to be unskilled/poor when adult, choose $l_{t+1}^{p,m}$ and $\Psi_{t+1}^{p,m}$ under status certainty. At the same time, all agents face uncertainty about reaching the old age.

[13] Since this probability does not depend on private decisions, like private spending on health or private income, there is nothing that the household can do to affect survival (see also e.g. Quadrini and Rios-Rull (2014)). A generalization could be to assume that this probability depends on the household's health capital which is increasing in, say, private spending on health; the latter could allow this probability to be different between rich and poor households.

Households act competitively taking prices, policy instruments and aggregate outcomes as given. Households' optimality conditions are presented in detail in Appendix A.

3.3 Firms

There are $f = 1, 2, ..., N_t^f$ firms. Each firm chooses capital and the two labor inputs, denoted as k_t^f, $l_t^{r,f}$ and $l_t^{p,f}$, to maximize net profits given by:

$$\pi_t^f \equiv y_t^f - (1+\tau_t^w)(w_t^r l_t^{r,f} + w_t^p l_t^{p,f}) - r_t^k k_t^f - \tau_t^f (y_t^f - w_t^r l_t^{r,f} - w_t^p l_t^{p,f}) \tag{12}$$

where y_t^f is the firm's output, τ_t^w is the social security contribution paid by employers and τ_t^f is a tax rate on the firm's gross profit, where the latter is defined as sales minus wage payments (see e.g. Turnovsky (1995, chapter 10)).[14] Notice that our modeling allows for double taxation on capital, since households also pay taxes on their personal capital income; however, capital's double taxation is a usual phenomenon in several countries and, in any case, we report that our results are not sensitive to whether we have double taxation or not.

Following e.g. Stokey (1996), Krusell et al (2000), Acemoglu (2009, chapter 15), He (2012) and Angelopoulos et al (2017), the production function is:

$$y_t^f = A\left(\lambda(A^u l_t^{p,f})^{\alpha} + (1-\lambda)[\mu(A^k k_t^f)^{\psi} + (1-\mu)(A^s l_t^{r,f})^{\psi}]^{\frac{\alpha}{\psi}}\right)^{\frac{1-\varepsilon}{\alpha}} \left(\frac{K_t^g}{N_t^f}\right)^{\varepsilon} \tag{13}$$

where capital, k_t^f, and skilled labor, $l_t^{r,f}$, are combined into a composite CES technology, so that $0 < \mu < 1$ is the importance of capital vis-a-vis skilled labor and $\frac{1}{1-\psi}$ measures the elasticity of substitution between these two factors, $0 < \lambda < 1$ is the importance of unskilled labor, $l_t^{p,f}$, relative to the composite of capital-skilled labor, $\frac{1}{1-a}$ measures the elasticity of substitution between unskilled labor and the composite of capital-skilled labor, where $0 < \psi < \alpha < 1$, the coeffcient $0 \leq \varepsilon < 1$ is a measure of the contribution of public infrastructure in production. Finally, A^k, A^s and A^u are separate productivity terms as in Acemoglu (2009, pp. 501-2); these parameters will allow us to study the effects of capital-augmenting technological change as captured by the relative size of A^k. This production function captures the idea that skilled labor is relatively more complementary to capital than unskilled labor, so that any technological progress that favors capital accumulation is more beneficial to skilled labor. At the same time, production exhibits CRS with respect to all inputs.

Firms' optimality conditions are in Appendix C.

[14] We assume that an unskilled worker becomes skilled when his effective human capital is higher than an exogenous threshold, S^*. Following e.g. Angelopoulos et al (2017), we assume that the effective human capital is the outcome of the endogenoulsly chosen $h_t^{p,m}$ and an exogenous idiosyncratic shock, ϕ_t^i, that follows for simplicity the uniform diistribution with a pdf $\frac{1}{\phi^h - \phi^l}$, where $\phi^h > \phi^l$ are the maximum and minimum levels of the distribution (see e.g. DeGroot (1989, chapter 3)). That is, the unskilled worker becomes skilled when the effective level of his human capital, $\phi_t^i h_t^{p,m}$ is higher than S^*. Then, caluculating the cumulative distribution function, we get the simple formula in (7).

3.4 Government

3.4.1 Policy instruments

On the revenue side, the government uses personal capital income taxes, τ_t^k, personal labor income taxes, $\tau_t^{n,i}$, which are also progressive, consumption taxes, τ_t^c, corporate income taxes, τ_t^f, social security contributions paid by employees and employers, τ_t^s and τ_t^w respectively, as well as taxes on bequests, τ_t^b. There can also be government revenues from the issuance of bonds purchased by the rich households, where D_{t+1} denotes the end-of-period total stock of one-period maturity bonds issued by the government.

On the expenditure side, we have public spending on education, G_t^e, health, G_t^h, work-supplement services, G_t^w, pensions, G_t^s, transfer payments, G_t^t, infrastructure, G_t^i, and utility-enhancing activities, G_t^c (see below in the empirical part of the paper for the categories included in G_t^c). Equivalently, if these public spending items are expressed as shares of GDP, we have $s_t^{g^e}$, $s_t^{g^h}$, $s_t^{g^w}$, $s_t^{g^s}$, $s_t^{g^t}$, $s_t^{g^i}$ and $s_t^{g^c}$ Recall that each one of these spending items plays a distinct and natural role in our model.

All the above fiscal policy instruments can affect different types of agents differently even if they are common. In our model, this happens because the same policy or the same shock affect differently the rich and the poor. But we will also allow for targeted or redistributive policies that target one group at the expense of the other ex ante. In particular, we will allow all items of public spending that do not have a clear public good feature (like G_t^i and G_t^c) to be allocated in favor of the poor. To do so, we assume that each of G_t^j, where $j = e, h, w, s, t$, can be divided into two parts, $G_t^j \equiv \zeta_t^j G_t^j$ and $G_t^j \equiv (1-\zeta_t^j)G_t^j$, where the former is earmarked for the rich and the latter is earmarked for the poor, so that the fraction ζ_t^j is a measure of targeted or redistributive policy. If $\zeta_t^j = n_t^r$ so that $(1-\zeta_t^j) = n_t^p$, the use of G_t^j is neutral ex ante in the sense that it is allocated according to population shares or, in simple words, all agents receive the same amount of G_t^j. If, on othe hand, $\zeta_t^j < n_t^r$ so that $(1-\zeta_t^j) > n_t^p$, it is the poor that get the lion's share of G_t^j. This modelling will enable us to study the implications of targeted policies and examine whether some popular policies that look redistributive at first sight are actually so once general equilibrium effects have been taken into account. Thus, we add ζ_t^e, ζ_t^h, ζ_t^w, ζ_t^s, and ζ_t^t to the menu of public policy instruments.

3.4.2 Government budget identities

The above instruments are linked to each other via the government budget constraint. When the PAYG system is fully funded, which means that this system is self-financed via adjustments in one of the two social security contributions (τ_t^s or τ_t^w), we have two separate budget constraints, one for the general budget and one for the social security system:

$$[s_t^{g^h} + s_t^{g^e} + s_t^{g^t} + s_t^{g^w} + s_t^{g^i} + s_t^{g^c}]n_t^f y_t^f + \left(1 + r_t^d\right)\frac{D_t}{N_t} = \frac{N_{t+1}}{N_t}\frac{D_{t+1}}{N_{t+1}} + \frac{T_t}{N_t} \tag{14a}$$

$$s_t^{g^s} n_t^f y_t^f = \tau_t^s[n_t^{r,m}(g_t^{w,r,m})^{\chi} w_t^r l_t^{r,m} h_t^{r,m} + n_t^{p,m}(g_t^{w,p,m})^{\chi} w_t^p l_t^{p,m} h_t^{p,m}] + \tau_t^w n_t^f (w_t^r l_t^{r,f} + w_t^p l_t^{p,f}) \tag{14b}$$

If however this is not the case, meaning that there can be a difference between public spending on pensions and social security contributions to it (simply because, although both τ_t^s and τ_t^w are positive as in the data, neither of them is flexible enough to adjust to close the budget constraint of the public pension system in each period), and that any such difference is financed out of the general government budget, then we merge the two separate budget constraints of the government and the public pension system into a single consolidated government budget constraint:

$$[s_t^{g^h} + s_t^{g^e} + s_t^{g^t} + s_t^{g^w} + s_t^{g^i} + s_t^{g^c}]n_t^f y_t^f + \left(1 + r_t^d\right)\frac{D_t}{N_t} + \tag{15}$$

$$+s_t^{g^s} n_t^f y_t^f - \tau_t^s [n_t^{r,m}(g_t^{w,r,m})^{\chi} w_t^r l_t^{r,m} h_t^{r,m} + n_t^{p,m}(g_t^{w,p,m})^{\chi} w_t^p l_t^{p,m} h_t^{p,m}] - \tau_t^w n_t^f (w_t^r l_t^{r,f} + w_t^p l_t^{p,f}) =$$

$$= \frac{N_{t+1}}{N_t}\frac{D_{t+1}}{N_{t+1}} + \frac{T_t}{N_t}$$

Public investment spending, G_t^i, is used to augment public capital. Thus, in total terms,

$$K_{t+1}^g = (1 - \delta^g)\, K_t^g + G_t^i \tag{16}$$

where $0 \leq \delta^g \leq 1$ is the depreciation rate of public infrastructure.

Tax revenues (except social security contributions) are defined as:

$$\frac{T_t}{N_t} \equiv \tau_t^c (n_t^{r,y} c_t^{r,y} + n_t^{r,m} c_t^{r,m} + n_t^{r,o} c_t^{r,o} + n_t^{p,y} c_t^{p,y} + n_t^{p,m} c_t^{p,m} + n_t^{p,o} c_t^{p,o}) +$$

$$+\tau_t^{n,r} n_t^{r,m} (g_t^{w,r,m})^{\chi} w_t^r l_t^{r,m} h_t^{r,m} + \tau_t^{n,p} n_t^{p,m} (g_t^{w,p,m})^{\chi} w_t^p l_t^{p,m} h_t^{p,m} + \tau_t^k \frac{N_{t-1}}{N_t} n_{t-1}^{r,m} (r_t k_{t-1}^{r,m} + \pi_t^{r,o}) +$$

$$+\tau_t^b n_t^{r,y} b_{t-1}^{r,y} + \tau_t^f n_t^f (y_t^f - w_t^r l_t^{r,f} - w_t^p l_t^{p,f}) \tag{17}$$

3.5 Macroeconomic equilibrium

Collecting all the above equations, the equilibrium system is presented in Appendix D. This is given the exogenously set policy instruments and initial conditions for the state variables. The latter include the 6 population shares and the 6 human capital stocks of the three age groups for each of the two social groups, the stocks of private and public physical capital, the stock of public debt and the stock of bequests inherited by the newly-born rich. Note that this applies to the special case in which the public pension system is self-financed via adjustments in $\{\tau_t^s\}_{t=0}^{\infty}$. If this is not the case (meaning that there can be a difference between public spending on pensions and social security contributions to it and that this difference is financed out of the general budget), we use the consolidated government budget constraint only (see equation 15), instead of two separate ones (see equations 14a and 14b), and at the same time move ϕ_t to the list of the exogenously set policy variables.

3.6 Solution methodology

In what follows, we work as follows. We will first present parameter values and data averages for fiscal variables from the Eurozone over 2001-2019. Using them, we will solve the model to quantify the aggregate and distributional implications of various hypothetical permanent reforms in the fiscal policy mix. Specifically, in our simulations, we will assume that the economy starts from an initial steady state in which fiscal variables are set as in the data averages and then study the implications of various reforms over time. For the solutions, we use a Newton-type non-linear method implemented in DYNARE. Note that, since the model is kept deterministic, transition dynamics will be driven by policy reforms only.

4 Parameters and data used in the solutions

Subsections 4.1 and 4.2 present respectively parameter values and average eurozone data for fiscal policy variables over 2001-2019. Subsection 4.3 will in turn present the resulting initial steady state solution.

4.1 Parameter values

We use conventional values and choose the rest so as the model's steady state solution is consistent with averages of annual data for the EZ over 2001-2019. Baseline parameter values are listed in Table 2a. Before we discuss these values, we report that our main results are robust to changes in these values at least within reasonable ranges. Thus, although our numerical simulations reported below are not meant to provide a rigorous quantitative study, they illustrate the qualitative dynamic features of the model in a robust way.

Utility function The time unit is meant to be a period consisting of 25 years. The time preference rate, β, is set at 0.985^{25} so as to be consistent with a value for the real annual interest rate around 1.5%. The weight given to utility-enhancing public goods/services, χ_g, and the associated exponent, η_g, are set at 0.1 and 1 respectively as in the similar specification of e.g. Baier and Glomm (2001). The elasticity of intertemporal substitution, σ, is set at 1. The preference parameter related to effort time, χ_n, and the inverse of the Frisch elasticity, η, are set at 10 and 1 respectively which give work hours within data ranges. Regarding bequests, the weight given to them, χ_b, and the associated exponent, η_b, are set at 3.5 and 1 respectively so as to get bequests as share of output around 20% (see e.g. Alvaredo et al (2017)).

Technology Regarding the production function, the scale parameter, A, is set at 1. The substitutability parameters, ψ and α, are set at -0.49 and 0.45 based on Krusell et al (2000) implying elasticities of substitution between capital and skilled labor and between the composite of capital-skilled labor and unskilled labor about 0.67 and 1.67 respectively. The parameters μ (namely, the importance of capital vis-a-vis skilled labor) and λ (namely, the importance of unskilled labour vis-a-vis the composite of capital-skilled labor) are set at 0.28 and 0.3 respectively, so that the skill premium and the share of skilled workers are close to data averages. The coefficient on public capital, ε is set at 0.02 which is close to the public investment to GDP ratio in the data as is usual practice (see e.g. Baxter and King (1993) and Ramey (2020)). The separate productivity coefficients A^k, A^s and A^u are all set at 1. Finally, as is usual in the OLG literature (see e.g. Heer (2019)), we set the values of the depreciation rates of physical and public capital at 0.75.

Human capital In the human capital production function, the values of B^r, B^p and θ are set at 10, 8 and 0.75 respectively; these values imply hours of education within usual ranges, in particular, $e^{r,y}$=0.3 and $e^{p,y}$=0.25 for the skilled and unskilled respectively Following e.g. Stokey (1996), the elasticity parameter, ν, is set at 0.5 (this implies an elasticity of $1/(1-\nu)=2$) so that private and public education spending are good substitutes. The importance given to private vis-a-vis public spending in the same function, γ, is set at 0.25; this implies that private education spending as share of GDP is around 0.5% which is as in the European data. As regards the parameter κ measuring the contribution of public spending on health relative to public spending on education in the creation of new human capital, we set it at the relatively neutral value of 0.5. Finally, as is usual in the OLG literature we set the value of the depreciation rates of human capital, $\delta^{r,h}$ and $\delta^{p,h}$, at 0.75 which is the same as that of physical capital (see e.g. Heer (2019)).

Social mobility probability We normalize the range of the distribution of the idiosyncratic abilities, ϕ^l and ϕ^h, to be 0 and 1. The exogenous thresholds, S^r and S^p, are set at 1 and 0.78 respectively, which, along with the skill acquisition elasticities, ξ^p and ξ^r, imply that the probability of changing economic status is around 10% in the case of upward mobility, q_t^s, and around 1% in the case of downward mobility, q_t^u; although direct data on these probabilities do not exist, our values capture that mobility in the bottom quantiles of the income/wealth distribution is higher than mobility in the upper/top quantiles as is the case in the data (see e.g. OECD (2018, chapter 1)). Finally, we set the population growth rate at $n=0.005$.

Table 2a: Baseline parameter values

4.2 Fiscal policy data used in the solutions

We use fiscal data from the Eurozone. Most of the data are from Eurostat.

Public spending Regarding public spending, we make use of the disaggregation of the international Classification of the Functions of Government (COFOG) in the framework of the European System of National Accounts (ESA, 2010), which is comprised by the functional categories listed in Table 1a. To solve our model, we need data on education, health, old age pensions and survivors (the latter is the main subcategory of social protection expenditure in Table 1a), infrastructure (this is part of spending on economic affairs in Table 1a) and other social expenditures such work-complements and transfers. The averages values of these spending items as shares of GDP ($s_t^{g^e}$, $s_t^{g^h}$, $s_t^{g^c}$, $s_t^{g^s}$, $s_t^{g^i}$, $s_t^{g^w}$, $s_t^{g^{tr}}$) over 2001-2019 are listed in Table 2b. The rest of spending items are included in s^{g^c}, as defined above.

Tax rates The tax rates on consumption, personal capital income and profits (τ^c, τ^k and τ^f) are the average values of the effective tax rates in the Eurozone (the source is European Commission (2014b, 2018b) or they have been constructed by us applying the same (Mendoza-Razin-Tesar) methodology by using Eurostat data). Recall that the value of the progressive labor income tax rate, $\tau_t^{n,i}$, where $i = r, p$, is endogenously determined. Also recall that the value of the labor tax rate does not include the social security contributions, τ_t^s and τ_t^w, paid by employees and employers respectively. Setting s^{g^s} and τ_t^w at their data average values, the value of τ_t^s that follows residually from the PAYG budget constraint is close to its average value in the data and the same applies in turn to the total tax burden on labor income, $\tau^{n,i} + \tau_t^s + \tau_t^w$.[15] The value of the tax rate on bequests is set at 0.05 (our results are not sensitive to this).[16] The values of these tax instruments over 2001-2019 are also listed in Table 2b.

Progressive taxation In the formula for progressive taxation, we set Φ at 0.87 and φ at 0.1 respectively; this implies that the tax rates on the labour income of the rich, $\tau^{n,r}$, and the labor income of the poor, $\tau^{n,p}$, are 15% and 10% respectively (for φ=0 we get an equal flat tax rate of 13%. Recall however that this does include social security contributions by employers and employees - see next subsection). In the formula for work supplement public services, the power coefficient χ is set at 0.1.

Allocation of public spending Finally, we need to decide how those items of public spending that have to do with social policy (education, health, work-supplement, pension and transfers) are allocated between the two economic groups, skilled and unskilled or equivalently the rich and the poor. Since there are no data on this, in our baseline solutions, we will assume that these spending items are allocated equally, namely, according to the population fractions of the two groups so that we set $\zeta_t^e = \zeta_t^h = \zeta_t^w = \zeta_t^s = \zeta_t^t = n_t^p$. By contrast, when we study targeted redistributive policies, we will assume that the unskilled/poor get the lion's share, namely, they get more than their population fraction as explained above, so that we set $\zeta_t^e = \zeta_t^h = \zeta_t^w = \zeta_t^s = \zeta_t^t \to 0$.

Table 2b: Fiscal policy variables used in the solutions

4.3 Initial steady state solution (status quo)

Using the parameter values in Table 2a and the fiscal variables in Table 2b, we now solve the model numerically. Its steady state solution (or the status quo) is presented in Table 3. As can be seen, the solution makes sense and the GDP ratios of the key macro variables can mimic reasonably well their values in the data.

Table 3: Status quo solution using 2001-19 data averages

We will now depart from this solution to simulate the aggregate and distributional effects of various policy reforms.

[15] For example, in the model above, the value of 0.13 for τ^n does not include the social security contributions, τ_t^w and τ_t^s, paid by workers and employers respectively. Setting s^{g^s} and τ_t^{ws} at their data average values (0.115 and 0.12 respectively over 2001-18), the value of τ_t^w that follows residually from the PAYG budget constraint is 0.09, which is close to its average value in the data over 2001-18. Then, the implied total tax burden on labor income, $\tau^n + \tau_t^s + \tau_t^s$, is 0.35 which is also close to its data average value.

[16] Inheritance taxes have fallen in most countries and nowadays constitute a very small share of total government revenues, less that 1%. See the anecdotal evidence in *The Economist*, November 25th, 2017, pp. 21-23.

5 Policy reforms

We will first define the reforms studied (subsection 5.1) and then embed them into the model and quantify their implications over time (subsection 5.2).

5.1 Policy reforms defined

Our aim is to study the implications of a rich menu of hypothetical policy reforms. We assume that the economy is at its status quo at the time of one-off permanent changes in the mix of public spending and taxes and then study the time-paths of the endogenous variables of the model as the economy travels towards its the new steady state. To solve for the dynamics of the model, we work as described above.

We will focus on policy changes that have been at the heart of the policy debate on redistribution. We will thus start with one-off permanent 1% increases in public spending on education, health, work complements, pensions and transfers, all expressed as shares of GDP. In this first type of experiments, spending increases are assumed to be population-wide in the sense that both types of agents (poor and rich) take equal advantage of policy changes, although of course the same policy can affect different agents differently because of agent heterogeneity (for instance, in initial conditions). In the second type of experiments, we will keep total public spending as well as its various items as in the status quo solution but we will assume that they target the poor in the sense that now the poor get a share of public spending that exceeds their fraction in population. Finally, we will study the effects of higher taxes on capital income and corporate profits as well as a higher degree of progressivity of labor income taxes, all of which are common policies in redistributive policy agendas.

Before we proceed, it is worth clarifying the following. First, in all experiments, the reference will be the associated status quo solution, namely, what would have happened if the policy variables had remained for ever as in the data at the time of departure. Second, we will assume that any exogenous change in policy instruments is permanent so we will end up at a new reformed steady state. This is as in the literature on policy reforms mentioned in the Introductory section above. Third, in all cases studied, any exogenous changes in policy will be accommodated by endogenous changes in the end-of-period public debt, d_t^m. Fourth, to understand the logic of our results, and following usual practice in the literature, we will study one policy reform at a time.

5.2 A first pass of the model

Before we provide a quantitative assessment of social policies, in order to check our model, we examine the implications of a capital-augmenting technological change and an increase in spending on public infrastructure. The former is considered to be the main form of directed technological change in the last 60 years and one of the key drivers of the skill pemium and thereby of income inequality (see e.g. the reviews in Acemoglu (2009, chapter 15), Aghion and Howitt (2009, chapter 8), Autor (2014), Bourguignon (2015, 2018)). The latter is considered to be a key driver of economic growth and this belief is reflected in both the EU's Recovery Fund and President Biden's fiscal stimulus. It is thus interesting to see what our model implies.

A capital-augmenting technological change can be captured by an increase in A^k. Impulse response functions are shown on Figure 1 which, like all other figures below, plots the transition paths of the key endogenous variables of the model in percent deviations from the status quo solution. As can be seen, a capital-augmenting technological change is skill-biased. This means that, although an increase in A^k is associated with higher capital and labor accumulation, an increase in GDP and higher social mobility, the net income of the skilled rises by more so that net income of the unskilled so that net income inequality worsens. This is as in the related literature. We thus feel confident that the model is consistent to the literature and can now move on to new policy experiments.

Figure 1: The effects of capital-augmenting technology

Figure 2 shows the implications of a 1 percentage permanent increase in public spending on infrastructure. The results are qualitatively similar to those from an increase in A^k. Namely, an increase in $s_t^{g^i}$ enhances total productivity so that factor accumulation and GDP increase, which is beneficial to both the skilled and the unskilled and is also good for social mobility, but, as the path of the ratio of the net income of the skilled to the net income of the unskilled reveals, it is again the skilled that benefit more. Notice however that quantitatively the rise in net income inequality is much smaller relative to Figure 1 so that an increase in public infrastructure spending seems to be a good policy.

Figure 2: The effects of public infrastructure spending

5.3 Social policies

Figures 3, 4, 5, 6 and 7 plot the IRFs in the case of a 1 percentage point increase in public spending on education, health, work-supplement services, pensions and transfers respectively ($s_t^{g^e}$, $s_t^{g^h}$, $s_t^{g^w}$, $s_t^{g^s}$, $s_t^{g^{tr}}$). As said above, we start by assuming that these increases are distributed equally between the poor and the rich meaning that each of these public spending items is allocated to the two groups according to their fractions in population ($\zeta_t^e = \zeta_t^h = \zeta_t^w = \zeta_t^s = \zeta_t^t = n_t^p$). In turn, Figures 8, 9, 10, 11 and 12 study what happens when there is a reallocation of the existing resources so that ($s_t^{g^e}$, $s_t^{g^h}$, $s_t^{g^w}$, $s_t^{g^s}$, $s_t^{g^{tr}}$) go to the unskilled only. Specifiaclly, now $\zeta_t^e = \zeta_t^h = \zeta_t^w = \zeta_t^s = \zeta_t^t \to 0$). Finally, Figures 13, 14 and 15 show respectively the effects of three traditional redistributive tax policies. Specifically, Figures 13 and 14 illustrate respectively the cases of an increase in the tax rate on capital income and corporate income, while Figure 15 plots the case in which there is an increase in the degree of progressivity of labor income taxes.

Figures 3-15: The effects of redistributive fiscal policies

The main messages are as follows. There are non-targeted public spending policies that are Pareto-efficient (and hence growth-enhancing) and, in addition, enhance social mobility and reduce income inequality. This is the case with an increase in public spending on education, health and work-complement services when provided to all members of the society. These cases are illustrated in Figures 3, 4 and 5. In other words, the public perception that growth-enhancing policies are necessarily regressive is not always true. Notice that this is particulary so in the case of public education spending whose beneficial effects are quantitatively stronger both in terms of the aggregate economy and income equality.

On the other hand, there are non-targeted public spending policies that hurt everybody (and hence the aggregate economy) and, at the same time, worsen social mobility and (at least in the medium run) increase income inequality. This is the case with an increase in public pensions and transfers to working adults when provided to everybody. These cases are shown in Figures 6 and 7. Intuitively, It is disincentives to work and save that shape these adverse general equilibrium effects.

There are targeted redistributive public spending policies that reduce income inequality, also making the poor better off and the rich worse off in absolute terms, without hurting the aggregate economy. This applies to a redistribution of the existing public spending on health in favor of the poor only (Figure 9) and a redistribution of the existing public spending on pensions in favor of the poor only (Figure 11). There are also targeted redistributive policies that reduce income inequality in relative terms also benefiting the unskilled in absolute terms but they hurt the aggregate economy. This applies to a redistribution of the existing public spending on education in favor of the poor only (Figure 8) and to a redistribution of the existing public spending on work-supplement services in favor of the poor only (Figure 10). Finally, there are targeted redistributive policies that reduce income inequality in relative terms but make everybody worse off in absolute trems, even those targeted, and of course also hurt the aggregae economy. This applies to transfers to unskilled working adults (Figure 12).

Finally consider the tax reforms in Figures 13, 14 and 15. Increases in the capital tax rate (Figure 13), the corporate tax rate (Figure 14) and the degree of labor income tax progressivity (Figure 15) all hurt the aggregate economy. Inequality may fall (at least temporarily) in these cases but this is at the cost of lower net incomes in absolute terms for everybody even the unskilled. Notice the strong fall in the net income of the unskilled in Figure 15 where tax progressivity has increased. Intuitively, there are strong complementarities in general equilbrium so that the skilled people should not be dis-incentivised to work. This can be thought of as a reminder of Mirrlees optimal taxation formula.

6 Conclusions

In this paper, we evaluated the aggregate and distributional implications of a wide range of fiscal policies typically used to combat inequality. The main drivers of inequality were unequal initial conditions/opportunities which, in turn, were reflected into differences in human capital and hence in economic status. Since the results have already been listed in the Introduction, we close with caveats and extensions. It would be interesting, and perhaps useful, to calibrate and solve the model to a particualr country. It would also be interesting to study the same issues in an open economy context and in particular in a setup where two countries/regions participate in an economic or currency union. We leave these extensions for future work.

References

[1] Abington C. and W. Blankenau (2013): Government education expenditures in early and late childhood, Journal of Economic Dynamics and Control, 37, 854-874.

[2] Acemoglu D. (1998): Why do new technologies complement skills? Directed technical change and wage inequality, Quarterly Journal of Economics, 113, 1055-'1090.

[3] Acemoglu D. (2003): Cross-country inequality trends, Economic Journal, 113, F121-F149.

[4] Acemoglu D. (2009): Introduction to Modern Economic Growth, Princeton University Press.

[5] Acemoglu D. and D. Autor (2011): Skills, tasks and technology: Implications for employment and earnings, in Handbook of Labor Economics, vol. 4B, edited by O. Ashenfelter and D. Card, North-Holland.

[6] Acosta-Ormaechea S. and A. Morozumi (2013): Can a government enhance long-run growth by changing the composition of public expenditure?, IMF Working Paper, 13/162.

[7] Aghion P. and P. Howitt (1998): Endogenous Growth Theory, MIT Press.

[8] Aghion P. and P. Howitt (2009): The Economics of Growth, MIT Press.

[9] Alesina A., C. Favero and F. Giavazzi (2019): Austerity, Princeton University Press.

[10] Altig D., A. Auerbach, L. Kotlikoff, K. Smetters and J. Walliser (2001): Simulating fundamental tax reforms in the US, American Economic Review, 91, 574-594.

[11] Alvaredo, F., Garbinti, B., and Piketty, T. (2017): On the share of inheritance in aggregate wealth: Europe and the USA, 1900–2010. Economica, 84(334), 239-260.

[12] Angelopoulos K., J. Malley and A. Philippopoulos (2012): Tax structure, growth and welfare in the UK, Oxford Economic Papers, 64, 237-258.

[13] Angelopoulos K., J. Malley and A. Philippopoulos (2017): Human capital accumulation and transition to skilled employment, Journal of Human Capital, 11, 72-105.

[14] Arcalean C. and I. Schiopu (2010): Public versus private investment and growth in a hierarchical education system, Journal of Economic Dynamics and Control, 34, 604-622.

[15] Auerbach A. and R. Lee (2011); Welfare and generational equity in sustainable unfunded pension systems, Journal of Public Economics, 95, 16-27.

[16] Autor D., L. Katz and A. Krueger (1998): Computing inequality: Have computers changed the labor market? Quarterly Journal of Economics, 113, 1169-1214.

[17] Autor D. (2014): Skills, education and the rise of earnings inequality among the "other 99 percent", Science, 344, 843-851.

[18] Barro R. and X. Sala-i-Martin (2004): Economic Growth, second edition, MIT Press.

[19] Bassetto M. (2008): Political economy of taxation in an overlapping-generations economy, Review of Economic Dynamics, 11, 18-43.

[20] Benabou R. (1996): Heterogeneity, stratification and growth: Macroeconomic implications of community structure and school finance, American Economic Review, 86, 584-609.

[21] Bhattacharya J. and X. Qiao (2007): Public and private expenditure on health in a growth model, Journal of Economic Dynamics and Conrol, 31, 2519-2535.

[22] Blankenau W.F. and N. B. Simpson (2004): Public education expenditures and growth, Journal of Development Economics, 73, 583-605.

[23] Blankenau W.F. (2005): Public schooling, college subsidies and growth, Journal of Economic Dynamics and Control, 29, 487-507.

[24] Bleaney M.F., N. Gemmell and R. Kneller (1999): Fiscal policy and growth: evidence from OECD countries, Journal of Public Economics, 74, 171-190.

[25] Bleaney M.F., N. Gemmell and R. Kneller (2001): Testing the endogenous growth model: public expenditure, taxation, and growth over the long run, Canadian Journal of Economics, 34, 36-57.

[26] Bouzahzah M., D. De la Croix and F. Docquier (2002): Policy reforms and growth in computable OLG economies, Journal of Economic Dynamics and Control, 26, 2093-2113.

[27] Bruce N. and S. Turnovsky (2013): Social security, growth and welfare in overlapping generations economies with or without annuities, Journal of Public Economics, 101, 12-24.

[28] Bourguignon F. (2015): The Globalization of Inequality, Princeton University Press.

[29] Bourguignon F. (2018): World changes in inequality: An overview of facts, causes, consequences and policies, CESifo Economic Studies, 64, 345-370.

[30] Chakraborty S. (2004): Endogenous lifetime and economic growth, Journal of Economic Theory, 116, 119-137.

[31] Chen S.-H.and J.-T. Guo (2013): Progressive taxation and macroeconomic (in)stability with productive government spending, Journal of Economic Dynamics and Control, 37, 951-963.

[32] Coenen G., M. Mohr and R. Straub (2008): Fiscal consolidation in the euro area: Long-run benefits and short-run costs, Economic Modelling, 25, 912-932.

[33] Coenen G., C.J. Erceg, C. Freedman, D. Furceri, M. Kumhof, R. Lalonde, D. Laxton, J. Lind, A. Mourougane, D. Muir, S. Mursula, C. de Resende, J. Roberts, W. Roeger, S. Snudden, M. Trabandt and Jan in't Veld (2012): Effects of fiscal stimulus in structural models, American Economic Journal: Macroeconomics, 4, 22-68.

[34] Coenen G., R. Straub and M. Trabandt (2013): Gauging the effects of fiscal stimulus packages in the euro area, Journal of Economic Dynamics and Control, 37, 367-386.

[35] Coeurdacier N., S. Guibaud and K. Jin (2015): Credit constraints and growth in a global economy, American Economic Review, 105, 2838-2881.

[36] Conesa J.C. and C. Garriga (2008): Optimal fiscal policy in the design of social security reforms, Internationall Economic Review, 49, 291-318.

[37] Cooley T. (1992): Tax distortions in a neoclassical monetary economy, Journal of Economic Theory, 58, 290-316.

[38] Cooley T. and J. Soares (1999): A positive theory of social security based on reputation, Journal of Political Economy, 107, 135-160.

[39] Cunha F. and J. Heckman (2007): The technology of skill formation, American Economic Review, 97, 31-47.

[40] de la Croix D. and P. Michel (2002): A Theory of Economic Growth: Dynamics and Policy in Overlapping Generations, Cambridge University Press.

[41] Del Rey E. and M.A. Lopez-Garcia (2013): Optimal education and pensions in an endogenous growth model, Journal of Economic Theory, 148, 1737-1750.

[42] Dettmer A., J. Heckman, J. Pantano, V. Ronda and S. Suomi (2020): Inter-generational effects of early-life advantages: Lessons from a primate study, University of Chicago, Working Paper, no 2020-064.

[43] Diamond P. (1965): National debt in a neoclassical growth model, American Economic Review, 55, 1126-1150.

[44] Dioikitopoulos E. (2014): Ageing, growth and the allocation of public expenditures on health and education, Canadian Journal of Economics, forthcoming.

[45] Docquier F. and O. Paddison (2003): Social security benefit rules, growth and inequality, Journal of Macroeconomics, 25, 47-71.

[46] European Commission (2019): Report on Public Finances in EMU - 2018.

[47] European Commission (2018a): Taxation Trends in the EU.

[48] European Commission (2018b): The 2018 Ageing Report, Economic and Budgetary Projections for the EU Member States, 2016-2070.

[49] Francesconi M. and J. Heckman (2016): Symposium on child development: The parental investment, IZA, Discussion Paper, no. 9977.

[50] Galor O. and O. Moav (2000): Ability-biased technological transition, wage inequality and economic growth, Quarterly Journal of Economics, 115, 469-497.

[51] Galor O. (2006): Das human-kapital: A theory of the demise of the class structure, Review of Economic Studies, 73, 85-117.

[52] Galor O. and J. Zeira (1993): Income distribution and macroeconomics, Review of Economic Studies, 60, 35-52.

[53] Garcia-Mila T., A. Marcet and E. Ventura (2010): Supply side interventions and redistribution. Economic Journal, 120, 105-130.

[54] Gemmell N., R. Kneller and I. Sanz (2012): The timing and persistence of fiscal policy impacts on growth: Evidence from OECD countries, Economic Journal, 121, F33-F58.

[55] Gemmell N., R. Kneller and I. Sanz (2014): Does the composition of government expenditure matter for long-run GDP levels?, Working Papers in Public Finance, Victoria Business School, no. 10/2014, August.

[56] Glomm G. and B. Ravikumar (1992): Public versus private investment in human capital: Endogeous growth and income inequality, Journal of Political Economy, 100, 818-834.

[57] Glomm G., J. Jung and C. Tran (2009): Macroeconomic implications of early retirement in the public sector: the case of Brasil, Journal of Economic Dynamics and Control, 33, 777-797.

[58] Goldin C. and L. Katz (2008): The Race Between Education and Technology, Harvard University Press.

[59] Guo J.-T. and K. Lansing (1998): Indeterminacy and stabilization policy, Journal of Economic Theory, 82, 481-490.

[60] He H. (2012): What drives the skill premium: Technological change or demographic variation?, European Economic Review, 56, 1546-1572.

[61] He H. and Z. Liu (2008): Investment-specific technological change, skill accumulation and wage inequality, Review of Economic Dynamics, 11, 314-344.

[62] Heathcote J., K. Storesletten and G. Violante (2009): Quantitative macroeconomics with heterogeneous households, Federal Reserve Bank of Minneapolis, Research Department Staff Report, no. 420.

[63] Heckman J. and S. Mosso (2014): The economics of human development and social mobility, IZA, Discussion Paper, no. 8000.

[64] Heer, B. (2001): Wealth Distribution and Optimal Inheritance Taxation in Life-cycle Economies with Intergenerational Transfers. Scandinavian Journal of Economics, 103(3), 445-465.

[65] Holmlund H., M. Lindahl and E. Plug (2011): The causal effect of parents' schooling: A comparison of estimation methods, Journal of Economic Literature, 49, 615-651.

[66] Hornstein A., P. Krusell, and G. Violante (2005): The effects of technical change on labor market inequalities, in Handbook of Economic Growth, volume 1, edited by P. Aghion and S. Durlauf, North-Holland.

[67] Huggett M., G. Ventura and A. Yaron (2011): Sources of lifetime inequality, American Economic Reviews, 101, 2923-54.

[68] Kaganovich M. and I. Zilcha (1999): Education, social security and growth, Journal of Public Economics, 71, 289-309.

[69] Kaganovich M. and I. Zilcha (2012): Pay-as-you-go or funded social security? A general equilibrium comparison, Journal of Economic Dynamics and Control, 36, 455-467.

[70] Krueger D. and F. Kubler (2006): Pareto-improving social security reform when financial markets are inomplete!?, American Economic Review, 96, 737-755.

[71] Krueger D. and F. Perri (2011): Public versus private risk sharing, Journal of Economic Theory, 146, 920-956.

[72] Krusell P., L. Ohanian, J. Rios-Rull and G. Violante (2000): Capital-skill complementarity and inequality: A macroeconomic analysis, Econometrica, 68, 1029-1053.

[73] Krusell P., T. Mukoyama, R. Rogerson and A. Sahin (2011): A three-state model of worker flows in general equilibrium, Journal of Economic Theory, 146, 1107-1133.

[74] Lansing K. (1998): Optimal fiscal policy in a business cycle model with public capital, Canadian Journal of Economics, 31, 337-364.

[75] Lucas R. (1990): Supply-side economics, Oxford Economic Papers, 42, 293-316.

[76] Mendoza E. and L. Tesar (1998): The international ramifications of tax reforms: Supply-side economics in a global economy, American Economic Review, 88, 226-245.

[77] Masuch K., R. Anderton, R. Setzer and N. Benalal (2018): Structural policies in the euro area, Occasional Paper Series, no. 210, European Central Bank.

[78] OECD (2017): Revenue Statistics 2017: Tax Revenue Trends in the OECD.

[79] Philippopoulos A., V. Vassilatos and P. Varthalitis (2017a): Fiscal consolidation in an open economy with sovereign premia and without monetary policy independence, International Journal of Central Banking, 13, 259-306.

[80] Philippopoulos A., V. Vassilatos and P. Varthalitis (2017b): Fiscal consolidation and its cross-country effects, Journal of Economic Dynamics and Control, 83, 55-106.

[81] Quadrini V. and J.V. Rios-Rull (2014): Inequality in macroeconomics, in Handbook of Income Distribution, vol. 2B, edited by A.B. Atkinson and F.J. Bourguignon, North-Holland.

[82] Shelton C. (2007): The size and composition of government expenditure, Journal of Public Economics, 91, 2230-2260.

[83] Soares J. (2006): A dynamic general equilibrium analysis of the political economy of public education, Journal of Population Economics, 19, 367-389.

[84] Stokey N. (1996): Free trade, factor returns and factor accumulation, Journal of Economic Growth, 1, 421-447.

[85] Turnovsky S. (1995): Methods of Macroeconomic Dynamics, MIT Press.

[86] Viaene J.M. and I. Zilcha (2013): Public funding of higher education, Journal of Public Economics, 108, 78-89.

[87] Yew S.L. and J. Zhang (2009): Optimal social security in a dynastic model with human capital externalities, fertility and endogenous growth, Journal of Public Economics, 93, 605-619.

Appendix

This appendix includes algebraic details.

Appendix A: Households' decisions

Optimality conditions of rich-born under social mobility Given the assumed timing, a rich-born young person chooses $e_t^{r,y}$ and $z_t^{r,y}$ subject to:

$$\left(1+\tau_t^c\right)c_t^{r,y}+z_t^{r,y}=\left(1-\tau_t^b\right)b_{t-1}^{r,y}+\Psi_t^{r,y} \tag{A1}$$

$$q_{t+1}^u[\left(1+\tau_{t+1}^c\right)c_{t+1}^{p,m}+\frac{n_{t+1}^{p,y}}{n_{t+1}^{p,m}}\Psi_{t+1}^{p,y}]+(1-q_{t+1}^u)[\left(1+\tau_{t+1}^c\right)c_{t+1}^{r,m}+k_{t+1}^{r,m}+d_{t+1}^{r,m}+\frac{n_{t+1}^{r,y}}{n_{t+1}^{r,m}}\Psi_{t+1}^{r,y}]=$$
$$=q_{t+1}^u[\left(1-\tau_{t+1}^{n,p}-\tau_{t+1}^s\right)w_{t+1}^p h_{t+1}^{p,m}l_{t+1}^{p,m}\left(g_{t+1}^{w,p,m}\right)^\chi+g_{t+1}^{t,p,m}]+(1-q_{t+1}^u)[\left(1-\tau_{t+1}^{n,r}-\tau_{t+1}^s\right)w_{t+1}^r h_{t+1}^{r,m}l_{t+1}^{r,m}\left(g_{t+1}^{w,r,m}\right)^\chi+g_{t+1}^{t,r,m}] \tag{A2}$$

where the motion of human capital is:

$$n_{t+1}^{r,m}h_{t+1}^{r,m}=\frac{N_t}{N_{t+1}}n_t^{r,y}\left[\left(1-\delta^{r,h}\right)h_t^{r,y}+B\left(e_t^{r,y}\right)^\theta\left[\gamma\left(z_t^{r,y}\right)^\nu+(1-\gamma)\left(g_t^{r,e}+\kappa g_t^{r,h}\right)^\nu\right]^{\frac{1-\theta}{\nu}}\right] \tag{A3}$$

The optimality conditions for $e_t^{r,y}$ and $z_t^{r,y}$ are:

$$\chi_n\left(e_t^{r,y}\right)^\eta=\frac{\beta q_{t+1}^u\left(c_{t+1}^{p,m}\right)^{-\sigma}\left(1-\tau_{t+1}^{n,p}-\Phi\varphi\left(\frac{w_{t+1}(g_{t+1}^{w,m})^\chi h_{t+1}^m l_{t+1}^m}{w_{t+1}^p(g_{t+1}^{w,p,m})^\chi h_{t+1}^{p,m}l_{t+1}^{p,m}}\right)^\varphi-\tau_{t+1}^s\right)\left(g_{t+1}^{w,p,m}\right)^\chi w_{t+1}^p l_{t+1}^{p,m}\frac{\partial h_{t+1}^{r,m}}{\partial e_t^{r,y}}}{\left(1+\tau_{t+1}^c\right)}+$$
$$+\frac{\beta(1-q_{t+1}^u)\left(c_{t+1}^{r,m}\right)^{-\sigma}\left(1-\tau_{t+1}^{n,r}-\Phi\varphi\left(\frac{w_{t+1}(g_{t+1}^{w,m})^\chi h_{t+1}^m l_{t+1}^m}{w_{t+1}^r(g_{t+1}^{w,r,m})^\chi h_{t+1}^{r,m}l_{t+1}^{r,m}}\right)^\varphi-\tau_{t+1}^s\right)\left(g_{t+1}^{w,r,m}\right)^\chi w_{t+1}^r l_{t+1}^{r,m}\frac{\partial h_{t+1}^{r,m}}{\partial e_t^{r,y}}}{\left(1+\tau_{t+1}^c\right)} \tag{A4}$$

$$\frac{\left(c_t^{r,y}\right)^{-\sigma}}{\left(1+\tau_t^c\right)}=\frac{\beta q_{t+1}^u\left(c_{t+1}^{p,m}\right)^{-\sigma}\left(1-\tau_{t+1}^{n,p}-\Phi\varphi\left(\frac{w_{t+1}(g_{t+1}^{w,m})^\chi h_{t+1}^m l_{t+1}^m}{w_{t+1}^p(g_{t+1}^{w,p,m})^\chi h_{t+1}^{p,m}l_{t+1}^{p,m}}\right)^\varphi-\tau_{t+1}^s\right)\left(g_{t+1}^{w,p,m}\right)^\chi w_{t+1}^p l_{t+1}^{p,m}\frac{\partial h_{t+1}^{r,m}}{\partial z_t^{r,y}}}{\left(1+\tau_{t+1}^c\right)}+$$

$$+\frac{\beta(1-q_{t+1}^u)\left(c_{t+1}^{r,m}\right)^{-\sigma}\left(1-\tau_{t+1}^{n,r}-\Phi\varphi\left(\frac{w_{t+1}(g_{t+1}^{w,m})^\chi h_{t+1}^m l_{t+1}^m}{w_{t+1}^r(g_{t+1}^{w,r,m})^\chi h_{t+1}^{r,m}l_{t+1}^{r,m}}\right)^\varphi-\tau_{t+1}^s\right)\left(g_{t+1}^{w,r,m}\right)^\chi w_{t+1}^r l_{t+1}^{r,m}\frac{\partial h_{t+1}^{r,m}}{\partial z_t^{r,y}}}{\left(1+\tau_{t+1}^c\right)} \tag{A5}$$

where we use:

$$\frac{\partial h_{t+1}^{r,m}}{\partial e_t^{r,y}}=B^r\theta\left(e_t^{r,y}\right)^{\theta-1}\left[\gamma\left(z_t^{r,y}\right)^\nu+(1-\gamma)\left(g_t^{r,e}+\kappa g_t^{r,h}\right)^\nu\right]^{\frac{1-\theta}{\nu}} \tag{A6}$$

$$\frac{\partial h_{t+1}^{r,m}}{\partial z_t^{r,y}}=\frac{B^r\left(e_t^{r,y}\right)^\theta\gamma\left(1-\theta\right)\left[\gamma\left(z_t^{r,y}\right)^\nu+(1-\gamma)\left(g_t^{r,e}+\kappa g_t^{r,h}\right)^\nu\right]^{\frac{1-\theta}{\nu}-1}}{\left(z_t^{r,y}\right)^{1-\nu}} \tag{A7}$$

Optimality conditions of skilled households In the beginning of adult life at $t+1$, status uncertainty is resolved so a skilled person will choose $l^{r,m}_{t+1}$, $k^{r,m}_{t+1}$, $d^{r,m}_{t+1}$, $b^{r,o}_{t+2}$ and $\Psi^{r,y}_{t+1}$ subject to the constraints (we use $\Psi^{r,m}_{t+1} = \frac{n^{r,y}_{t+1}}{n^{r,m}_{t+1}} \Psi^{r,y}_{t+1}$):

$$\left(1 + \tau^c_{t+1}\right) c^{r,m}_{t+1} + k^{r,m}_{t+1} + d^{r,m}_{t+1} + \frac{n^{r,y}_{t+1}}{n^{r,m}_{t+1}} \Psi^{r,y}_{t+1} = \left(1 - \tau^{n,r}_{t+1} - \tau^s_{t+1}\right) w^r_{t+1} h^{r,m}_{t+1} l^{r,m}_{t+1} \left(g^{w,r,m}_{t+1}\right)^{\chi} + g^{t,r,m}_{t+1} \tag{A8}$$

$$q^o_{t+2}[\left(1 + \tau^c_{t+2}\right) c^{r,o}_{t+2} + b^{r,o}_{t+2}] + (1 - q^o_{t+2}) \Omega^{r,o}_{t+2} = \left[1 - \delta^k + \left(1 - \tau^k_{t+2}\right) r^k_{t+2}\right] k^{r,m}_{t+1} + \left(1 - \tau^k_{t+2}\right) \pi^{r,o}_{t+2} + \left(1 + r^d_{t+2}\right) d^{r,m}_{t+1} + q^o_{t+2} s^{r,o}_{t+2} \tag{A9}$$

The optimality conditions for $l^{r,m}_{t+1}$, $k^{r,m}_{t+1}$, $d^{r,m}_{t+1}$, $b^{r,o}_{t+2}$ and $\Psi^{r,y}_{t+1}$ are:

$$\chi_n \left(l^{r,m}_{t+1}\right)^{\eta} = \frac{\left(c^{r,m}_{t+1}\right)^{-\sigma} \left(1 - \tau^{n,r}_{t+1} - \Phi\varphi \left(\frac{w_{t+1} (g^{w,m}_{t+1})^{\chi} h^m_{t+1} l^m_{t+1}}{w^r_{t+1} (g^{w,r,m}_{t+1})^{\chi} h^{r,m}_{t+1} l^{r,m}_{t+1}}\right)^{\varphi} - \tau^s_{t+1}\right) \left(g^{w,r,m}_{t+1}\right)^{\chi} w^r_{t+1} h^{r,m}_{t+1}}{\left(1 + \tau^c_{t+1}\right)} \tag{A10}$$

$$\frac{\left(c^{r,m}_{t+1}\right)^{-\sigma}}{\left(1 + \tau^c_{t+1}\right)} = \frac{\beta q^o_{t+2} \left(c^{r,o}_{t+2}\right)^{-\sigma} \left[1 - \delta^k + \left(1 - \tau^k_{t+2}\right) r^k_{t+2}\right]}{\left(1 + \tau^c_{t+2}\right)} \tag{A11}$$

$$\frac{\left(c^{r,m}_{t+1}\right)^{-\sigma}}{\left(1 + \tau^c_{t+1}\right)} = \frac{\beta q^o_{t+2} \left(c^{r,o}_{t+2}\right)^{-\sigma} \left(1 + r^d_{t+2}\right)}{\left(1 + \tau^c_{t+2}\right)} \tag{A12}$$

$$\frac{\left(c^{r,o}_{t+2}\right)^{-\sigma}}{\left(1 + \tau^c_{t+2}\right)} = \beta \chi_b \left(b^{r,o}_{t+2}\right)^{-\eta_b} \tag{A13}$$

$$\frac{n^{r,y}_{t+1}}{n^{r,m}_{t+1}} (c^{r,m}_{t+1})^{-\sigma} = (c^{r,y}_{t+1})^{-\sigma} \tag{A14}$$

Optimality conditions of poor-born under social mobility Working similarly, a poor-born young person chooses $e^{p,y}_t$ subject to:

$$(1 + \tau^c_t) c^{p,y}_t = \Psi^{p,y}_t \tag{A15}$$

$$q^s_{t+1}[\left(1 + \tau^c_{t+1}\right) c^{r,m}_{t+1} + k^{r,m}_{t+1} + d^{r,m}_{t+1} + \frac{n^{r,y}_{t+1}}{n^{r,m}_{t+1}} \Psi^{r,y}_{t+1}] + (1 - q^s_{t+1})[\left(1 + \tau^c_{t+1}\right) c^{p,m}_{t+1} + \frac{n^{p,y}_{t+1}}{n^{p,m}_{t+1}} \Psi^{p,y}_{t+1}] =$$

$$= q^s_{t+1}[\left(1 - \tau^{n,r}_{t+1} - \tau^s_{t+1}\right) w^r_{t+1} h^{r,m}_{t+1} l^{r,m}_{t+1} \left(g^{w,r,m}_{t+1}\right)^{\chi} + g^{t,r,m}_{t+1}] + (1 - q^s_{t+1})[\left(1 - \tau^{n,p}_{t+1} - \tau^s_{t+1}\right) w^p_{t+1} h^{p,m}_{t+1} l^{p,m}_{t+1} \left(g^{w,p,m}_{t+1}\right)^{\chi} + g^{t,p,m}_{t+1}] \tag{A16}$$

where the motion of human capital is:

$$n^{p,m}_{t+1} h^{p,m}_{t+1} = \frac{N_t}{N_{t+1}} n^{p,y}_t \left[\left(1 - \delta^{p,h}\right) h^{p,y}_t + B^p \left(e^{p,y}_t\right)^{\theta} \left[(1 - \gamma) \left(g^{p,e}_t + \kappa g^{p,h}_t\right)^{\nu}\right]^{\frac{1-\theta}{\nu}}\right] \tag{A17}$$

The optimality condition for $e_t^{p,y}$ is:

$$\chi_n \left(e_t^{p,y}\right)^{\eta} = \frac{\beta(1-q_{t+1}^{s})\left(c_{t+1}^{p,m}\right)^{-\sigma}\left(1-\tau_{t+1}^{n,p}-\Phi\varphi\left(\frac{w_{t+1}(g_{t+1}^{w,m})^{\chi}h_{t+1}^{m}l_{t+1}^{m}}{w_{t+1}^{p}(g_{t+1}^{w,p,m})^{\chi}h_{t+1}^{p,m}l_{t+1}^{p,m}}\right)^{\varphi}-\tau_{t+1}^{s}\right)\left(g_{t+1}^{w,p,m}\right)^{\chi}w_{t+1}^{p}l_{t+1}^{p,m}\frac{\partial h_{t+1}^{p,m}}{\partial e_t^{p,y}}}{\left(1+\tau_{t+1}^{c}\right)}+$$

$$+\frac{\beta q_{t+1}^{s}\left(c_{t+1}^{r,m}\right)^{-\sigma}\left(1-\tau_{t+1}^{n,r}-\Phi\varphi\left(\frac{w_{t+1}(g_{t+1}^{w,m})^{\chi}h_{t+1}^{m}l_{t+1}^{m}}{w_{t+1}^{r}(g_{t+1}^{w,r,m})^{\chi}h_{t+1}^{r,m}l_{t+1}^{r,m}}\right)^{\varphi}-\tau_{t+1}^{s}\right)\left(g_{t+1}^{w,r.m}\right)^{\chi}w_{t+1}^{r}l_{t+1}^{r,m}\frac{\partial h_{t+1}^{p,m}}{\partial e_t^{p,y}}}{\left(1+\tau_{t+1}^{c}\right)} \tag{A18}$$

where we use:

$$\frac{\partial h_{t+1}^{p,m}}{\partial e_t^{p,y}} = B^{p}\theta\left(e_t^{p,y}\right)^{\theta-1}\left[(1-\gamma)\left(g_t^{p,e}+\kappa g_t^{p,h}\right)^{\nu}\right]^{\frac{1-\theta}{\nu}}$$

Optimality conditions of unskilled households In the beginning of adult life at $t+1$, status uncertainty is resolved so an unskilled person will choose $l_{t+1}^{p,m}$ and $\Psi_{t+1}^{p,y}$ subject to the constraints (we use $\Psi_{t+1}^{p,m}=\frac{n_{t+1}^{r,y}}{n_{t+1}^{r,m}}\Psi_{t+1}^{p,y}$):

$$\left(1+\tau_{t+1}^{c}\right)c_{t+1}^{p,m}+\frac{n_{t+1}^{p,y}}{n_{t+1}^{p,m}}\Psi_{t+1}^{p,y}=\left(1-\tau_{t+1}^{n,p}-\tau_{t+1}^{s}\right)w_{t+1}^{p}h_{t+1}^{p,m}l_{t+1}^{p,m}\left(g_{t+1}^{w,p,m}\right)^{\chi}+g_{t+1}^{t,p,m} \tag{A19}$$

$$q_{t+2}^{o}\left(1+\tau_{t+2}^{c}\right)c_{t+2}^{p,o}=q_{t+2}^{o}s_{t+2}^{p,o} \tag{A20}$$

The optimality conditions for $l_{t+1}^{p,m}$ and $\Psi_{t+1}^{p,y}$ are:

$$\chi_n\left(l_{t+1}^{p,m}\right)^{\eta}=\frac{\left(c_{t+1}^{p,m}\right)^{-\sigma}\left(1-\tau_{t+1}^{n,p}-\Phi\varphi\left(\frac{w_{t+1}(g_{t+1}^{w,m})^{\chi}h_{t+1}^{m}l_{t+1}^{m}}{w_{t+1}^{p}(g_{t+1}^{w,p,m})^{\chi}h_{t+1}^{p,m}l_{t+1}^{p,m}}\right)^{\varphi}-\tau_{t+1}^{s}\right)\left(g_{t+1}^{w,p,m}\right)^{\chi}w_{t+1}^{p}h_{t+1}^{p,m}}{\left(1+\tau_{t+1}^{c}\right)} \tag{A21}$$

$$\frac{n_{t+1}^{p,y}}{n_{t+1}^{p,m}}(c_{t+1}^{p,m})^{-\sigma}=(c_{t+1}^{p,y})^{-\sigma} \tag{A22}$$

Appendix B: Population shares

Let us define by ν_r and ν_p the exogenous birth rates of the two groups and by δ_r and δ_p their exogenous death rates. Also, recall that q_t^{o} is the probability of an adult reaching the old age (assumed to be common for all agents), while q_t^{s} is the probability of moving upwards and q_t^{u} is the probability of moving downwards the social ladder. Then, we have the population levels for the 6 distinct groups:

$$N_t^{r,y}\equiv(1-\delta_r)X_{t-1}^{r}+\nu_r N_{t-1} \tag{B1}$$

$$N_t^{r,m}\equiv N_{t-1}^{r,y}+q_t^{s}N_{t-1}^{p,y}-q_t^{u}N_{t-1}^{r,y} \tag{B2}$$

$$N_t^{r,o} \equiv q_t^o N_{t-1}^{r,m} \tag{B3}$$

$$N_t^{p,y} \equiv (1-\delta_p) X_{t-1}^p + \nu_p N_{t-1} \tag{B4}$$

$$N_t^{p,m} \equiv N_{t-1}^{p,y} - q_t^s N_{t-1}^{p,y} + q_t^u N_{t-1}^{r,y} \tag{B5}$$

$$N_t^{p,o} \equiv q_t^o N_{t-1}^{p,m} \tag{B6}$$

Since the total population at t, N_t, is:

$$N_t \equiv N_t^{p,y} + N_t^{r,y} + N_t^{p,m} + N_t^{r,m} + N_t^{p,o} + N_t^{r,o} \tag{B7}$$

we have for the motion of N_t over time:

$$N_t = (1+\nu_r+\nu_p)N_{t-1} - N_{t-1}^{r,o} - N_{t-1}^{p,o} + (1-\delta_r)N_{t-1}^{r,y} + (1-\delta_p)N_{t-1}^{p,y} - (1-q_t^o)N_{t-1}^{r,m} - (1-q_t^o)N_{t-1}^{p,m} \tag{B8}$$

Appendix C: Firms' decisions

The firm's first-order conditions for k_t^f, $l_t^{r,f}$ and $l_t^{p,f}$ are respectively:

$$r_t^k = (1-\tau_t^f)\frac{\partial y_t^f}{\partial k_t^f} \tag{C1}$$

$$(1+\tau_t^w-\tau_t^f)w_t^r = (1-\tau_t^f)\frac{\partial y_t^f}{\partial l_t^{r,f}} \tag{C2}$$

$$(1+\tau_t^w-\tau_t^f)w_t^p = (1-\tau_t^f)\frac{\partial y_t^f}{\partial l_t^{p,f}} \tag{C3}$$

where we use:

$$\frac{\partial y_t^f}{\partial k_t^f} = \frac{(1-\varepsilon)y_t^f(1-\lambda)[\mu(k_t^f)^\psi + (1-\mu)(l_t^{r,f})^\psi]^{\frac{\alpha}{\psi}-1}\mu(k_t^f)^{\psi-1}}{\left(\lambda(l_t^{p,f})^\alpha + (1-\lambda)[\mu(k_t^f)^\psi + (1-\mu)(l_t^{r,f})^\psi]^{\frac{\alpha}{\psi}}\right)}$$

$$\frac{\partial y_t^f}{\partial l_t^{r,f}} = \frac{(1-\varepsilon)y_t^f(1-\lambda)[\mu(k_t^f)^\psi + (1-\mu)(l_t^{r,f})^\psi]^{\frac{\alpha}{\psi}-1}(1-\mu)(l_t^{r,f})^{\psi-1}}{\left(\lambda(l_t^{p,f})^\alpha + (1-\lambda)[\mu(k_t^f)^\psi + (1-\mu)(l_t^{r,f})^\psi]^{\frac{\alpha}{\psi}}\right)}$$

$$\frac{\partial y_t^f}{\partial l_t^{p,f}} = \frac{(1-\varepsilon)y_t^f\lambda(l_t^{p,f})^{\alpha-1}}{\left(\lambda(l_t^{p,f})^\alpha + (1-\lambda)[\mu(k_t^f)^\psi + (1-\mu)(l_t^{r,f})^\psi]^{\frac{\alpha}{\psi}}\right)}$$

Appendix D: Macroeconomic system

Collecting all equations and writing them as at time t, the final system is:

Rich-born and skilled households

$$(1+\tau_t^c)\,c_t^{r,y} + z_t^{r,y} = \left(1-\tau_t^b\right) b_{t-1}^{r,y} + \Psi_t^{r,y} \tag{D1}$$

$$q_t^u[(1+\tau_t^c)\,c_t^{p,m}+\frac{n_t^{p,y}}{n_t^{p,m}}\Psi_t^{p,y}]+(1-q_t^u)[(1+\tau_t^c)\,c_t^{r,m}+k_t^{r,m}+d_t^{r,m}+\frac{n_t^{r,y}}{n_t^{r,m}}\Psi_t^{r,y}]=$$

$$=q_t^u[(1-\tau_t^{n,p}-\tau_t^s)\,w_t^p h_t^{p,m} l_t^{p,m}\,(g_t^{w,p,m})^{\chi}+g_t^{t,p,m}]+(1-q_t^u)[(1-\tau_t^{n,r}-\tau_t^s)\,w_t^r h_t^{r,m} l_t^{r,m}\,(g_t^{w,r,m})^{\chi}+g_t^{t,r,m}] \tag{D2}$$

$$q_t^o[(1+\tau_t^c)\,c_t^{r,o}+b_t^{r,o}]+(1-q_t^o)\Omega_t^{r,o}=\left[1-\delta^k+\left(1-\tau_t^k\right)r_t^k\right]k_{t-1}^{r,m}+\left(1-\tau_t^k\right)\pi_t^{r,o}+\left(1+r_t^d\right)d_{t-1}^{r,m}+q_t^o s_t^{r,o} \tag{D3}$$

$$n_t^{r,m}h_t^{r,m}=\frac{N_{t-1}}{N_t}n_{t-1}^{r,y}\left[\left(1-\delta^{r,h}\right)h_{t-1}^{r,y}+B\left(e_{t-1}^{r,y}\right)^{\theta}\left[\gamma\left(z_{t-1}^{r,y}\right)^{\nu}+(1-\gamma)\left(g_{t-1}^{r,e}+\kappa g_{t-1}^{r,h}\right)^{\nu}\right]^{\frac{1-\theta}{\nu}}\right] \tag{D4}$$

$$\chi_n\left(e_t^{r,y}\right)^{\eta}=\frac{\beta q_{t+1}^u\left(c_{t+1}^{p,m}\right)^{-\sigma}\left(1-\tau_{t+1}^{n,p}-\Phi\varphi\left(\frac{w_{t+1}(g_{t+1}^{w,m})^{\chi}h_{t+1}^{m}l_{t+1}^{m}}{w_{t+1}^{p}(g_{t+1}^{w,p,m})^{\chi}h_{t+1}^{p,m}l_{t+1}^{p,m}}\right)^{\varphi}-\tau_{t+1}^{s}\right)\left(g_{t+1}^{w,p,m}\right)^{\chi}w_{t+1}^{p}l_{t+1}^{p,m}\frac{\partial h_{t+1}^{r,m}}{\partial e_t^{r,y}}}{\left(1+\tau_{t+1}^{c}\right)}+$$

$$+\frac{\beta(1-q_{t+1}^u)\left(c_{t+1}^{r,m}\right)^{-\sigma}\left(1-\tau_{t+1}^{n,r}-\Phi\varphi\left(\frac{w_{t+1}(g_{t+1}^{w,m})^{\chi}h_{t+1}^{m}l_{t+1}^{m}}{w_{t+1}^{r}(g_{t+1}^{w,r,m})^{\chi}h_{t+1}^{r,m}l_{t+1}^{r,m}}\right)^{\varphi}-\tau_{t+1}^{s}\right)\left(g_{t+1}^{w,r,m}\right)^{\chi}w_{t+1}^{r}l_{t+1}^{r,m}\frac{\partial h_{t+1}^{r,m}}{\partial e_t^{r,y}}}{\left(1+\tau_{t+1}^{c}\right)} \tag{D5}$$

$$\frac{\left(c_t^{r,y}\right)^{-\sigma}}{(1+\tau_t^c)}=\frac{\beta q_{t+1}^u\left(c_{t+1}^{p,m}\right)^{-\sigma}\left(1-\tau_{t+1}^{n,p}-\Phi\varphi\left(\frac{w_{t+1}(g_{t+1}^{w,m})^{\chi}h_{t+1}^{m}l_{t+1}^{m}}{w_{t+1}^{p}(g_{t+1}^{w,p,m})^{\chi}h_{t+1}^{p,m}l_{t+1}^{p,m}}\right)^{\varphi}-\tau_{t+1}^{s}\right)\left(g_{t+1}^{w,p,m}\right)^{\chi}w_{t+1}^{p}l_{t+1}^{p,m}\frac{\partial h_{t+1}^{r,m}}{\partial z_t^{r,y}}}{\left(1+\tau_{t+1}^{c}\right)}+$$

$$+\frac{\beta(1-q_{t+1}^u)\left(c_{t+1}^{r,m}\right)^{-\sigma}\left(1-\tau_{t+1}^{n,r}-\Phi\varphi\left(\frac{w_{t+1}(g_{t+1}^{w,m})^{\chi}h_{t+1}^{m}l_{t+1}^{m}}{w_{t+1}^{r}(g_{t+1}^{w,r,m})^{\chi}h_{t+1}^{r,m}l_{t+1}^{r,m}}\right)^{\varphi}-\tau_{t+1}^{s}\right)\left(g_{t+1}^{w,r,m}\right)^{\chi}w_{t+1}^{r}l_{t+1}^{r,m}\frac{\partial h_{t+1}^{r,m}}{\partial z_t^{r,y}}}{\left(1+\tau_{t+1}^{c}\right)} \tag{D6}$$

$$\chi_n\left(l_t^{r,m}\right)^{\eta}=\frac{\left(c_t^{r,m}\right)^{-\sigma}\left(1-\tau_t^{n,r}-\Phi\varphi\left(\frac{w_t(g_t^{w,m})^{\chi}h_t^{m}l_t^{m}}{w_t^{r}(g_t^{w,r,m})^{\chi}h_t^{r,m}l_t^{r,m}}\right)^{\varphi}-\tau_t^{s}\right)\left(g_t^{w,r,m}\right)^{\chi}w_t^{r}h_t^{r,m}}{(1+\tau_t^c)} \tag{D7}$$

$$\frac{\left(c_t^{r,m}\right)^{-\sigma}}{(1+\tau_t^c)}=\frac{\beta q_{t+1}^o\left(c_{t+1}^{r,o}\right)^{-\sigma}\left[1-\delta^k+\left(1-\tau_{t+1}^k\right)r_{t+1}^k\right]}{\left(1+\tau_{t+1}^c\right)} \tag{D8}$$

$$\frac{\left(c_t^{r,m}\right)^{-\sigma}}{(1+\tau_t^c)}=\frac{\beta q_{t+1}^o\left(c_{t+1}^{r,o}\right)^{-\sigma}\left(1+r_{t+1}^d\right)}{\left(1+\tau_{t+1}^c\right)} \tag{D9}$$

$$\frac{\left(c_t^{r,o}\right)^{-\sigma}}{(1+\tau_t^c)}=\beta\chi_b\left(b_t^{r,o}\right)^{-\eta_b} \tag{D10}$$

$$\frac{n_t^{r,y}}{n_t^{r,m}}\left(c_t^{r,m}\right)^{-\sigma}=\left(c_t^{r,y}\right)^{-\sigma} \tag{D11}$$

where we use:

$$\frac{\partial h_{t+1}^{r,m}}{\partial e_t^{r,y}}=B^r\theta\left(e_t^{r,y}\right)^{\theta-1}\left[\gamma\left(z_t^{r,y}\right)^{\nu}+(1-\gamma)\left(g_t^{r,e}+\kappa g_t^{r,h}\right)^{\nu}\right]^{\frac{1-\theta}{\nu}}$$

$$\frac{\partial h_{t+1}^{r,m}}{\partial z_t^{r,y}}=\frac{B^r\left(e_t^{r,y}\right)^{\theta}\gamma\,(1-\theta)\left[\gamma\left(z_t^{r,y}\right)^{\nu}+(1-\gamma)\left(g_t^{r,e}+\kappa g_t^{r,h}\right)^{\nu}\right]^{\frac{1-\theta}{\nu}-1}}{\left(z_t^{r,y}\right)^{1-\nu}}$$

The definitions for $\tau_t^{n,p}$, $\tau_t^{n,r}$ and $w_t(g_t^{w,m})^{\chi}h_t^m l_t^m$ are provided below.

In this block, we have 11 equations for the paths of $c_t^{r,y}$, $c_{t,}^{r,m}$, $c_t^{r,o}$, $e_t^{r,y}$, $z_t^{r,y}$, $l_t^{r,m}$, $k_t^{r,m}$, $d_t^{r,m}$, $b_t^{r,o}$, $h_t^{r,m}$, $\Psi_t^{r,y}$.

Poor-born and unskilled households

$$(1+\tau_t^c)\,c_t^{p,y} = \Psi_t^{p,y} \tag{D12}$$

$$q_t^s[(1+\tau_t^c)\,c_t^{r,m} + k_t^{r,m} + d_t^{r,m} + \frac{n_t^{r,y}}{n_t^{r,m}}\Psi_t^{r,y}] + (1-q_t^s)[(1+\tau_t^c)\,c_t^{p,m} + \frac{n_t^{p,y}}{n_t^{p,m}}\Psi_t^{p,y}] =$$
$$= q_t^s[(1-\tau_t^{n,r}-\tau_t^s)\,w_t^r h_t^{r,m} l_t^{r,m}\,(g_t^{w,r,m})^{\chi} + g_t^{t,r,m}] + (1-q_t^s)[(1-\tau_t^{n,p}-\tau_t^s)\,w_t^p h_t^{p,m} l_t^{p,m}\,(g_t^{w,p,m})^{\chi} + g_t^{t,p,m}] \tag{D13}$$

$$(1+\tau_t^c)\,c_t^{p,o} = s_t^{p,o} \tag{D14}$$

$$n_t^{p,m}h_t^{p,m} = \frac{N_{t-1}}{N_t}n_{t-1}^{p,y}\left[\left(1-\delta^{p,h}\right)h_{t-1}^{p,y} + B^p\left(e_{t-1}^{p,y}\right)^{\theta}\left[(1-\gamma)\left(g_{t-1}^{p,e}+\kappa g_{t-1}^{p,h}\right)^{\nu}\right]^{\frac{1-\theta}{\nu}}\right] \tag{D15}$$

$$\chi_n\,(e_t^{p,y})^{\eta} = \frac{\beta(1-q_{t+1}^s)\left(c_{t+1}^{p,m}\right)^{-\sigma}\left(1-\tau_{t+1}^{n,p}-\Phi\varphi\left(\frac{w_{t+1}(g_{t+1}^{w,m})^{\chi}h_{t+1}^m l_{t+1}^m}{w_{t+1}^p(g_{t+1}^{w,p,m})^{\chi}h_{t+1}^{p,m}l_{t+1}^{p,m}}\right)^{\varphi}-\tau_{t+1}^s\right)\left(g_{t+1}^{w,p,m}\right)^{\chi}w_{t+1}^p l_{t+1}^{p,m}\frac{\partial h_{t+1}^{p,m}}{\partial e_t^{p,y}}}{\left(1+\tau_{t+1}^c\right)}+$$
$$+\frac{\beta q_{t+1}^s\left(c_{t+1}^{r,m}\right)^{-\sigma}\left(1-\tau_{t+1}^{n,r}-\Phi\varphi\left(\frac{w_{t+1}(g_{t+1}^{w,m})^{\chi}h_{t+1}^m l_{t+1}^m}{w_{t+1}^r(g_{t+1}^{w,r,m})^{\chi}h_{t+1}^{r,m}l_{t+1}^{r,m}}\right)^{\varphi}-\tau_{t+1}^s\right)\left(g_{t+1}^{w,r,m}\right)^{\chi}w_{t+1}^r l_{t+1}^{r,m}\frac{\partial h_{t+1}^{p,m}}{\partial e_t^{p,y}}}{\left(1+\tau_{t+1}^c\right)} \tag{D16}$$

$$\chi_n\,(l_t^{p,m})^{\eta} = \frac{\left(c_t^{p,m}\right)^{-\sigma}\left(1-\tau_t^{n,p}-\Phi\varphi\left(\frac{w_t(g_t^{w,m})^{\chi}h_t^m l_t^m}{w_t^p(g_t^{w,p,m})^{\chi}h_t^{p,m}l_t^{p,m}}\right)^{\varphi}-\tau_t^s\right)(g_t^{w,p,m})^{\chi}w_t^p h_t^{p,m}}{(1+\tau_t^c)} \tag{D17}$$

$$\frac{n_t^{p,y}}{n_t^{p,m}}(c_t^{p,m})^{-\sigma} = (c_t^{p,y})^{-\sigma} \tag{D18}$$

where we use:

$$\frac{\partial h_{t+1}^{p,m}}{\partial e_t^{p,y}} = B^p\theta\,(e_t^{p,y})^{\theta-1}\left[(1-\gamma)\left(g_t^{p,e}+\kappa g_t^{p,h}\right)^{\nu}\right]^{\frac{1-\theta}{\nu}}$$

The definitions for $\tau_t^{n,p}$, $\tau_t^{n,r}$ and $w_t(g_t^{w,m})^{\chi}h_t^m l_t^m$ are provided right below.

In this block, we have 7 equations for the paths of $c_t^{p,y}$, $c_{t,}^{p,m}$, $c_t^{p,o}$, $e_t^{p,y}$, $l_t^{p,m}$, $h_{t+1}^{p,m}$, $\Psi_t^{p,y}$.

Fomulas for the progressive labor income tax rates

$$\tau_t^{n,r} \equiv 1-\Phi\left(\frac{w_t(g_t^{w,m})^{\chi}h_t^m l_t^m}{w_t^r(g_t^{w,r,m})^{\chi}h_t^{r,m}l_t^{r,m}}\right)^{\varphi} \tag{D19}$$

$$\tau_t^{n,p} \equiv 1-\Phi\left(\frac{w_t(g_t^{w,m})^{\chi}h_t^m l_t^m}{w_t^p(g_t^{w,p,m})^{\chi}h_t^{p,m}l_t^{p,m}}\right)^{\varphi} \tag{D20}$$

where the numerator is defined as:

$$w_t(g_t^{w,m})^{\chi} h_t^m l_t^m \equiv \frac{w_t^r(g_t^{w,r,m})^{\chi} h_t^{r,m} l_t^{r,m} + w_t^p(g_t^{w,p,m})^{\chi} h_t^{p,m} l_t^{p,m}}{2}$$

In this block, we have 2 equations for the paths of $\tau_t^{n,r}$ and $\tau_t^{n,p}$.

Market-clearing conditions in the market for bequests and the probability of reaching the old age

$$n_t^{r,y} b_{t-1}^{r,y} = \frac{N_{t-1}}{N_t} n_{t-1}^{r,o} [b_{t-1}^{r,o} + \frac{(1-q_{t-1}^o)\Omega_{t-1}^{r,o}]}{q_{t-1}^o}] \tag{D21}$$

where

$$\Omega_{t-1}^{r,o} \equiv \left[1-\delta^k + \left(1-\tau_{t-1}^k\right) r_{t-1}^k\right] k_{t-2}^{r,m} + \left(1-\tau_{t-1}^k\right) \pi_{t-1}^{r,o} + \left(1+r_{t-1}^d\right) d_{t-2}^{r,m} \tag{D22}$$

$$q_t^o \equiv \Xi \left(1 + \frac{s_t^{g^h}}{1+s_t^{g^h}}\right) \tag{D23}$$

In this block, we have 3 equations for the paths of $b_t^{r,y}$, $\Omega_t^{r,o}$, q_t^o.

Probabilities of changing status The probability of switching from unskilled to skilled is:

$$q_t^s = 1 - \frac{\left(\frac{S^p}{(h_t^{p,m})^{\xi^p}}\right) - \phi^l}{\phi^h - \phi^l} \tag{D24}$$

and the probability of switching from skilled to unskilled is:

$$q_t^u = 1 - \frac{\left(\frac{(h_t^{r,m})^{\xi^r}}{S^r}\right) - \phi^l}{\phi^h - \phi^l} \tag{D25}$$

In this block, we have 2 equations for the paths of q_t^s and q_t^u, namely, the probabilities of moving up and moving down.

Equations for the motion of human capital in all other ages In addition to the two motions from young to adult which were defined above, we model, for each group, the motion from adult to old and the motion for old to young (so we capture the persistence of human capital from one generation to another). Thus,

$$n_t^{r,o} h_t^{r,o} = \left(1-\delta^{r,h}\right) q_t^o \frac{N_{t-1}}{N_t} n_{t-1}^{r,m} h_{t-1}^{r,m} \tag{D26}$$

$$n_t^{r,y} h_t^{r,y} = \left(1-\delta^{r,h}\right) \frac{N_{t-1}}{N_t} n_{t-1}^{r,o} h_{t-1}^{r,o} \tag{D27}$$

$$n_t^{p,o} h_t^{p,o} = \left(1-\delta^{p,h}\right) q_t^o \frac{N_{t-1}}{N_t} n_{t-1}^{p,m} h_{t-1}^{p,m} \tag{D28}$$

$$n_t^{p,y} h_t^{p,y} = \left(1-\delta^{p,h}\right) \frac{N_{t-1}}{N_t} n_{t-1}^{p,o} h_{t-1}^{p,o} \tag{D29}$$

In this block, we have 4 equations for the paths of $h_t^{r,o}$, $h_t^{r,y}$, $h_t^{p,o}$ and $h_t^{p,y}$.

Firms

$$n_t^f y_t^f = A\left(\lambda(n_t^f l_t^{p,f})^\alpha + (1-\lambda)[\mu(n_t^f k_t^f)^\psi + (1-\mu)(n_t^f l_t^{r,f})^\psi]^{\frac{\alpha}{\psi}}\right)^{\frac{1-\varepsilon}{\alpha}} (k_t^g)^\varepsilon \tag{D30}$$

$$r_t^k = (1-\tau_t^f)\frac{\partial y_t^f}{\partial k_t^f} \tag{D31}$$

$$(1+\tau_t^w - \tau_t^f)w_t^r = (1-\tau_t^f)\frac{\partial y_t^f}{\partial l_t^{r,f}} \tag{D32}$$

$$(1+\tau_t^w - \tau_t^f)w_t^p = (1-\tau_t^f)\frac{\partial y_t^f}{\partial l_t^{p,f}} \tag{D33}$$

$$\pi_t^f \equiv y_t^f - (1+\tau_t^w)(w_t^r l_t^{r,f} + w_t^p l_t^{p,f}) - r_t^k k_t^f - \tau_t^f(y_t^f - w_t^r l_t^{r,f} - w_t^p l_t^{p,f}) \tag{D34}$$

where we use:

$$\frac{\partial y_t^f}{\partial k_t^f} = \frac{(1-\varepsilon)y_t^f(1-\lambda)[\mu(k_t^f)^\psi + (1-\mu)(l_t^{r,f})^\psi]^{\frac{\alpha}{\psi}-1}\mu(k_t^f)^{\psi-1}}{\left(\lambda(l_t^{p,f})^\alpha + (1-\lambda)[\mu(k_t^f)^\psi + (1-\mu)(l_t^{r,f})^\psi]^{\frac{\alpha}{\psi}}\right)}$$

$$\frac{\partial y_t^f}{\partial l_t^{r,f}} = \frac{(1-\varepsilon)y_t^f(1-\lambda)[\mu(k_t^f)^\psi + (1-\mu)(l_t^{r,f})^\psi]^{\frac{\alpha}{\psi}-1}(1-\mu)(l_t^{r,f})^{\psi-1}}{\left(\lambda(l_t^{p,f})^\alpha + (1-\lambda)[\mu(k_t^f)^\psi + (1-\mu)(l_t^{r,f})^\psi]^{\frac{\alpha}{\psi}}\right)}$$

$$\frac{\partial y_t^f}{\partial l_t^{p,f}} = \frac{(1-\varepsilon)y_t^f\lambda()^\alpha(l_t^{p,f})^{\alpha-1}}{\left(\lambda(l_t^{p,f})^\alpha + (1-\lambda)[\mu(k_t^f)^\psi + (1-\mu)(l_t^{r,f})^\psi]^{\frac{\alpha}{\psi}}\right)}$$

In this block, we have 5 equations for the paths of y_t^f, r_t, w_t^r, w_t^p, π_t^f.

Market-clearing conditions in the factor markers

$$l_t^{p,f} = \frac{n_t^{p,m}(g_t^{w,p,m})^\chi l_t^{p,m} h_t^{p,m}}{n_t^f} \tag{D35}$$

$$l_t^{r,f} = \frac{n_t^{r,m}(g_t^{w,r,m})^\chi l_t^{r,m} h_t^{r,m}}{n_t^f} \tag{D36}$$

$$k_t^f = \frac{\frac{N_{t-1}}{N_t} n_{t-1}^{r,m} k_{t-1}^{r,m}}{n_t^f} \tag{D37}$$

$$\pi_t^f = \frac{\frac{N_{t-1}}{N_t} n_{t-1}^{r,m} \pi_t^{r,o}}{n_t^f} \tag{D38}$$

In this block, we have 4 equations for the pathhs of $l_t^{p,f}$, $l_t^{r,f}$, k_t^f, $\pi_t^{r,o}$.

Government budget constraint Under a fully funded PAYG system, there are two separate government budget constraints:

$$[s_t^{g^h} + s_t^{g^e} + s_t^{g^t} + s_t^{g^w} + s_t^{g^i} + s_t^{g^c}]n_t^f y_t^f + \left(1 + r_t^d\right) \frac{D_t}{N_t} = \frac{N_{t+1}}{N_t} \frac{D_{t+1}}{N_{t+1}} + \frac{T_t}{N_t} \tag{D39}$$

$$s_t^{g^s} n_t^f y_t^f = \tau_t^s [n_t^{r,m} \left(g_t^{w,r,m}\right)^\chi w_t^r l_t^{r,m} h_t^{r,m} + n_t^{p,m} \left(g_t^{w,p,m}\right)^\chi w_t^p l_t^{p,m} h_t^{p,m}] + \tau_t^w n_t^f (w_t^r l_t^{r,f} + w_t^p l_t^{p,f}) \tag{D40}$$

By contrast, under a partial PAYG system, we have a single budget constraint:

$$[s_t^{g^h} + s_t^{g^e} + s_t^{g^t} + s_t^{g^w} + s_t^{g^i} + s_t^{g^c}]n_t^f y_t^f + \left(1 + r_t^d\right) \frac{D_t}{N_t} +$$
$$+ s_t^{g^s} n_t^f y_t^f - \tau_t^s [n_t^{r,m} \left(g_t^{w,r,m}\right)^\chi w_t^r l_t^{r,m} h_t^{r,m} + n_t^{p,m} \left(g_t^{w,p,m}\right)^\chi w_t^p l_t^{p,m} h_t^{p,m}] - \tau_t^w n_t^f (w_t^r l_t^{r,f} + w_t^p l_t^{p,f}) =$$
$$= \frac{N_{t+1}}{N_t} \frac{D_{t+1}}{N_{t+1}} + \frac{T_t}{N_t} \tag{D41}$$

so that now we can move both τ_t^s and τ_t^w to the list of exogenous set variables.

In the above we use the market-clearing condition in the bond market:

$$\frac{D_t}{N_t} = \frac{N_{t-1}}{N_t} n_{t-1}^{r,m} d_{t-1}^{r,m} \tag{D42}$$

$$\frac{N_{t+1}}{N_t} \frac{D_{t+1}}{N_{t+1}} = n_t^{r,m} d_t^{r,m} \tag{D43}$$

The tax revenues are:

$$\frac{T_t}{N_t} \equiv \tau_t^c (n_t^{r,y} c_t^{r,y} + n_t^{r,m} c_t^{r,m} + n_t^{r,o} c_t^{r,o} + n_t^{p,y} c_t^{p,y} + n_t^{p,m} c_t^{p,m} + n_t^{p,o} c_t^{p,o}) +$$
$$+ \tau_t^{n,r} n_t^{r,m} \left(g_t^{w,r,m}\right)^\chi w_t^r l_t^{r,m} h_t^{r,m} + \tau_t^{n,p} n_t^{p,m} \left(g_t^{w,p,m}\right)^\chi w_t^p l_t^{p,m} h_t^{p,m} + \tau_t^k \frac{N_{t-1}}{N_t} n_{t-1}^{r,m} (r_t k_{t-1}^{r,m} + \pi_t^{r,o}) +$$
$$+ \tau_t^b n_t^{r,y} b_{t-1}^{r,y} + \tau_t^f n_t^f \left(y_t^f - w_t^r l_t^{r,f} - w_t^p l_t^{p,f}\right) \tag{D44}$$

The law of motion for public capital (per capita):

$$\frac{N_{t+1}}{N_t} k_{t+1}^g = \left(1 - \delta^g\right) k_t^g + s_t^{g^i} n_t^f y_t^f \tag{D45}$$

In this block, under a fully funded PAYG system (in which we have 2 separate budget constaints), we have 4 equations for the paths of $d_t^{r,m}$, τ_t^s, $\frac{T_t}{N_t}$, k_{t+1t}^g. Or, under a partial PAYG system (in which we have a merged budget constraint), we have 3 equations for the paths of $d_t^{r,m}$, $\frac{T_t}{N_t}$, k_{t+1t}^g.

Government spending items and their allocation to the two groups

$$g_t^{r,h} \equiv \frac{G_t^{r,h}}{N_t^{r,y}} = \frac{\zeta_t^h G_t^h}{N_t^{r,y}} = \frac{\zeta_t^h s_t^{g^h} N_t^f y_t^f}{N_t^{r,y}} = \frac{\zeta_t^h s_t^{g^h} n_t^f y_t^f}{n_t^{r,y}} \tag{D46}$$

$$g_t^{p,h} \equiv \frac{G_t^{p,h}}{N_t^{p,y}} = \frac{(1-\zeta_t^h) s_t^{g^h} N_t^f y_t^f}{N_t^{p,y}} = \frac{(1-\zeta_t^h) s_t^{g^h} n_t^f y_t^f}{n_t^{p,y}} \tag{D47}$$

$$g_t^{r,e} \equiv \frac{G_t^{r,e}}{N_t^{r,y}} = \frac{\zeta_t^e G_t^e}{N_t^{r,y}} = \frac{\zeta_t^e s_t^{g^e} N_t^f y_t^f}{N_t^{r,y}} = \frac{\zeta_t^e s_t^{g^e} n_t^f y_t^f}{n_t^{r,y}} \tag{D48}$$

$$g_t^{p,e} \equiv \frac{G_t^{p,e}}{N_t^{p,y}} = \frac{(1-\zeta_t^e) s_t^{g^e} N_t^f y_t^f}{N_t^{p,y}} = \frac{(1-\zeta_t^e) s_t^{g^e} n_t^f y_t^f}{n_t^{p,y}} \tag{D49}$$

$$g_t^{w,r,m} \equiv \frac{G_t^{r,w}}{N_t^{r,m}} = \frac{\zeta_t^w G_t^w}{N_t^{r,m}} = \frac{\zeta_t^w s_t^{g^w} N_t^f y_t^f}{N_t^{r,m}} = \frac{\zeta_t^w s_t^{g^w} n_t^f y_t^f}{n_t^{r,m}} \tag{D50}$$

$$g_t^{w,p,m} \equiv \frac{G_t^{p,w}}{N_t^{p,m}} = \frac{(1-\zeta_t^w) G_t^w}{N_t^{p,m}} = \frac{(1-\zeta_t^w) s_t^{g^w} N_t^f y_t^f}{N_t^{p,m}} = \frac{(1-\zeta_t^w) s_t^{g^w} n_t^f y_t^f}{n_t^{p,m}} \tag{D51}$$

$$g_t^{t,r,m} \equiv \frac{\zeta_t^t G_t^t}{N_t^{r,m}} = \frac{\zeta_t^t s_t^{g^t} N_t^f y_t^f}{N_t^{r,m}} = \frac{\zeta_t^t s_t^{g^t} n_t^f y_t^f}{n_t^{r,m}} \tag{D52}$$

$$g_t^{t,p,m} \equiv \frac{(1-\zeta_t^t) G_t^t}{N_t^{p,m}} = \frac{(1-\zeta_t^t) s_t^{g^t} N_t^f y_t^f}{N_t^{p,m}} = \frac{(1-\zeta_t^t) s_t^{g^t} n_t^f y_t^f}{n_t^{p,m}} \tag{D53}$$

$$s_t^{r,o} \equiv \frac{\zeta_t^s G_t^s}{N_t^{r,o}} = \frac{\zeta_t^s s_t^{g^s} N_t^f y_t^f}{N_t^{r,o}} = \frac{\zeta_t^s s_t^{g^s} n_t^f y_t^f}{n_t^{r,o}} \tag{D54}$$

$$s_t^{p,o} \equiv \frac{(1-\zeta_t^s) G_t^s}{N_t^{p,o}} = \frac{(1-\zeta_t^s) s_t^{g^s} N_t^f y_t^f}{N_t^{p,o}} = \frac{(1-\zeta_t^s) s_t^{g^s} n_t^f y_t^f}{n_t^{p,o}} \tag{D55}$$

This block gives 10 formulas for the group-specific public spending items, $g_t^{r,h}$, $g_t^{p,h}$, $g_t^{r,e}$, $g_t^{p,e}$, $g_t^{w,r,m}$, $g_t^{w,p,m}$, $g_t^{t,r,m}$, $g_t^{t,p,m}$, $s_t^{r,o}$ and $s_t^{p,o}$.

Population fractions From Appendix B above, we have:

$$n_t^{r,y} \equiv \frac{N_t^{r,y}}{N_t} = \frac{N_{t-1}}{N_t}[(1-\delta_r)x_{t-1}^r + \nu_r] \tag{D56}$$

$$n_t^{r,m} \equiv \frac{N_t^{r,m}}{N_t} = \frac{N_{t-1}}{N_t}\left[(1-q_t^u)n_{t-1}^{r,y} + q_t^s n_{t-1}^{p,y}\right] \tag{D57}$$

$$n_t^{r,o} \equiv \frac{N_t^{r,o}}{N_t} = q_t^o \frac{N_{t-1}}{N_t} n_{t-1}^{r,m} \tag{D58}$$

$$n_t^{p,y} \equiv \frac{N_t^{p,y}}{N_t} = \frac{N_{t-1}}{N_t}[(1-\delta_p)x_{t-1}^p + \nu_p] \tag{D59}$$

$$n_t^{p,m} \equiv \frac{N_t^{p,m}}{N_t} = \frac{N_{t-1}}{N_t}\left[(1-q_t^s)n_{t-1}^{p,y} + q_t^u n_{t-1}^{r,y}\right] \tag{D60}$$

$$n_t^{p,o} \equiv \frac{N_t^{p,o}}{N_t} = q_t^o \frac{N_{t-1}}{N_t} n_{t-1}^{p,m} \tag{D61}$$

$$\frac{N_t}{N_{t-1}} = 1 + \nu_r + \nu_p - n_{t-1}^{r,o} - n_{t-1}^{p,o} + (1-\delta_r)x_{t-1}^{r} + (1-\delta_p)x_{t-1}^{p} - (1-q_t^o)n_{t-1}^{r,m} - (1-q_t^o)n_{t-1}^{p,m} \quad \text{(D62)}$$

In this block, we have 7 equations in the above 7 variables. As said in Appendix A, $x_{t-1}^{r} \equiv n_{t-1}^{r,y}$ and $x_{t-1}^{p} \equiv n_{t-1}^{p,y}$. In our numerical solutions, we set $\delta_r = 0.03$, δ_p =0.03, ν_r =0.002, ν_p =0.01.

Economy's resource constraint (which holds residually by Walras' law)

$$n_t^{r,y}c_t^{r,y} + n_t^{r,y}z_t^{r,y} + n_t^{r,m}c_t^{r,m} + n_t^{r,o}c_t^{r,o} + n_t^{p,y}c_t^{p,y} + n_t^{p,m}c_t^{p,m} + n_t^{p,o}c_t^{p,o} +$$

$$+ \left[n_t^{r,m}k_t^{r,m} - \left(1-\delta^k\right)\frac{N_{t-1}}{N_t}n_{t-1}^{r,m}k_{t-1}^{r,m} \right] + \left(s_t^{g^h} + s_t^{g^e} + s_t^{g^w} + s_t^{g^i} + s_t^{g^c} \right) n_t^f y_t^f = n_t^f y_t^f \quad \text{(D63)}$$

Tables and Graphs

Table 1a: Structure of public spending in the Eurozone (2001-19)

Type of spending	% of GDP	% of total public spending
Social protection	19.20	40.00
Health	7.00	14.50
General public services	7.00	14.60
Education	4.70	9.80
Economic affairs	4.6	9.50
Public order and safety	1.70	3.50
Defence	1.20	2.60
Environmental protection	0.80	1.70
Housing and community amenities	0.70	1.50
Recreation and culture	1.10	2.40
Total	48.00	

Source: Eurostat, Government finance statistics, Government expenditures by function - COFOG

Table 1b: Structure of tax revenues in the Eurozone (2001-18)

Type of tax	% of GDP	% of total tax revenues
SSC (Employers)	8.00	19.80
SSC (Employees)	6.00	15.00
Consumption	10.70	27.8
Labour income (excluding SSC)	5.00	12.00
Capital income (non-corporate)	5.45	13.40
Capital income (corporate)	2.70	6.82
Total		

(i) Source: European Commission, Taxation Trends in the EU; author's calculations

(ii) "Total' indicates the total tax revenues of the listed types of tax in the tables.

(iii) We do not include other types of taxes like environmental taxes, property taxes, etc.

Table 2a: Baseline parameter values

Parameter	Value	Description
β	0.982^{25}	Time discount rate
σ	1	Elasticity of inter-temporal substitution in utility
χ_n	12	Preference parameter related to effort
χ_b	3.5	Preference parameter related to bequests
χ_g	1	Preference parameter related to public consumption
η	1	Inverse of Frisch labour supply elasticity in utility
η_g	1	Exponent on public consumption in utility
η_b	1	Exponent on bequests in utility
A	1	Total factor productivity
$\frac{1}{1-\alpha}$	1.86	Elasticity of unskilled labor in production ($\alpha = 0.45$)
$\frac{1}{1-\psi}$	0.67	Elasticity of capital in production ($\psi = -0.49$)
ε	0.03	Elasticity of public capital in production
μ	0.28	Importance of physical capital in production
λ	0.4	Importance of unskilled labor in production
χ	0.05	Elasticity of work supplement public services
B^r, B^p	10, 8	Productivity of education (skilled, unskilled)
θ	0.75	Productivity of education time
κ	0.5	Contribution of health to human capital
$\frac{1}{1-\nu}$	2	Elasticity of human capital ($\nu = 0.5$)
γ	0.25	Importance of private spending for human capital
$\delta^k, \delta^h, \delta^g, \delta^{r,h}, \delta^{p,h}$	0.75	Depreciation rates of physical/human/public capital
ϕ^h, ϕ^l	1, 0	Max and min bounds in probabilities
S^r, S^p	1, 0.78	Skill thresholds in social mobility functions
ξ^r, ξ^p	0.025, 0.24	Human capital elasticities in probabilities
n	0.005	Population growth rate

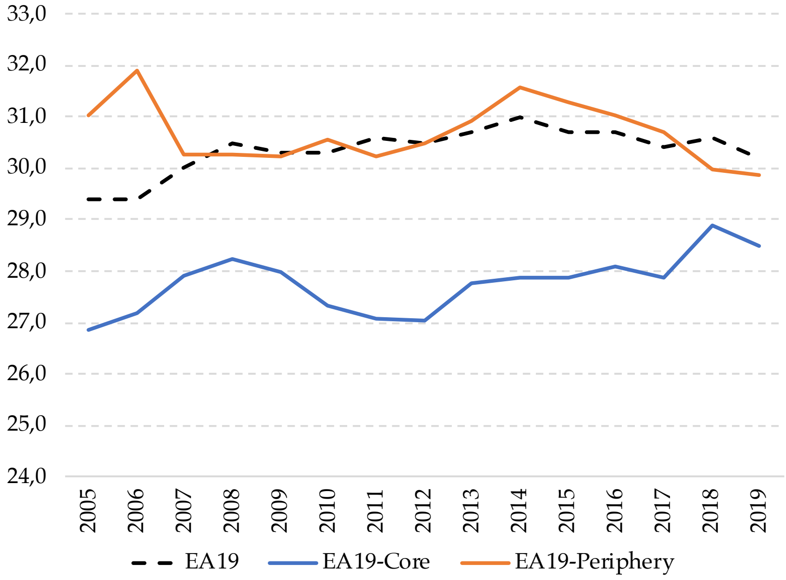
Table 2b: Fiscal policy variables (EA-19 data averages)

Policy instruments	Value	Description
τ^c	0.19	consumption tax rate
τ^k	0.3	personal capital income tax rate
τ_t^s	0.08	social security contributions paid by employees
τ_t^w	0.16	social security contributions paid by employers
τ^f	0.15	corporate tax rate
τ^b	0.05	bequest tax rate
Φ	0.85	constant in progressive tax formula
φ	0.1	power coeffient in progressive tax formula
$s_t^{g^e}$	0.04	government spending on education as share of output
$s_t^{g^h}$	0.06	government spending on health as share of output
$s_t^{g^i}$	0.04	government spending on infrastructure as share of output
$s_t^{g^s}$	0.14	government spending on social protection (pensions) as share of output
$s_t^{g^w}$	0.017	government spending on social protection (family/work supplement) as share of output
$s_t^{g^c}$	0.15	government spending on the rest as share of output
$\zeta_t^{g^e}$	0.2	fraction of public spending on education allocated to the rich (set)
$\zeta_t^{g^h}$	0.2	fraction of public spending on health allocated to the rich (set)
$\zeta_t^{g^w}$	0.2	fraction of public spending on work-supplements allocated to the rich (set)
$\zeta_t^{g^s}$	0.2	fraction of public spending on pensions allocated to the rich (set)
$\zeta_t^{g^{tr,p}}$	0.2	fraction of public spending on transfers allocated to the poor (set)

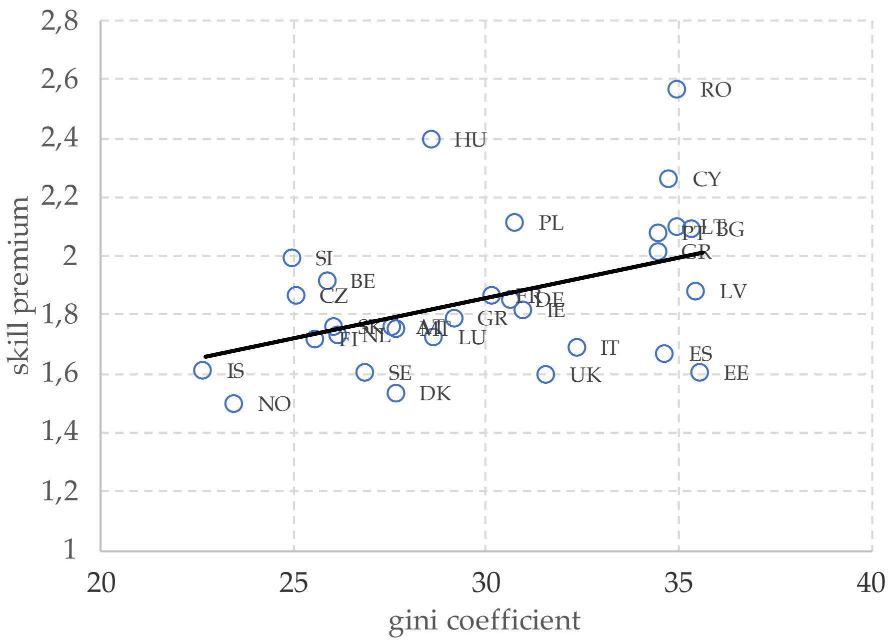
Table 3: Status quo solution using 2001-19 data averages

Variable	Solution	Data
$C/Y = \frac{n^{r,y}(c^{r,y}+c^{r,m}+c^{r,o})+n^{p,y}(c^{p,y}+c^{p,m}+c^{p,o})}{n^{r,m}y}$	0.619	0.548
$K/Y = \frac{n^{r,m}k^{r,m}}{n^{r,m}y}$	0.1809	0.204
$Z/Y = \frac{n^{r,y}z^{r,y}+n^{p,y}z^{p,y}}{n^{r,m}y}$	0.009	0.006
$D/Y = \frac{n^{r,m}d^{r,m}}{n^{r,m}y}$	0.875	0.897
$1+\rho$	1.008	
w^r/w^p	1.66	1.58
n^r	0.210	0.243
n^p	0.790	0.757
$y^{net,r}$	4.685	-
$y^{net,p}$	0.875	-
$y^{net,r}/y^{net,p}$	4.09	-
$\tau^{n,r}$	0.14	-
$\tau^{n,p}$	0.10	-

Graph 1. Gini coefficient (2005-2019)

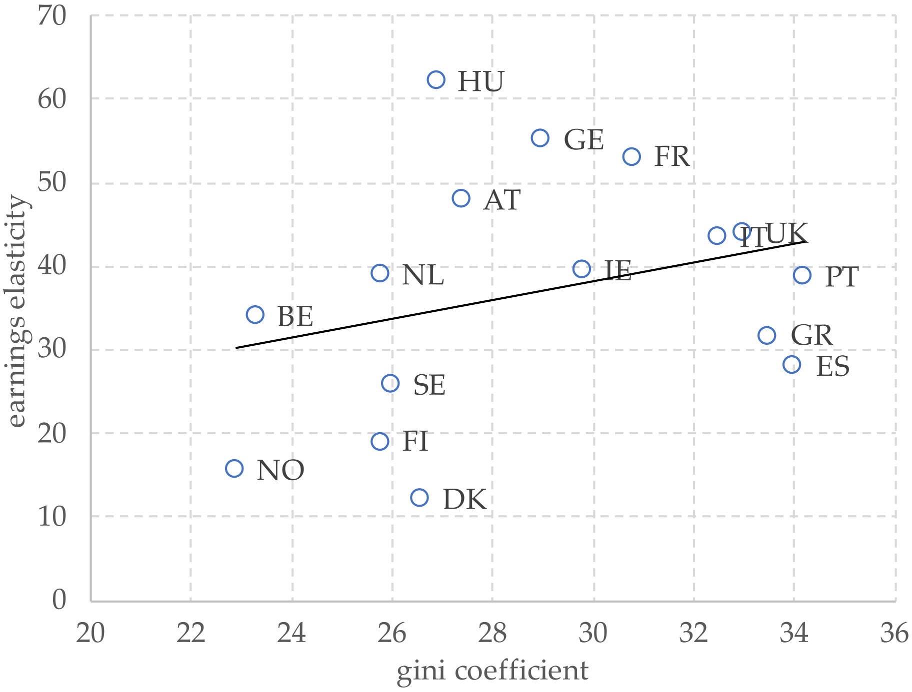


Graph 2. Skill premium and income inequality (2014)



Source: Eurostat, EU-SES and EU-SILC surveys

Graph 3. Earnings elasticity (mobility) and income inequality (late 2000s)



Note (i): selected European countries
Source: OECD (2008); Eurostat EU-SILC survey

Figure 1: A capital-augmenting change A^k

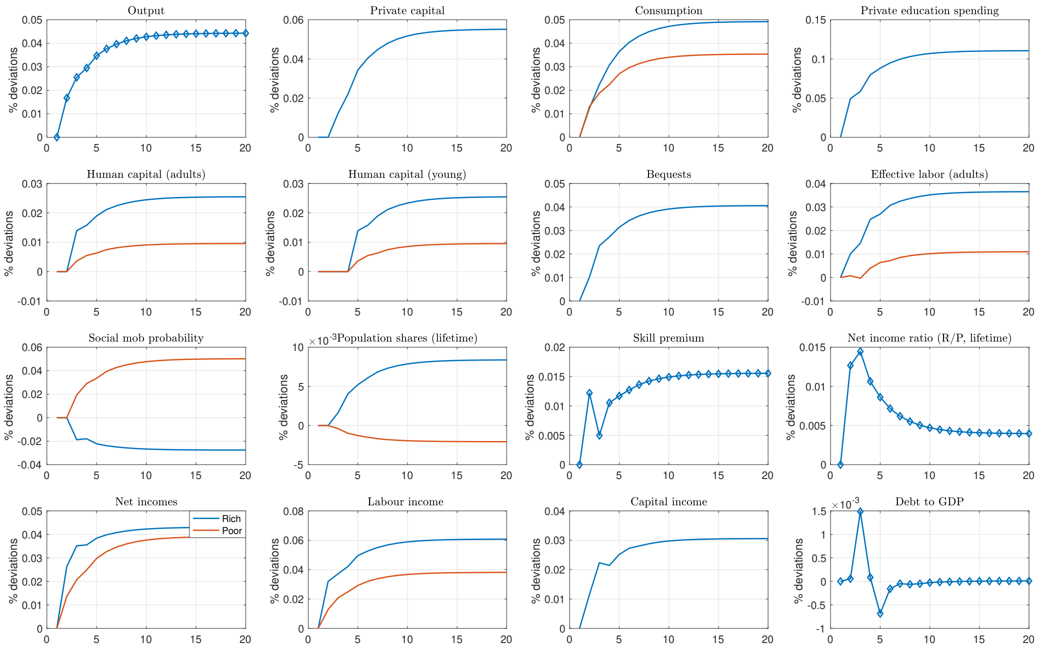


Figure 2: An increase in public infrastructure spending s^{g^i}

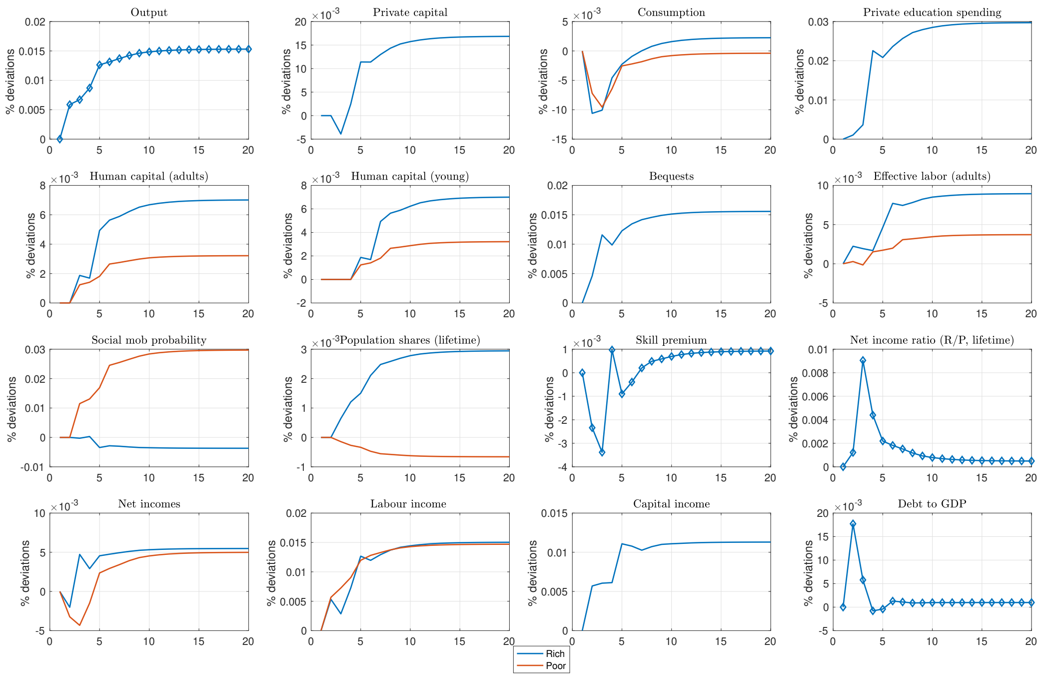


Figure 3: An increase in total public education spending s^{g^e}

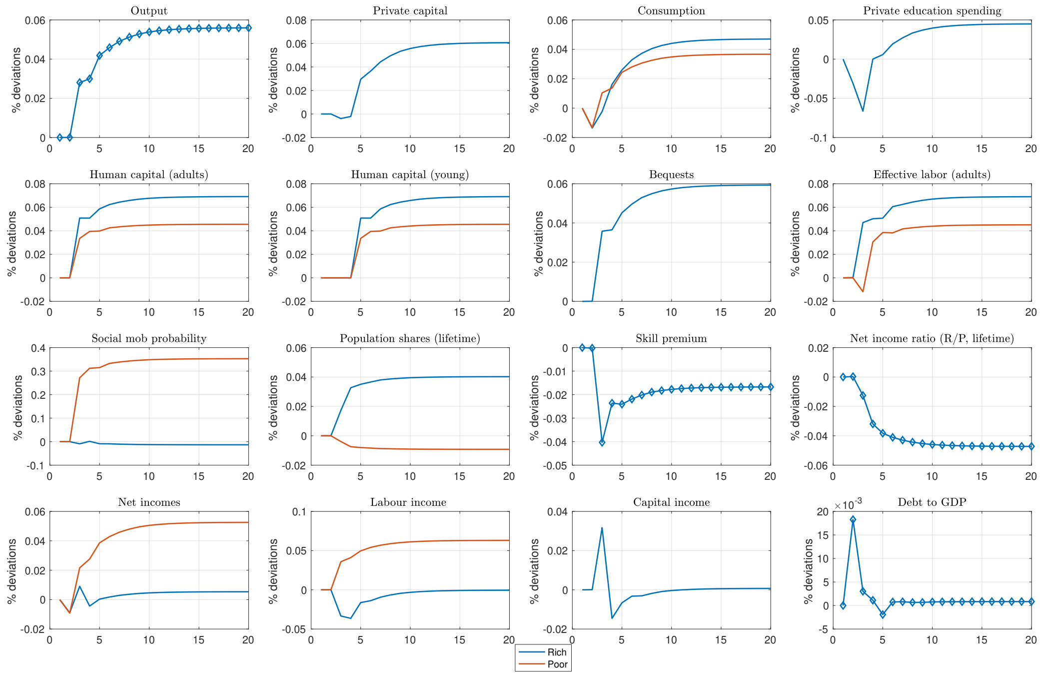


Figure 4: An increase in total public health spending s^{g^h}

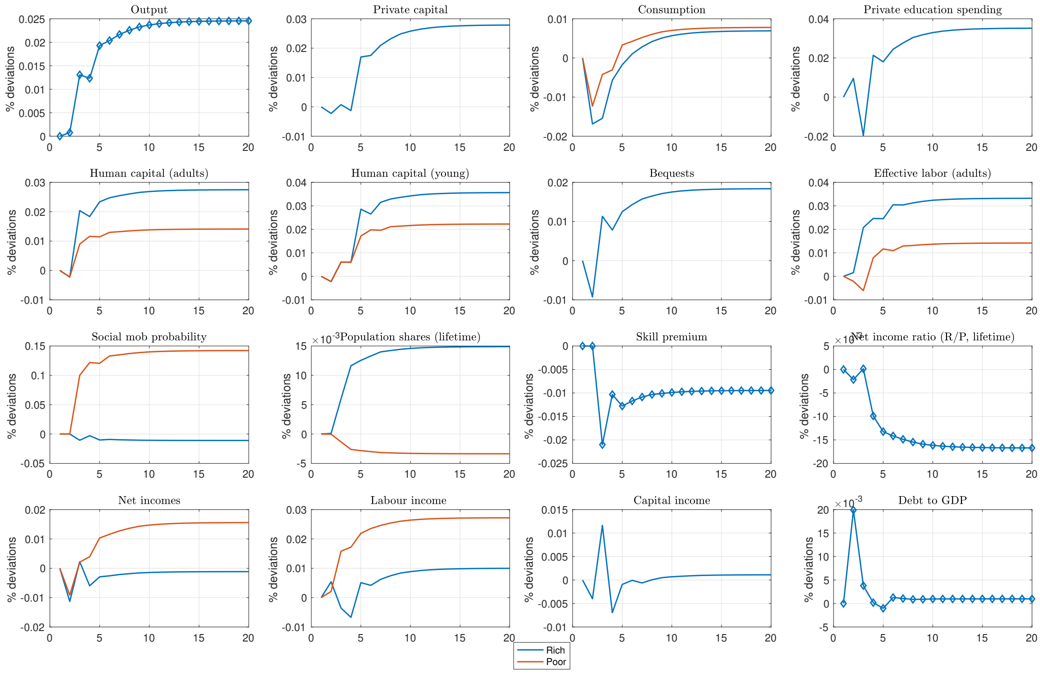


Figure 5: An increase in total public spending on work-complements s^{g^w}

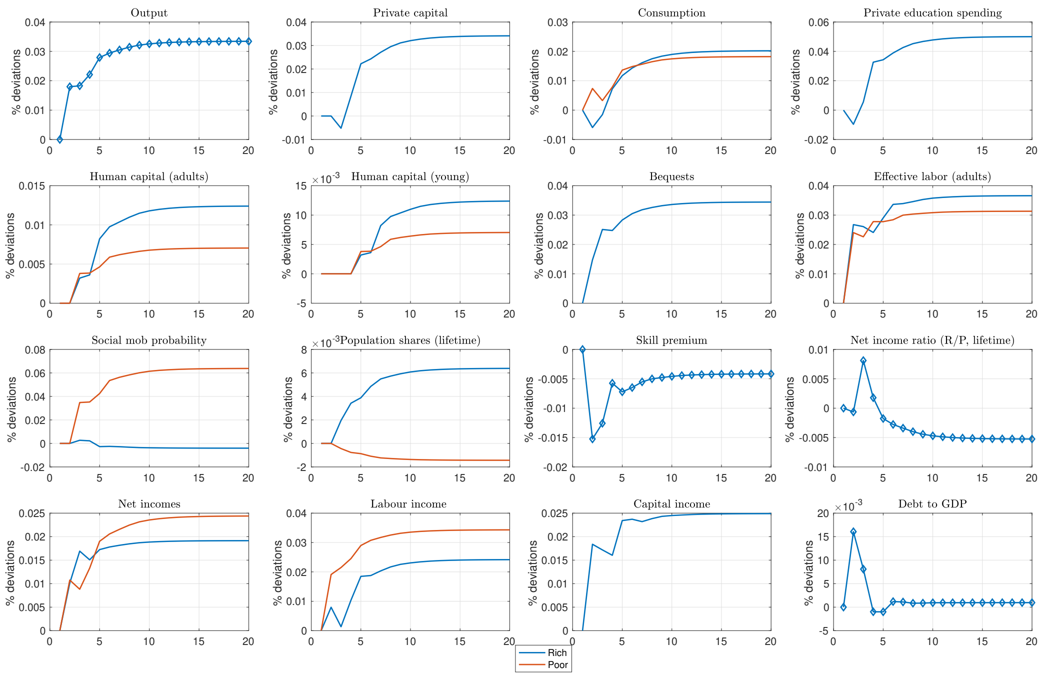


Figure 6: An increase in total public spending on pensions s^{g^s}

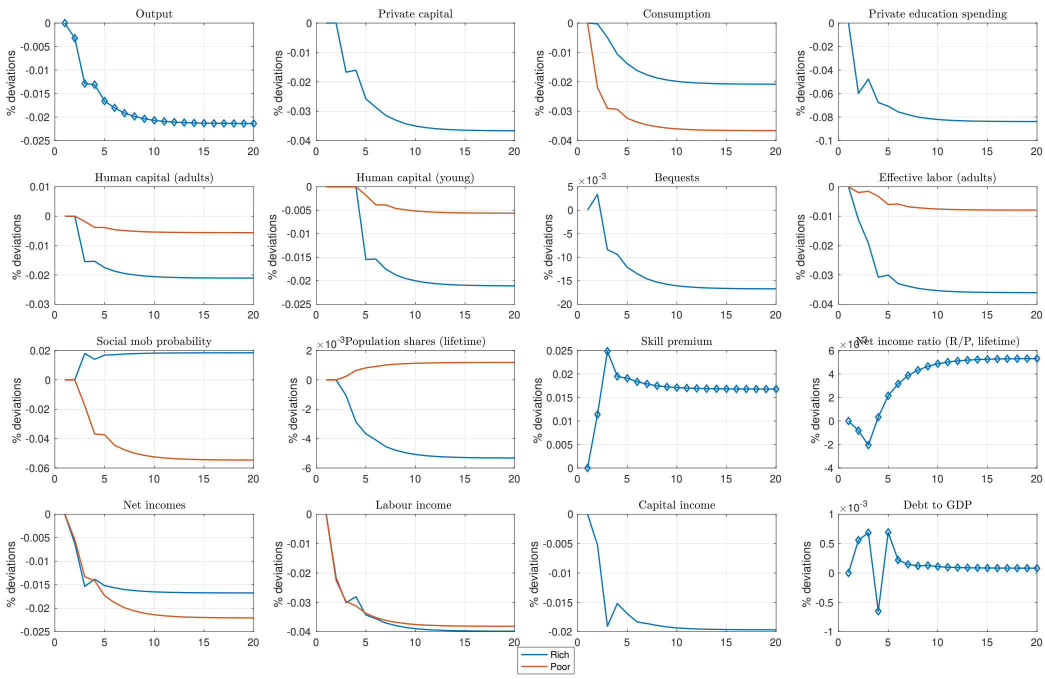


Figure 7: An increase in total public spending on transfers $s^{g^{tr}}$

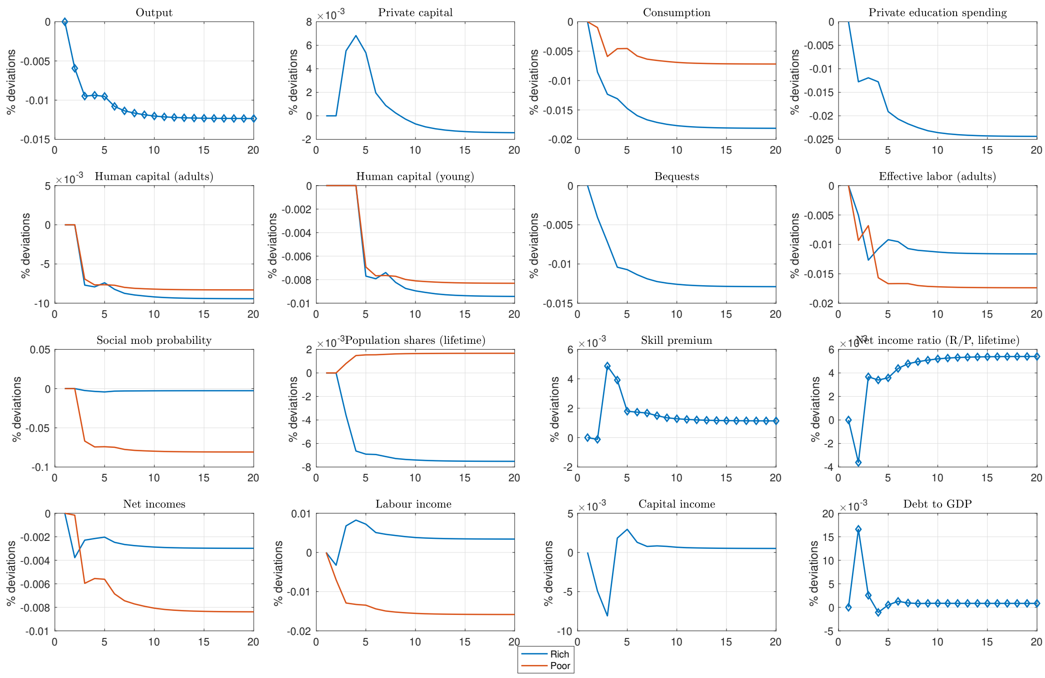


Figure 8:An increase in the share of public education spending going to poor $(1-\zeta^{g^e})$

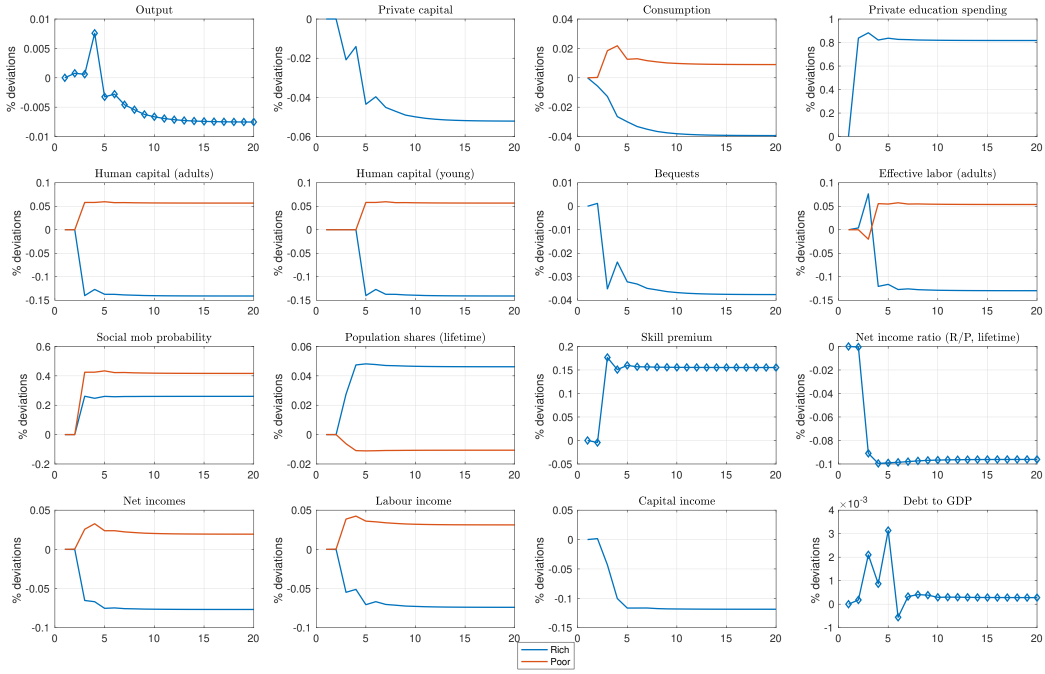


Figure 9: An increase in the share of public health spending going to poor $(1-\zeta^{g^h})$

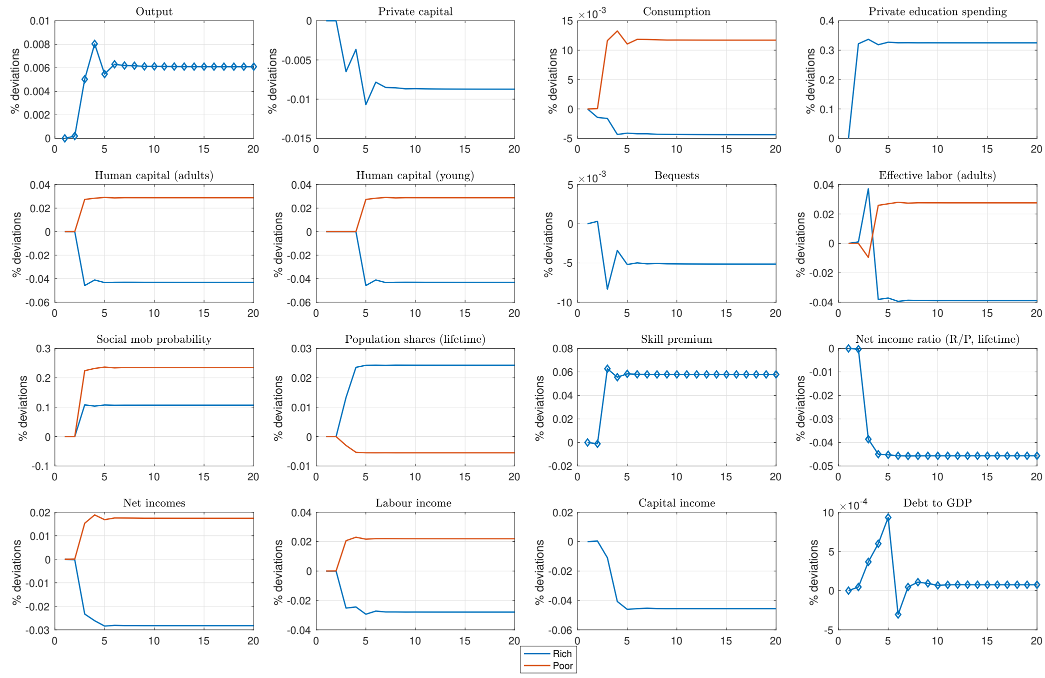


Figure 10: An increase in the share of work-complements going to poor $(1-\zeta^{g^w})$

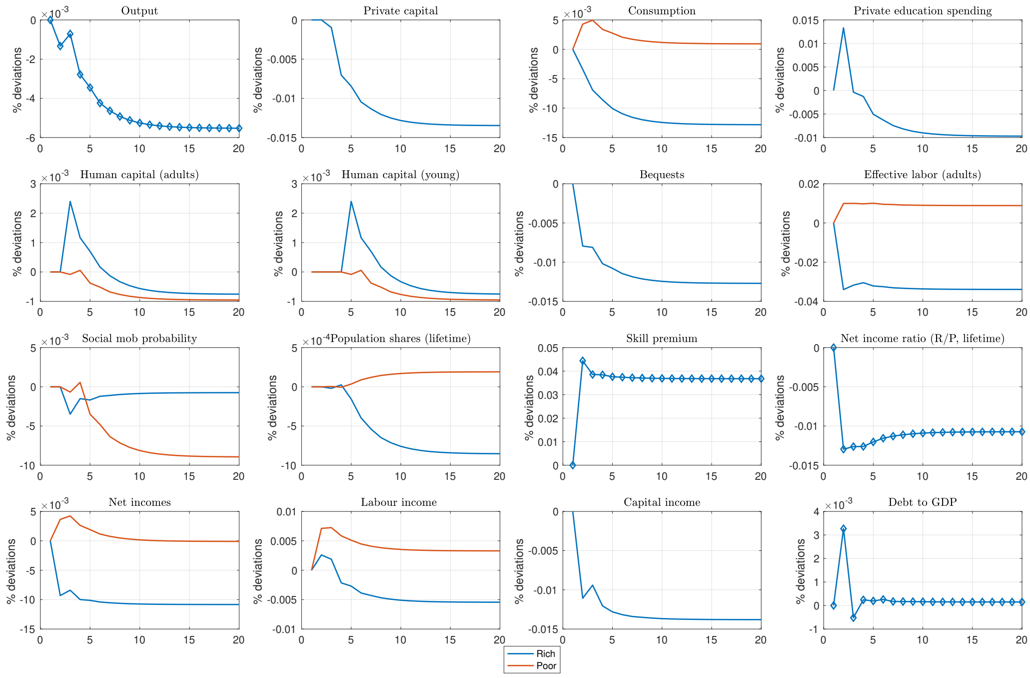


Figure 11: An increase in the share of public pensions spending going to poor $(1-\zeta^{g^s})$

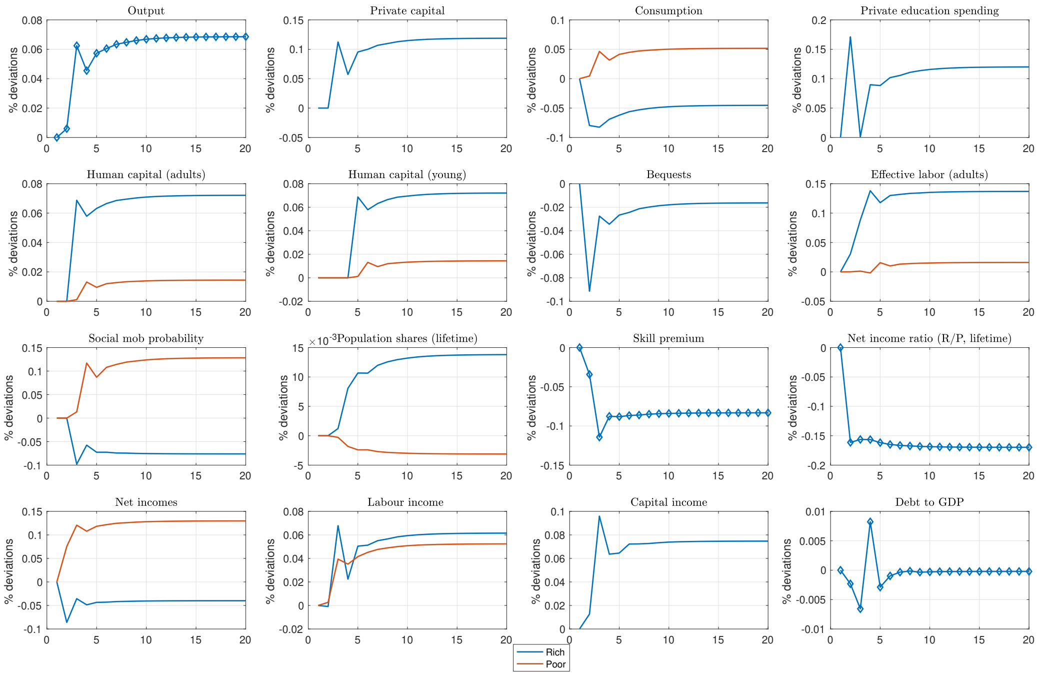


Figure 12:An increase in the share of transfers going to poor $(1-\zeta^{g^{t,p,m}})$

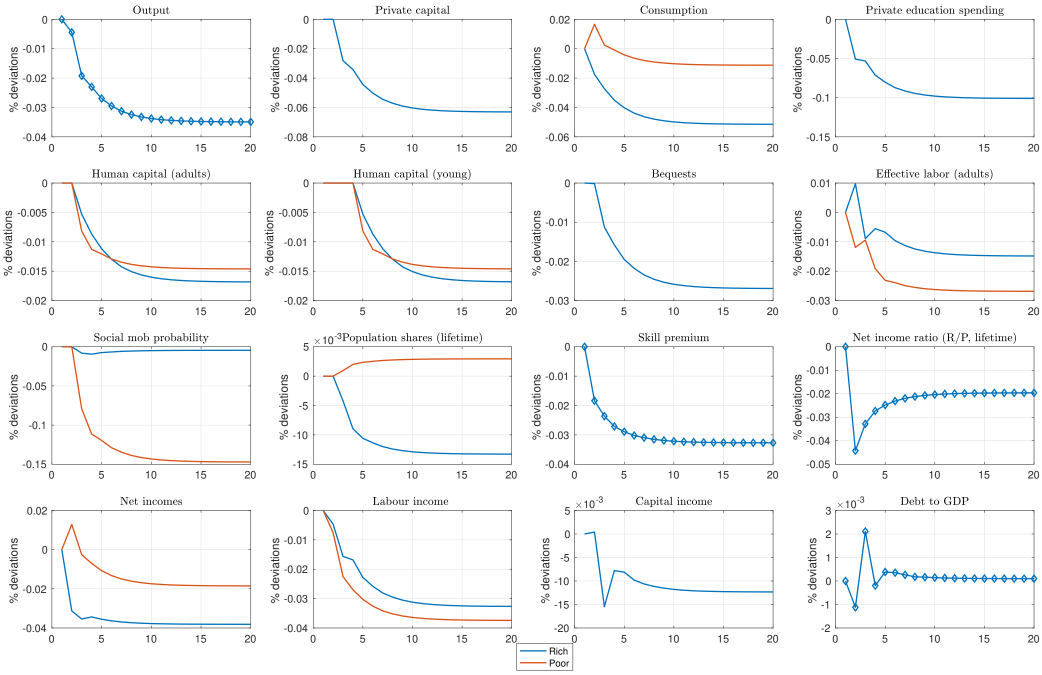


Figure 13: An increase in the tax rate of capital income τ^k

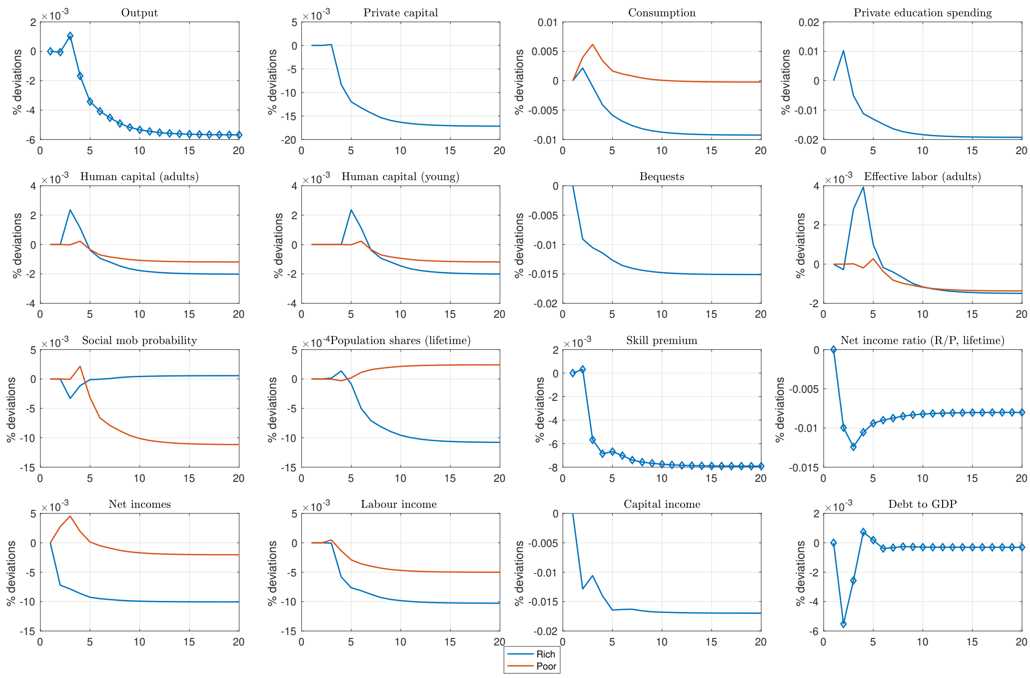


Figure 14: An increase in the tax rate of corporate income τ^f

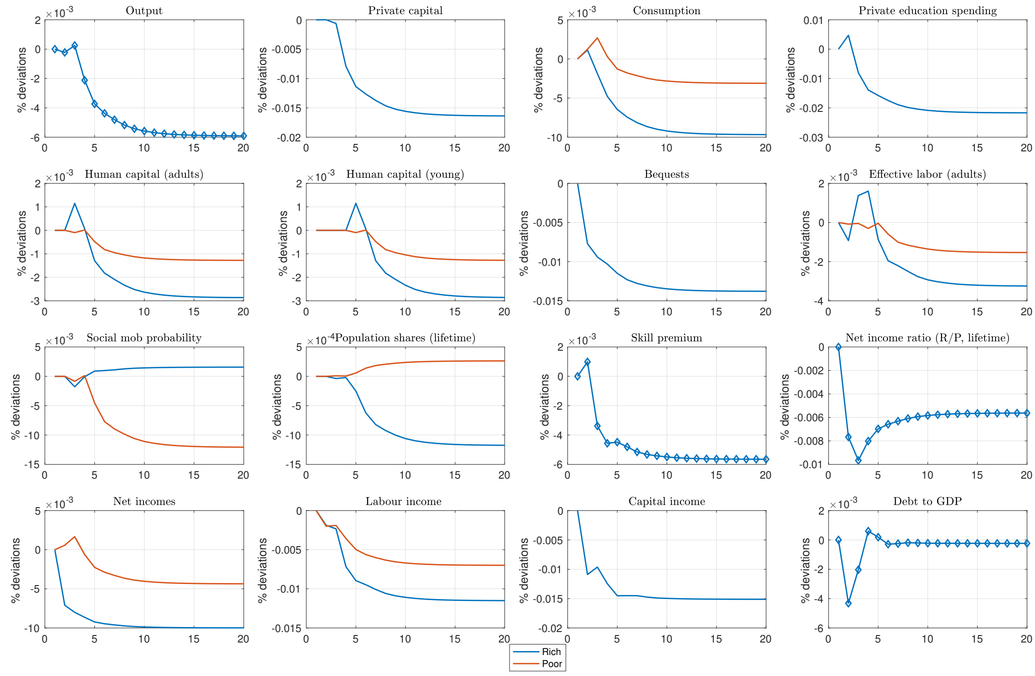
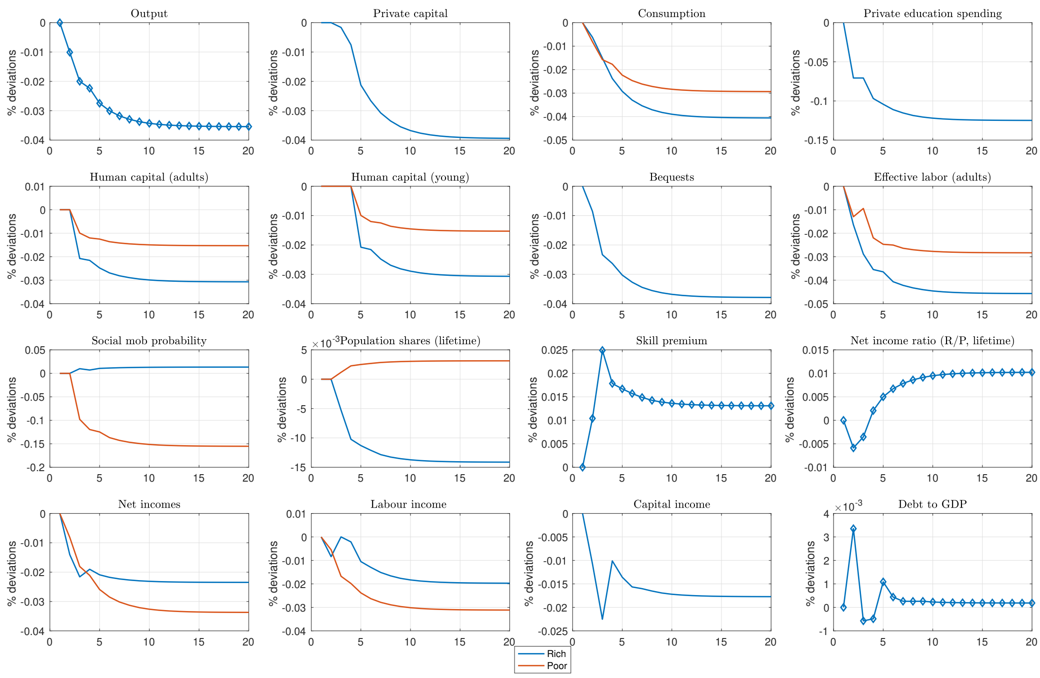


Figure 15: An increase in the degree of progressivity of labour income ϕ



The European Commission's science and knowledge service

Joint Research Centre

JRC Mission

As the science and knowledge service of the European Commission, the Joint Research Centre's mission is to support EU policies with independent evidence throughout the whole policy cycle.

EU Science Hub
ec.europa.eu/jrc

@EU_ScienceHub

EU Science Hub - Joint Research Centre

Joint Research Centre

EU Science Hub